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Growth, Water Resilience, and Sustainability: A DSGE Model Applied to South Africa*

Chuan-Zhong Li and Ranjula Bali Swain†

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Abstract

In this paper, we analyze a dynamic stochastic general equilibrium model on how water resilience affects economic growth and dynamic welfare with special reference to South Africa. While water may become a limiting factor for future development in general, as a drought prone and water poor country with rapid population growth, South Africa may face more serious challenges for sustainable development. Using the model, we conduct numerical simulations for different parameter configurations with varying discount rate, climate change scenario, and the degree of uncertainty in future precipitation. We find that with sufficient capital accumulation, development may still be sustainable despite increased future water scarcity and decreased long-run sustainable welfare; While stochastic variation in precipitation has a negative effect on water resilience and the expected dynamic welfare, the effect is mitigated by persistence in the precipitation pattern. With heavier time discounting and lower capital formation, however, the current welfare may not be sustained.

JEL: D6, O4, Q25, Q55

Keywords: Water resilience, growth, dynamic welfare, sustainability

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1 Introduction

In the recent decade, the theory of dynamic welfare analysis for sustainability measurement has been greatly advanced (cf Weitzman 2001; Arrow et al., 2003; Dasgupta, 2004; and Löfgren and Li, 2011). The theory attempts to incorporate natural resource depletion and environmental costs into economic figures such as gross domestic product (GDP) and national wealth. This involves adding the flow value of non-marketed consumption services on the conventional measures and adjust the value of gross investments by taking into account the effects of current stock changes on future consumption. For example, consumption may include not only market goods such food, clothing and housing, but also environmental amenities such as fresh air, clean water and other ecosystem services. Capital stocks need not necessarily be man-made, and they may also contain natural and environmental resources, and social and cultural assets. The main idea is that if a welfare measure which is constructed on sound economic theory and comprehensive accounting can be kept non-declining over time, then social welfare is improving and the development is sustainable. Similarly, if the welfare measure in one region is higher than others, then the residents in the region are better-off.

The recent literature also takes into account the value of "resilience" for sustainability measurement. Resilience is the capacity for a system to cope with disturbances, such as extreme weather conditions caused by climatic changes, without shifting from a normal into a qualitatively different and less desirable state (Holling, 1973; Serrao et al., 1996; Carpenter et al, 2001; Walkers et. al., 2004). A system with very low resilience may simply lose its stability and functioning by a small perturbation while that with higher resilience may absorb larger shocks without any dramatic changes. This implies that policies that improve the resilience of a system should promote sustainability and improve human well-being. The idea is that when the state of nature undergoes changes across a threshold, which lies beyond a society's ability to respond, the current social welfare may not be supported. With a buffer of resilience in the systems, adaptive environmental assessment and management actions can provide robust responses to the loss. A couple of recent papers have formalized the idea of resilience valuation in a growth-theoretic framework (cf Mäler and Li, 2010; Walker et al., 2010). In addition to the conventional capital stocks, the resilience is treated as an asset, i.e. a stock variable, in its own right, and thus the ecosystem resilience may be valued according to its marginal contribution to social well-being by its role in maintaining ecosystem functioning and stabilities.

In this paper, we analyze a dynamic stochastic general equilibrium (DSGE) model on how the resilience of water, in particular groundwater, and their implications for economic growth and sustainability with special reference to the case of South Africa. First, evidence indicates that water scarcity in general would be more serious in the future both due to the steadily growing world population and the global warming effect from climate changes (EEA, 2009; USGS, 2009; Job, 2010). Among the different freshwater sources, groundwater is the largest source and often the only reliable one in watersheds away from surface water. According to Llamas and Custodio (2003), UNESCO (2003) and Brown (2004), groundwater supplies half of the world's population with drinking water and serves as the fastest growing source of additional irrigation water for food production. Moreover, it contributes to alleviating poverty and public health challenges by providing clean drinking water and an alternative water source at a low cost (Llamas and Custodio, 2003). Even at times without precipitation, it helps maintain the flow of rivers and streams. However, as groundwater is stored away from sight and has easy accessibility for everyday use, it has been ignored and treated as a free resource. Although groundwater may be considered a renewable resource by some, its temporal and spacial availability is becoming increasingly vulnerable. Second, South Africa provides an interesting starting point for water resilience analysis due to its specific geophysical and demographic characteristics (South Africa has absolute water scarcity characterized by low precipitation, high evaporation and rapid population growth). Over 90 per cent of the aquifers occur in hard rock with relatively low recharge rate[1], and it is believed that climate change would lead to a further reduction in the precipitation by some 10% in the future (Statistics South Africa,. 2010). Together with the projected population growth from 51.77 to 65.67 millions, it is conceivable that the water availability per capita would be significantly smaller[2].

In the DSGE model, we have three types of state variables, namely labor, physical capital, and the stochastic groundwater stock, and we explore the optimal trade-offs between consumption and investment, between water extraction and resilience service, and between industrial and residential use of water. Using the Bellman equation, we derive the optimality conditions for the optimal sequence of decisions, present formulas for the shadow (resilience) value of the surface and groundwater stock, and the dynamic average utilitarian measure for sustainability measurement. We also calibrate the model to the initial state of the economy, and numerically solve the model to study the growth and welfare effects of different parameter configurations such as the discount rate, climate change assumption, and the different degree of uncertainty in water availability. The results indicate, among other things, that with sufficient capital accumulation over time, the development can still be made sustainable despite of increased future water scarcity, but as expected the scenario with climate change damages leads to lower long-run

[1] Only 18% of the South African aquifers are high-yielding ones producing good quality groundwater.

[2] Details on groundwater statistics are provided in the Appendix.

sustainable welfare. Concerning the effect of stochastic variation in surface water flow and groundwater recharge, we find that the magnitude of the variation has a negative effect on the water resilience and social welfare, but the effect is mitigated by the positive correlation over time. In other words, when the precipitation pattern is somewhat persistent rather than stochastic over time, the society would be better off. However, if the discount rate is too high, it seriously discourages investment, present welfare cannot be sustained and therefore development becomes unsustainable.

