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Cooperation or Conflict in Common Pools*

Ingela Ternström†

Abstract

Many of the world's common-pool resources are located in poor countries, where consumption levels may be sufficiently low to have an adverse effect on the users' health. Under these circumstances, an agent's utility function may be described as an S-shaped function of consumption. Using non-cooperative game theory, very poor groups of users are shown to have a lower probability of cooperative management of common pool resources, than groups with adequate consumption levels. However, the chances for cooperation are greatest for users that are only moderately poor. If there is a variation in resource productivity for this group, cooperation may break down in periods of low productivity. The theoretical results concur with empirical evidence of cooperation in common pool resources.

Keywords: Common-Pool Resources, Developing Countries, Dynamic Game, Irrigation, Natural Resources, Non-linear Utility.

JEL Classification: C72, O13, Q15, Q25

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1 Introduction

A common-pool resource is a resource with a well-defined group of co-users. There is no individual ownership, but the group can exclude outsiders from the use of the resource. A large part of the third world's natural resources are managed as small local common-pool resources, for example irrigation systems, village forests, grazing grounds and fishing waters. There is a large literature on why and when we may expect cooperative management of such resources to be successful; what is lacking is an explicit consideration of the socioeconomic status of the people managing them.

Non-cooperative game theory has been used for analysing the common-pool resource problem for a long period of time. The infinitely repeated prisoner's dilemma is usually the preferred parable, since in a simple way, it captures how community incentives can keep short-run incentives to take more than one's share of the resource in check. This paper also uses non-cooperative game theory but includes the fact that in the third world, users of the resource are generally poor, and dependent on the resource for their survival or well-being. The paper explores how the users' incentives to cooperate are affected by their level of well-being under the following assumptions:

First, the marginal utility of consumption is highly dependent on the level of consumption, and there is thus a non-linear relationship between consumption and individual well-being. Second, credit markets are imperfect and therefore, do not compensate for the non-linearity of the utility function by smoothing consumption. Third, the state of the resource is controlled by exogenous factors such as weather conditions which, together with the users' actions, determine the level of output from the common-pool resource. The paper does not discuss the technology or dynamics of the resource.

Under these circumstances, how would we expect the users to act? On the one hand, an agent can gain more by cheating when the resource is large than when it is small. On the other hand, what is gained by cheating may not be worth as much when the resource is large.

This paper predicts a non-monotonic relationship between the size of the resource and the chances of cooperation within the group of users. Cooperation will be more difficult when the users are starving than in a well-fed group, but easiest of all in a group of people whose health would be seriously affected by a slight decrease in consumption. If we accept that the utility of an individual is closely related to her health, the model gives clear-cut implications: When the state of the resource is such that the users' body mass index[1] (BMI) is close to 20 if they cooperate, they will have the greatest chances of sustaining cooperation. From this point, both increases and decreases in the size of the resource will make cooperation more difficult.

Furthermore, changes in complementary income sources, as well as the introduction of markets for goods or capital, can make cooperation more difficult. When there is seasonal or stochastic variation in the exogenous factor determining the state of the resource, cooperation will be more difficult in periods with low resource-levels for most groups. However, the very poor will find the periods with high resource-levels to be the ones most prone to failed cooperation. I also find that cooperating part of the time can be a both possible and welfare improving alternative, when cooperating all the time is impossible. When I combine these results, the model is strongly supported by the empirical finding that in functioning common-pool resources, the relatively less productive period is the greatest challenge to cooperation.

Baland and Platteau (1996, Ch. 12) give a summary of the characteristics found to be important for successful cooperation in the empirical literature (mainly Ostrom (1990), Wade (1988) and McKean (1986)). One of these characteristics is that users should be highly dependent on the common-pool resource. There are also many empirical examples relating the breakdown of cooperation to resource scarcity. Regarding irrigation systems, Ostrom (1990) gives several examples of the connection between water scarcity and the temptation to cheat, and between bad times and actual rule-breaking.[2] Ostrom, Gardner and Walker (1994) state that "As the availability of water decreases, temptation increases for irrigators to break rules that limit water allocations".[3] Regarding irrigation systems in India, Baland and Platteau (1996) point to the high correlation between the degree of water scarcity and the level of activity of informal water users' organisations.[4] Ternström (2002), examining time-series data from irrigation system in Nepal, find a positive correlation between food sufficiency and cooperation, but also evidence that deviations from cooperation occur both in times of high and low water supply. Contrary to these observations are e.g. Cardenas' (2003) field experiments, where the contribution to cooperation in the common-pool experiment was negatively correlated with the participants' real wealth. For a more recent review of factors that affect cooperation in locally-

[1] The body mass index is a measurement of weight relative to height, BMI = weight/height^2.

[2] See Ostrom (1990) pp. 69, 73 and 99 for examples.