The remaining part of the paper is structured as the following: Section II formalizes the dynamic stochastic model, derives the optimality conditions from the Bellman equation, and present the mathematical formulas for the shadow prices of water, and the dynamic average utilitarian welfare. Section III calibrates the model based on the official statistics in South Africa, and sets up other model parameters for our numerical analysis. In section IV, we present the numerical simulation results for different scenarios, and discuss their growth and welfare implications. Section V sums up the study.

2 The Model

We consider a dynamic stochastic general equilibrium (DSGE) model with three stock variables: population size, physical capital and groundwater stock. To focus on the role of groundwater for economic growth and welfare for the overall economy, we abstract from the detailed spatial issues across the different administrative areas. Alternatively, we may conceive the model as a model for resource management in a typical administrative area or a given aquifer conditioning on certain calibration of the parameters.

Let N_t be the population size, K_t and X_t the physical capital and groundwater stock, respectively, in time period $t = 0, 1, 2...\infty$. Then, the corresponding per capita physical capital and groundwater stock can be written as $k_t = K_t/N_t$ and $x_t = X_t/N_t$, respectively. We consider an aggregate Cobb-Douglas production function for the economy i.e.

$$Y_t = AK_t^{\gamma_1} N_t^{\gamma_2} W_t^{\gamma_3} \tag{1}$$

where A is the total factor productivity[3] with W_t as the total fresh water use in the production sector, and γ_1, γ_2, and γ_3 are positive coefficients.

[3]In this paper, we do not explicitly model technological progress and accordingly we apply a somewhat lower discount rate in our analysis. We do not differentiate between population and labor supply in order to focus on the role of water in the model. This implicitely assumes constant labor participation and the employment rates, with which the production function with labor and that with population can be made exactly the same by a certain calibration of the productivity parameter.

We abstract from the distribution of water use among agricultural, manufacturing, mining and tourism sectors etc. These issues have been widely studied elsewhere (cf Hassan, 2003; Nieuwoudt et al., 2004), which are more concerned with inefficient allocations of water across the different industrial sectors. Irrigation, for example, is the least efficient sector where the marginal revenue product is the lowest, although much water is still allocated in this sector for equity considerations, among other things. In this study, we only differentiate between productive and residential water use in order to sharply focus on the overall growth and welfare effects of groundwater in the presence of rapid population growth. Let S_t be the *stochastic* withdraw of surface water in period t, Z_t that of groundwater and H_t the respective residential use. The productive water use[4] can thus be expressed as $W_t = S_t + Z_t - H_t$.

Concerning groundwater extraction, we consider two types of costs, namely, the unit cost of normal extraction $a > 0$, and the expected cost incurred from crossing a stochastic threshold $\tilde{X}$ with $\tilde{X} \geq 0$. Whenever $X_t \geq \tilde{X}$, this extra cost is 0 but when $X_t < \tilde{X}$, there would be an additional cost $d > 0$ per unit of water extraction. For example, when a lower stock implies that the water table is beyond 30-50 meters deep, then drilling and extraction costs, as well as the wildcat drilling failure rates (MacDonalds et. al 2011) may rise abruptly due to geological, technical and even institutional reasons[5]. It is also possible for coastal aquifers to suffer from saltwater intrusion for some too low groundwater stock, which can lead to contamination of drinking water sources and other consequences.

Assume that the probability of crossing the threshold can be described by $\Pr\left(X_t < \tilde{X}\right) = \exp\left(-rX_t\right)$ for some positive parameter r, which implies that at $X_t = 0$ the threshold-crossing probability is 1, and with X_t sufficiently large the probability of the event is virtually 0. Under these assumptions, we can express the total unit cost function by $\phi\left(X_t\right) = a + d\exp\left(-rX_t\right)$. In the normal case, the unit extraction cost is simple a, but this jumps to $a + d$, after a threshold crossing, and the event of threshold crossing is almost sure when the aquifers were almost dry. We now describe the dynamics of the stock variables as the following:

$$\begin{aligned} K_{t+1} &= Y_t + (1-\delta) K_t - C_t - \phi\left(X_t\right) Z_t \\ N_{t+1} &= b_1 N_t - b_2 N_t^2 \\ X_{t+1} &= X_t - Z_t + m_t + \varepsilon_t \end{aligned} \quad (2)$$

[4] Note that we regard groundwater and surface water as perfect substitute to each other in this model.

[5] Rural communities that rely on handpump or small motor pump for water extraction, might be severely impacted once the water table is below 40 meters. Discussions with the experts at the Department of Water Affairs (Republic of South Africa) revealed that management thresholds may vary between 5 - 40 meters, but on average it is usually around 25 meters (Discussion notes, Bali Swain, May 2014).

where $\delta \in (0,1)$ denotes the capital depreciation rate, $(1-\delta) K_t$ the undepreciated physical capital, and K_{t+1} the physical capital stock in the next time period. We describe population dynamics by the Gordon-Schaefer model where N_t denotes the population size in period t and N_{t+1} that in period $t+1$. The parameter b_1 is the intrinsic growth rate with $(b_1 - 1)/b_2$ as the asymptotic population size. The third difference equation represents the dynamics of the groundwater stock where X_t is the groundwater stock in period t and X_{t+1} that in period $t+1$ with m_t and Z_t as the natural recharge and extraction rates, respectively. The last term ε_t is a stochastic disturbance in period t, with zero mean and constant variance, which may be autocorrelated over time. It is worth mentioning that we do not impose Z_t to be strictly positive, and thus allow artificial recharge through engineering with $Z_t < 0$. As compared to Koundouri (2000), we use the water stock rather than the head level from the surface to the water table which was mostly used in the literature. For given aquifer size and storability, however, the difference is simply up to a scale. The society derives instantaneous utility $U(C_t, H_t, N_t)$ from consuming the composite good C_t and water service H_t in period t according to

$$U(C_t, H_t, N_t) = N_t \left[\ln\left(\frac{C_t}{N_t}\right) + \alpha \frac{(H_t/N_t)^{1-\theta}}{1-\theta} \right]$$

i.e. the aggregated utility over all individuals with $c_t = C_t/N_t$ and $h_t = H_t/N_t$ as the per capita consumption and residential water use, respectively. The per capita utility both from water use and other consumption is thus $u_t = U(C_t, H_t, N_t)/N_t = \ln(c_t) + \alpha h_t^{1-\theta}/(1-\theta)$. Note that the parameter α is the relative welfare weight attached to the sub-utility from residential water service, and the parameter θ denotes the corresponding relative risk aversion. As argued in Golosov et al (2014), the use of logarithmic preference for the part of composite consumption is common and standard. In an infinite time horizon model, the risk aversion and intertemporal elasticity of substitution implied by logarithmic curvature are considered as rather reasonable and confirmed in their numerical optimal carbon taxes analyses.

The intertemporal welfare at the start of the planning horizon is

$$V_0 = \max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t, N_t) \tag{3}$$

where $\beta \in (0,1)$ is the discount factor, and E_0 the mathematical expectation of the discounted sum of future utilities conditional on the given information at period 0. We are now interested in optimizing the intertemporal welfare (3) subject to the three state dynamics equations (2). The standard Bellman equation (with the time subscript suppressed for the concurrent period stock variables and with a prime for the next period) then reads

$$V(K, N, X) = \max_{(C,Z,H)} \left[U(C, H, N) + \beta E_t V(K', N', X') \right]$$

The first-order conditions are:

$$\left(\frac{C}{N}\right)^{-1} = \beta E_t\left(V_{K'}\right) \text{ (w.r.t. } C_t) \tag{4}$$

$$\beta E_t\left(V_{X'}\right) = \beta E_t\left(V_{K'}\right)\left[\frac{\gamma_3 Y}{S+Z-H} - \phi'(X)\right] \text{ (w.r.t. } Z_t) \tag{5}$$

$$\alpha\left(\frac{H}{N}\right)^{-\theta} = \beta E_t\left(V_{K'}\right)\frac{\gamma_3 Y}{S+Z-H} \text{ (w.r.t. } H_t) \tag{6}$$

Equation (4) is the well-known Keynes-Ramsey rule indicating that the marginal utility of consumption should be equal to the present value of future utilities that would be generated by the marginal unit if it were invested. From this relationship, we can interpret the right-hand-side of equation (5) as the marginal productivity of water in utility units, which should at the optimum be equal to the expected shadow price of water in present value. The third optimality condition (6) shows the equality between the marginal utility of water for residential service (in the left-hand-side) and that derived from the industrial use.

After a few manipulations, we arrive at the following first-order optimality conditions:

$$\begin{aligned} K' &= \beta E_t \frac{C/N}{C'/N'}\left[\gamma_1 Y' + (1-\delta) K'\right] \\ \frac{\gamma_3 Y}{S+Z-H} - \phi(X) &= \frac{\phi'(X') Z' + \frac{\gamma_3 Y'}{S+Z'-H'} - \phi(X')}{\frac{\gamma_1 Y'}{K'} + (1-\delta)} \\ \alpha\left(\frac{H}{N}\right)^{-\theta} &= \beta \frac{1}{C/N} \cdot \frac{\gamma_3 Y}{S+Z-H} \end{aligned} \tag{7}$$

which together with the three state dynamics equations (2) constitute the modified Hamiltonian dynamic system. In the system, we have three state variables K, N, X, and three control variables C, Z, and H. Let $\{C_t^*, Z_t^*, H_t^*, K_t^*, N_t^*, X_t^*\}|_{t=0}^{\infty}$ denote the optimal sequence, the resulting optimal value function at any period s then becomes

$$V_s\left(K_s, N_s, X_s\right) = E_s \sum_{t=s}^{\infty} \beta^{(t-s)} U\left(C_t^*, H_t^*, N_t^*\right) \tag{8}$$

with E_s as the mathematical expectation conditional on information up to period s. The shadow value of the groundwater stock, in monetary units, can then be expressed as

$$\lambda_s\left(K_s, N_s, X_s\right) = \frac{\partial V_s\left(K_s, N_s, X_s\right)/\partial X_s}{\partial U\left(C_s, H_s, N_s\right)/\partial C_s} \tag{9}$$

Following Walker et al (2010) and Mäler and Li (2011), we may also interpret this measure as the resilience price of groundwater where resilience is conceived as the distance between the actual water stock level to the potential threshold i.e. $X_t - \tilde{X}$. As the potential threshold $\tilde{X}$ with constant mean and constant variance is independent of the actual water stock, the partial derivative with respect to such a distance variable is the same as that with respect to the observable groundwater stock X_t. A larger groundwater stock implies more resilience, and a lower marginal value per unit of groundwater. Thus this resilience value may also serve as an indicator of groundwater scarcity. The shadow price of extracted water for residential (or industrial) use at year s can be expressed as

$$\begin{aligned} p_s &= \frac{\partial U\left(C_s, H_s, N_s\right)/\partial H_s}{\partial U\left(C_s, H_s, N_s\right)/\partial C_s} \\ &= \left(\frac{C_s}{N_s}\right)\alpha\left(\frac{H_s}{N_s}\right)^{-\theta} \end{aligned} \tag{10}$$

which is equal to the sum of shadow value and the marginal extraction cost i.e. $p_s = \lambda_s + \phi'\left(X_s\right)$.

For sustainability analysis with population growth, we apply the following dynamic average utilitarian criterion (Dasgupta, 2004, p301; Arrow et al., 2012)

$$\bar{v}_s\left(K_s, N_s, X_s\right) = E_s\frac{\sum_{t=s}^{\infty}\beta^{(t-s)}U\left(C_t^*, H_t^*, N_t^*\right)}{\sum_{t=s}^{\infty}\beta^{(t-s)}N_t^*} \tag{11}$$

in which the numerator is the present discounted value of the aggregated utility stream from year s onwards, the denominator denotes the present discounted value of future population sizes. Loosely speaking, this is a forward-looking measure of per-capita wealth. If the value of this measures does not decline over time, then on average, the future generations would be able to derive at least the same utility level per capita as compared to the present generation, and in other words, development would be sustainable.

3 Parameter selection

We now analyze our model for the case of South Africa. First, we characterize the South-African economy at the start of the planning horizon. In our quantitative analysis, we consider 2011 as the starting period and we take each subsequent period to be one year. According to the official statistics, the capital value in South Africa is 9.7857 (in 100 billions 2005 USD) or approximately $K_0 = 15.1849$ (100 billion 2011 USD) in 2011, and the population size in the same year is 51.77 (million) with a per-capita capital stock about $k_0 = \$29331$ (2011 dollars). The groundwater stock regarded as utilizable in the country is estimated to be in the range of 7.5 to 10.3 billion cubic meters (DWA 2010), and for our analysis we assume this to be $X_0 = 8.8$ (billion m^3) corresponding to $x_0 = 169.98$ m^3 per capita. The annual recharge rate is estimated to be $10 - 30\%$ of the total groundwater stock[6] i.e. $0.88 - 2.64$ (billion m^3). The total freshwater withdrawal is about 12.5 billion cubic meters per year, where surface water takes up about $\bar{S}_t = 10$ on average and groundwater about $m_t = 2$ (DWA 2010). Concerning the allocation of freshwater, it is estimated that about 20% is allocated for residential use and 80% for industrial use in a broad sense.

To calibrate the parameters for the population dynamics, we made a simple logistic regression on the population projection (http://esa.un.org/unpd/wpp/index.htm), and obtained the parameters $b_1 = 1.0440$ and $b_2 = 0.0067$ with an asymptotic population size $N_\infty = 65.67$ million as shown in Figure 1. The total GDP in 2011 is $Y_0 = 3.4510$ (100 billion 2011 USD). To calibrate for the total factor productivity A in the production function for the initial year, we scale up the estimated exponents γ_1 through γ_2 in Juana (2008) to be of constant return to scale (with $\gamma_1 = 0.41$; $\gamma_2 = 0.45$; and $\gamma_3 = 0.14$) and calculate A to satisfy $Y_0 = AK_0^{\gamma_1}N_0^{\gamma_2}W_0^{0.14}$ such that $A = 2.1$.

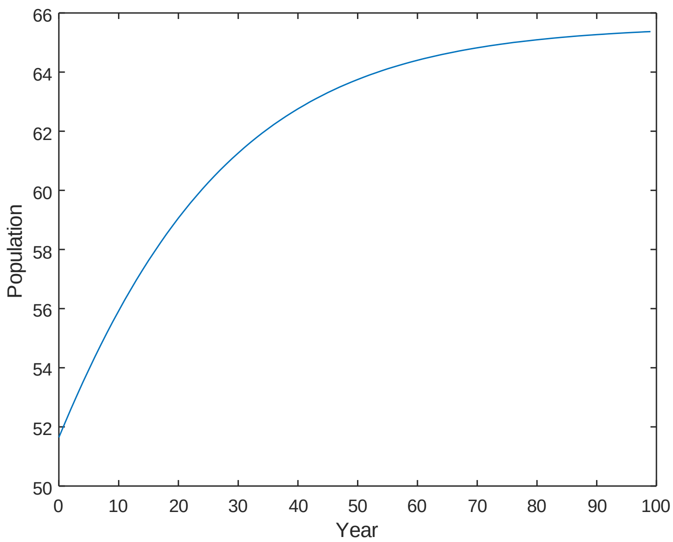

Figure 1: The Fitted Population Growth Curve

[6]Recharge figures are calculated in the Groundwater Resource Assessment phase II by the commonly used chloride mass-balance method. However, these figures are approximate and also subject to error. Second, South Africa has several geo-hydrological regions with their typical characteristics that result in a range of recharge rates. Discussions with the experts at CSIR and the Department of Water Affairs in South Africa suggested that the recharge rate is within the range of $10 - 30\%$ (Discussion notes, Bali Swain, May 2014).

Table 1: Model parameters

Y_0	K_0	X_0	N_0	S	m
3.4510	15.1848	8.8	53.0	10.0	2.5
γ_1	γ_2	γ_3	b_1	b_2	δ
0.41	0.45	0.14	1.044	0.0067	0.05
α	β	θ	a	d	r
0.03	0.98	0.5	0.28	1.12	0.1

For the augmented unit cost function $\phi(X_t) = a + d\exp(-rX_t)$, we set $a = 0.28$,which is the price in dollar per m^3 water for irrigation use - the lowest price among all other productive uses. Upon a possible "flip", we assume the additional cost to be $d = 1.12$ (i.e. 4 times higher) with $r = 0.1$ as the hazard rate. The stochastic term ε_t is assumed to be of mean 0 and constant variance σ^2 ranging from 0.1^2 to 0.4^2, which may be autocorrelated with different coefficient values (from $\rho = 0$ to $\rho = 0.9$ we will test in the sensitivity analysis).

For the economic parameters, we assume that the physical capital depreciates by $\delta = 5\%$ per year, and the pure rate of time preference[7] is 2% in the benchmark scenario. We set the risk-aversion parameter for residential water use to be $\theta = 0.5$, and we calibrate the welfare weight of the subutility from residential water use to be 0.03 such that the distribution between industrial and residential water use conforms with the observed values in the past years (about 80% and 20%, respectively).

4 Quantitative Results and Interpretation

In this section, we apply the Dynare software in Matlab (Stephane et al, 2011) to analyze the stochastic dynamic general equilibrium model to characterize the optimal plans, and their growth and welfare implications. To start with, we analyze the model in a deterministic setting for four different parameter configurations as shown in Table 2. In what follows, we consider *Model 1*, with parameter values $\beta = 0.98$, $S_0 = 10$ and $m = 2.5$, as the benchmark for our analysis. The resulting optimal time sequences of the relevant variables are depicted in Figure 2. It can be seen that capital (K_t), production (Y_t) and consumption (C_t) all increase monotonically over time from their initial level to the corresponding steady state value. The development of groundwater stock, (X_t), extraction rate (Z_t) and the residential water use (H_t) ,however, show the typical inverted U pattern over time.

[7] To focus on the role of groundwater, we do not assume any exogenous technical progress in productivity here, and thus we assume a low rate of pure rate of time preference. Otherwise, this rate may need to be increased for the case of South-Africa.

Table 2: The deterministic models to be analyzed

Model 1	Light Discounting *Without*	$\beta = 0.98$
	Climate Change Effect	$s = 10$, $m = 2.5$
Model 2	Light Discounting *With*	$\beta = 0.98$
	Climate Change Effect	$s \downarrow= 9$, $m \downarrow= 2.25$
Model 3	Heavy Discounting *Without*	$\beta = 0.95$
	Climate Change Effect	$s = 10$, $m = 2.5$
Model 4	Heavy Discounting *With*	$\beta = 0.95$
	Climate Change Effect	$s \downarrow= 9$, $m \downarrow= 2.25$

Initially, the extraction rate is lower than the recharge, which helps to build up the groudwater stock, but after after some peak level, the optimal water stock begins to gradually decline .

As expected, the groundwater extraction rate Z_t converges to the net recharge rate $m_t = 2.5$, and the steady-state residential water use $H = 0.5321$ takes up about 21% of the total extraction. The accounting price (the shadow value) per unit of groundwater calculated from (9) shows an increasing trend over time, which is mainly due to the increased population size and thereby a lower per-capita availability of groundwater in the future. In steady state, the water availability is only 190 m^3 per capita. Obviously, the extracted water has a higher price than the water under the surface due to the positive marginal cost of extraction.

In figure 2, we also depict the optimal sequence of per-capita utility u_t and the dynamic average utilitarian measure $\bar{v}_s$. We find that both measures monotonically increase over time and thus development is sustainable under the benchmark model parameter assumptions ($\bar{v}_s$ increases asymptotically from an initial value of -0.6466 to -0.5716 in steady state). The result indicates that in spite of the increasing per-capita water scarcity due to the projected population growth, the accumulation of capital and the increased overall consumption can more than compensate the loss in utility from the increased water scarcity. It is worth mentioning that by "development is sustainable" here, we mean that the initial per-capita welfare can be sustained over time, or in other words, the potential per-capita welfare level in the future would be at least as high as the present generation's.

In a strict sense, it is the dynamic average wealth $\bar{v}_s$, rather than the period-wise utility level u_s for a given year s, that is the correct dynamic welfare measure. For this scenario, however, we find that the two measures give the same sustainability conclusion. We can also see from the figure that the dynamic average wealth $\bar{v}_s$ starts higher than the "point-wise" utility u_s but grows in value less rapidly. The reason is simply that $\bar{v}_s$ is a forward-looking measure, which has been gauged by the higher future utilities while u_s is a static utility measure in year s only.

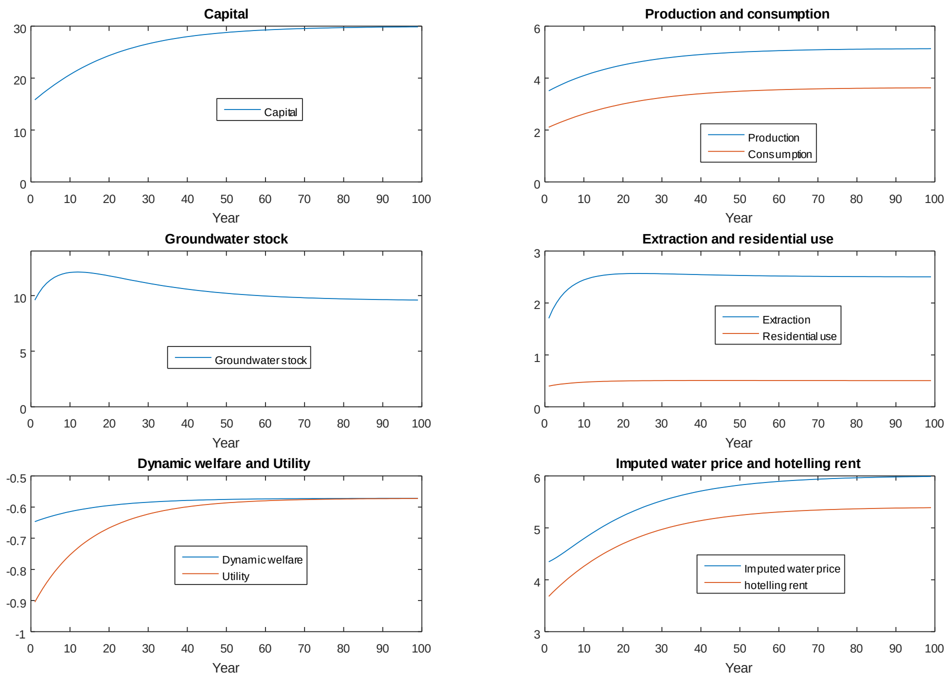

Figure 2: Optimal solution for the benchmark case

Next, we study the effect of climate changes on growth and sustainability with *Model 2*, where we assume that both the long-run future surface water flow and groundwater recharge rate would decrease by 10% from the present level due to climate changes (Statistics South Africa,. 2010). More exactly, we consider the following two difference equations

$$\begin{aligned} S_{t+1} &= 0.9S_t + 0.9 \\ m_{t+1} &= 0.9m_t + 0.225 \end{aligned} \tag{12}$$

with steady states $\bar{S} = 9$ and $\bar{m} = 2.25$. With the same discount rate $\beta = 0.98$, we run our computer program and find that the trends are similar to the benchmark case i.e. the present per capita wealth level can be sustained over the future. Although climate changes would reduce both the total and the per-capita water availability, the increased consumption due to economic growth can more than offset the utility loss from the water shortage. However, the long-run sustainable per-capita welfare becomes considerably lower (being -0.5976) than that in the benchmark case being -0.5716). The main reason for this is the loss in the sub-utility from reduced steady state residential water use with $H = 0.4115$ instead of the benchmark level 0.5043.

To study the effect of the discount rate, we now analyze *Model 3* with $\beta = 0.95$ and otherwise the same parameter values as *Model 1*. From the results depicted in Figure 3, we can see that, even without climate changes, the heavier time discounting would result in decreased groundwater stock over time, and thus lower the future dynamic welfare. When the future is valued less by the heavier discounting, more water is used today, which together with the increased current consumption would lead to a significantly lower steady state capital stock. For this scenario, development would not be sustainable. Both the resulting period-wise utility and dynamic average welfare are decreasing over time such that the future generations would be worse off than the present population.

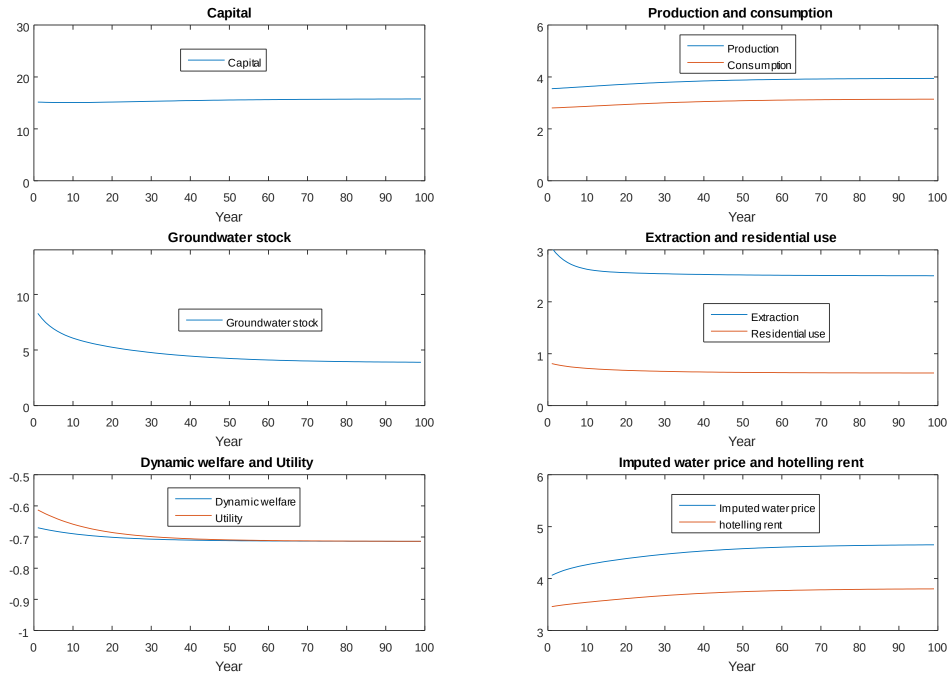

Figure 3: Optimal solution with heavier discounting

For *Model 4*, a scenario with both heavier discounting and climate changes (12), it is conceivable that development would not be sustainable. The groundwater stock would decrease asymptotically from its current level 8.8 to 2.1789 (billion m^3) in steady state, with a long-run per capita water availability about 170 m^3 per year. As compared to the benchmark model, both capital accumulation and the future industrial water use would be lower, which implies lower steady state consumption. The long-run dynamic welfare would be -0.7402 in contrast to -0.5716 as in the benchmark model. The asymptotic marginal value of groundwater is $\lambda =$

\$4.0532. As compared to that in *Model 3* with $\lambda = \$3.8087$, where the marginal value is larger here due to the increased future water shortage caused by climate changes. In other words, when groundwater becomes less resilient, the reduction in the "resilience" stock is reflected by the marginal value increase. Conditional on the given preference and production parameters, however, the long-run residential water use remains at the level about 0.5 (billion m^3) but the part for industrial use becomes smaller. Together with the lower capital stock, this would lead to lower per-capital consumption, and in turn lower utility and dynamic welfare in the future. Obviously, development would not be sustainable in this case.

After the deterministic analysis above, we now turn to the full-fledged model with stochastic disturbances taken into account. To focus on the growth and welfare effects of the stochastic disturbances[8], we limit our analysis here to the stochastic steady state. More exactly, we treat the steady state from the deterministic counterpart as the "initial" state and then examine how the uncertainty in surface water flow S_t and groundwater recharge rate m_t would affect growth, water resilience and the dynamic welfare. We do so with 8 different parameter configurations with standard deviations $\sigma = 0.1, 0.2, 0.3$,and 0.4 (billion m^3) for groundwater recharge (for surface water, the standard deviation is assumed to be 4σ as the average surface water flow is on avearge about $10/2.5 = 4$ times greater than groundwater recharge[9]). For the autocorrelation of the stochastic disturbances in neighbor years, we consider two different values, namely, $\rho = 0$ and 0.9. Unlike the deterministic case, where we can plot the entire time sequences of all the variables, the solution to the stochastic problem is simply a set of contingent plans in a feedback control form. With given initial state, we can only determine the optimal first period consumption, water extraction and uses etc, while the quantities for the remaining periods would depend on future realizations of the stochastic disturbance terms. To have a feel on the possible solution sequences, we plot a particular realization in Figure 4 for the optimal surface water flow, groundwater stock, and the intertemporal welfare, with $\sigma = 0.4$ and $\rho = 0.9$.

The averaged results over a large number of simulations are summarized in Table 3 for different parameter values. The $\bar{v}$ values in columns 3 and 4 indicate that a larger variation in surface water flow and groundwater recharge from year to year leads to lower (expected) intertemporal welfare, both for $\rho = 0$ and $\rho = 0.9$. This is a well-know result in economics due to the risk-averse preference structure and the jointly concave production function. With an increasing and a strictly concave optimal value function, this is simply a consequence from the Jensen equality, namely the expected optimal value from uncertain water supplies

[8] We assume that precipitation is stochastic, which both affacts the gorundwater recharge rate and surface water flow.

[9] This treatment is based on the consideration that the variability of both groundwater recharge and surface water flow is caused by variations in precipitation.

Table 3: The results from the stochastic model analysis

Standard Deviation (σ)	Steady State Mean Value	Correlation (ρ) 0.0	0.9
	$\bar{x}$	9.4589	9.5257
0.1	$\bar{\lambda}$	5.4410	5.3989
	$\bar{v}$	-187.4624	-187.4578
	$\bar{x}$	9.2745	9.5410
0.2	$\bar{\lambda}$	5.5540	5.3859
	$\bar{v}$	-187.5801	-187.5615
	$\bar{x}$	8.9689	9.5667
0.3	$\bar{\lambda}$	5.7423	5.3642
	$\bar{v}$	-187.7763	-187.7343
	$\bar{x}$	8.5449	9.6026
0.4	$\bar{\lambda}$	6.0055	5.3338
	$\bar{v}$	-188.0516	-187.9762

is always smaller than the optimal value of the expected water supply level. However, the effect on the expected groundwater stock and its marginal value depends on whether or not the variations are autocorrelated over time. With no correlation (for example, the precipitation this year is completely independent of that in other years), the expected groundwater stock would decrease in the variation of rainfall from year to year, and thus a larger shadow value (due the diminishing resilience). On the other hand, with strong autocorrelation, i.e. there is some strong persistence between rainfalls from year to year, the trend is opposite. Larger variations lead to slightly increased (average) steady state groundwater stock, and thereby smaller shadow value of water. The main reason for such a trend may be that with persistence, the precautionary principle applies to avoid longer periods of water shortage in extreme conditions. From the table, we can also see that, with positive correlation, the steady state groundwater stock is larger for any given standard deviation α, and the shadow value smaller. The intertemporal welfare, however, is larger with positive correlation as compared to the case with pure randomness. This is a particularly interesting result from an information and planning point of view. Although the (positive) autocorrelation implies a risk of persistent drought over time, it provides extra information for improved social planning. For example, with a large realized rainfall this year, we may expect with a higher confidence that the trend would continue in the next year. In other words, the conditional variance of future water availability would be smaller under positive autocorrelation, and thus a positive welfare effect.

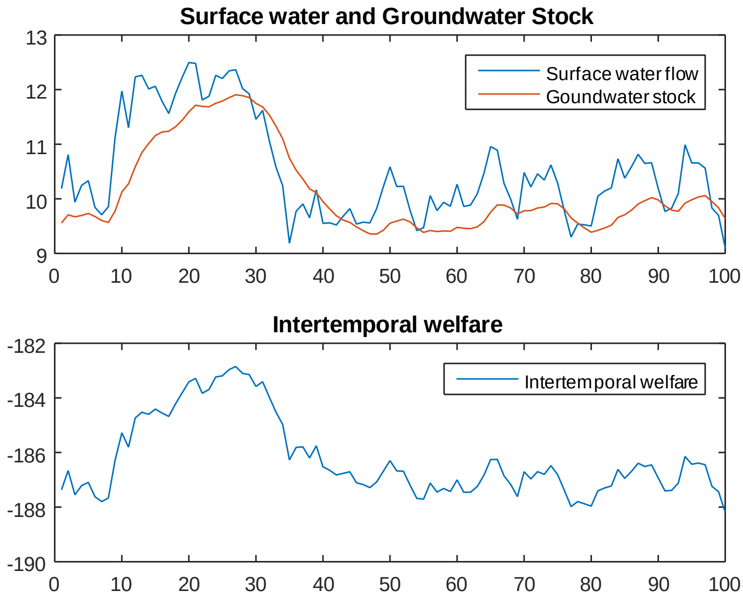

Figure 4. A realized sequence from the stochastic model

5 Concluding Remarks

This paper formalizes a DSGE model for analyzing the growth and welfare implications of water resilience with special reference to the South African economy. The country is a drought prone and water poor region facing serious future water shortage both due to the projected climate change effect and rapid population growth. We assume that there exist stochastic thresholds for the groundwater stock below which the expected extration cost would dramatically increase, and consider a generalized extraction cost function. In addition, we allow both the surface water flow and grodwater recharge to be stochastic and examine how the uncertainty affects future water resilience, economic growth and dynamic welfare.

With the official data from Statistics South Africa, we calibrate the model to conform the initial conditions in 2011, and then simulate the optimal future sequences of the interested variables under alternative parameter configurations. In a deterministic setting, where we assume that the future surface water flow and groundwater recharge are all known with certainty, we optimize the sequences for capital, production, consumption, groundwater extraction, and industrial as well as residential water use. We find, among other things, that with a 2% annual utility discount rate, development of the country can still be made sustainable although climate changes and the projected larger population size in the future would seriously reduce the per-capita water availability. The reason is that capital accumulation and the increased per-capita consumption can more than offset the loss caused by water shortage. On the other hand, even without climate change damage, an annual utility discount rate of 5% would render the development unsustainable. With the DSGE model, we have also examined the effect of precipitation variability and its autocorrelation over time on growth and dynamic welfare in the stochastic steady state. The main findings are that the variability would reduce the groundwater resilience and result in lower dynamic welfare; however, positive correlation between the variability level over time enhances groundwater resilience and improves dynamic welfare. We attribute these results to the value of the extra information from the correlation measure for improved planning.

It is worth mentioning the essential difference between optimality and sustainability. With heavier discounting of future utilities (say with 5% without assuming positive technical progress), we have seen that even along the optimal path, the dynamic welfare would still be declining over time. To improve sustainability, it is thus important to, in the first place, choose a somewhat lower discount rate to make better trade-offs between consumption and investment. With more accumulated productive capital, the future shortage in water availability can be mitigated for welfare improvement. Next, to overcome serious water shortage in the future, it would pay to develop improved surface water collection and storage techniques to buffer the precipitation variability over time[10]. Finally, to reduce global greenhouse gas emissions by international climate agreement, which would mitigate the global warming damages, is obviously important for sustainability in South Africa.

Although we have dealt with the South African economy in the quantitative analysis, the DSGE model is general enough for applications to other countries and regions as well. The advantage with South Africa is that the country has fairly detailed water accounts and all the data needed are available. Also, the country is classified as a poor country with future absolute water shortage. Despite the promising results from our model, we may need to extend the analysis along several directions in future research. First, the industrial use of water may be modeled more in detail for the diverse sectors such as agriculture, manufacturing, mining, and tourism etc, in which he marginal productivity of water may considerably differ due to market imperfections and equity considerations. Second, the variations between different water management areas and even aquifers may need to be taken into account concerning water quality and vulnerability. Finally, potential technical progess (e.g. converting saltwater to sweet water) and water

[10]Among other measures South Africa's Groundwater Strategy already entails components of water resources planning and sustainable groundwater management. In addition to implementation of existing strategies, regulations and guidelines on groundwater management, they are also developing an artificial recharge strategy (DWA 2010).

transportation issues should also be considered.

Appendix: Details on groundwater statistics in South Africa

Resilience of the water supplies that depend on groundwater, is a crucial issue for the absolute water scarce country like South Africa. With 80 per cent of its surface water [11] already allocated, it is further constrained by its low precipitation, high evaporation and rapid population growth. Groundwater is becoming strategically more important for the country to meet its food production, industry, household and environmental needs (DWAF 2004). The total volume of available, renewable groundwater in South Africa (the Utilisable Groundwater Exploitation Potential, or UGEP) is 10 343 million m^3/a (or 7500 million m^3/a under drought conditions)[12] (DWA 2010, Middleton and Bailey 2009). Of the total utilisable stock of groundwater, South Africa currently uses between 2000 and 4000 million m^3/a of groundwater[13](DWA 2010). At present, about two-thirds of country's surface area and 65 percent of its population are largely dependent on groundwater, and the future dependence due to climate changes would be more eminent, especially in the semi-desert to desert parts without perennial streams. About 98 per cent of South Africa its groundwater reserves are found in fractured, hard rock aquifer systems.

Boreholes are buffered against short term variations in climate. Unstressed aquifers in semi-arid areas are similar to the aquifers in humid areas, thus for areas where rainfall is generally greater than 600 mm), even if the climate becomes drier, many rural water supplies are likely to remain functional. However, below the critical threshold of 500 mm of mean annual rainfall there is a dramatic drop-off in recharge (MacDonald et al. 2011). Since the average annual rainfall in South Africa below this level experts believe that recharge is generally low (Turton et. al, 2006)[14]. Other thresholds include annual rainfalls (200 mm) and per-capita

[11] The total surface water estimated in South Africa is 12 000 m^3/a

[12] Figures derived from the Groundwater Resource Assessment Phase II. For details refer to Middleton and Bailey (2009). Utilisable Groundwater Exploitation Potential (UGEP) is the volume of groundwater that may be abstracted based on a defined maximum allowable water level draw down. It takes into account the limits imposed by recharge (including changes due to drought), variations in the borehole yield, groundwaters contribution to river base flow, and the ecological reserve. It however excludes the existing abstractions (including basic human needs), or limitations due to poor groundwater quality.

[13] These official figures are based on the groundwater licenses (WARMS data) and are approximations as the actual use may be very different from the information in the collected data.

[14] There is a lot of regional variation in rainfall in South Africa. Rainfall usually occurs during the summer from Novemberto March. In the south-west, around Cape Town rainfall occurs

water availability (in m^3 per capita) due to population growth etc. South Africa, is vulnerable to climate change and variability due to multiple stresses and low adaptive capacity. With Scholes and Biggs (2004) predicting a future scenario with hotter and drier conditions in southern Africa, global climate change will lead to a reduction in aquifer recharge, leading to a worsening of the groundwater situation and the vulnerability of the poor. This vulnerability is further augmented by the livelihoods of the people that are often directly linked to the climate of the area (CSIR, 2010). When the depth to the water table is beyond 50 meters, then drilling and extraction costs, as well as the failure rates (MacDonalds et. al 2011) may rise in an abrupt way due to geological, technical and institutional reasons[15].

About 62 per cent of South Africa's total water is used in the Agriculture for irrigation, with the urban requirement at about 23 per cent. The remaining 15 per cent is shared by rural users, mining and bulk industrial, power generation and afforestation[16] (Statistics South Africa 2010).

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in winter from June to August. In the north-west, annual rainfall often remains below 200 mm. Much of the eastern Highveld, in contrast, receives 500 to 900 mm of rainfall per year, occasionally exceeding 2 000 mm/annum. A large area of the central country is semi-arid and receives about 400 mm of rain on average, and there are wide variations closer to the coast. The land to the east of this region is suitable for growing crops,and land to its west is only for livestock grazing or crop cultivation on irrigated land, also known as dryland farming (Statistics South Africa 2010

[15]Rural communities that rely on handpump or small motor pump for water extraction, might be severely impacted once the water table is below 40 meters. Discussions with the experts at the Department of Water Affairs revealed that management thresholds may vary between 5 - 40 meters, but on average it is usually around 25 meters (Discussion notes, Bali Swain, May 2014)

[16]Irrigation constitutes over 64 per cent of the groundwater use. Mining and domestic consumption in urban and rural areas, each use 8Highveld and domestic use in the rural areas occurs in KwaZulu-Natal, the Eastern Cape, Mpumalanga, and Limpopo

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