[3] Ostrom, Gardner and Walker (1994) pp. 225-6.

[4] Baland and Platteau (1996) p. 210.

managed irrigation systems, see Dayton-Johnson (2003). Wade (1987) argues that villagers confronting crisis conditions tend to behave opportunistically, and gives examples of such incidences.[5]

In the literature, there are also examples of very old common-pool resources that cease to function altogether with the disappearance of an outside income source. Baland and Platteau (1996, p. 266) tell the story of fishermen in Gahavälla, traditionally living off a combination of common-pool resource fishery and wage-earnings from day labour. When the wage earnings decreased due to a reduction in the economic opportunities in agriculture, cooperation in fishery became more difficult to sustain and gradually, cooperation was replaced by violent competition for the fish; see also Jodha (1988) for a similar account. Berkes and Folke (1998) have given a number of other examples of the important links between resource availability and management regimes. Finally, the magnitude of the problem becomes evident when considering the degree of dependence on local resources in developing countries, as discussed in for example Dasgupta and Mäler (1997).

On the theoretical side, the most closely related contribution is Spagnolo (1998), who studies the effect of concave utility on the outcome of repeated prisoner's dilemma games. The present paper is also in some ways related to the problem of price wars in oligopolies, see for example Green and Porter (1984) and Rotemberg and Saloner (1986). While the former assume imperfect information, Rotemberg and Saloner make the same assumption as I do here, in that agents have full information regarding the state of the world, and come to similar conclusions. Their model predicts deviations in times of high demand, since that is when the gain from deviating is highest, while my model predicts deviations in bad periods, for exactly the same reason.

The paper proceeds as follows: The next section introduces the model and gives the optimal size of the resource. Section 3 introduces complementary income sources and markets for goods and capital to the model. In Section 4, I explore how variations in the size of the resource affects the chances of cooperation. In Section 5, I show that the chances of cooperation can be improved by introducing the possibility to cooperate in some periods only. Section 6 concludes.

[5] Wade (1987) describes how desperation caused by a severe drought in an Indian village made people seriously consider breaking the rules of their common irrigation system. Wade interprets the reason for the behaviour in a slightly different way from what I do here. The breakdown of cooperation was avoided by increasing fines.

2 The Model

Throughout the paper, my example will be that of farmers using an irrigation system to water their fields. The farmers are the agents in a dynamic prisoner's dilemma game over a common-pool resource, the irrigation system. To simplify the analysis, I assume that there are only two agents and that they are identical in all respects. The farmers' main source of food and income is the harvest from the fields that get water from the irrigation system. In the simplest version of the model, they have no access to markets for goods or credit and no storage facilities.[6] The amount of water in the irrigation system is given by the level of rainfall, which is perfectly observable. The benefit of the rain can be enhanced by the use of the irrigation system. The extent to which the use of the irrigation system benefits the farmers depends on whether they cooperate in its use. The farmers decide whether to cooperate by comparing the utility gained by taking different actions.

2.1 The Utility Function

The empirical examples given in the introduction indicate that there is a non-linearity in the cost-benefit ratio of deviating. There are at least three possible causes: The relationship between the amount of water and the size of the harvest may be non-linear, there may be a connection between nutrition and productivity that affects the harvest size in a non-linear fashion, and the utility gained from different levels of consumption may not be linear. If we have multiple sources of non-linearity, their combined effect depends on their relative location; they may either join forces or have a neutralising effect on each other.

Given that I am examining poor agents, I here choose to focus on the non-linearity in connection with the level of consumption. For poor people with mainly one source of food, the supply from this source will be crucial for their physical well-being. Figure 1 illustrates the S-shaped correlation between BMI and the probability of remaining in good health as presented by Dasgupta (1993, ch.14). Note that; *(i)* it takes a certain (above zero) BMI to have any chances at all of staying alive, (*ii*) the marginal health-benefit from food is increasing for low levels of food intake, and (*iii*) the marginal health-benefit from food is decreasing for

[6] I extend the model to allow for storage and markets for goods and credit in Section 3.2.

high levels of food intake.[7]

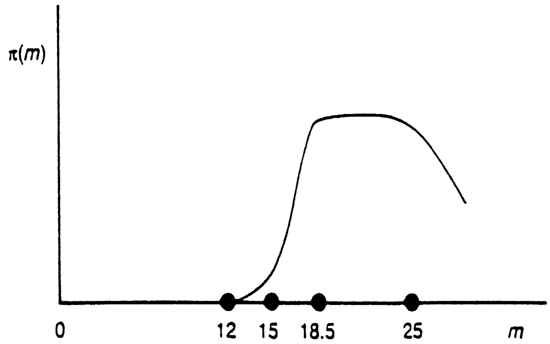

Figure 1: One minus the probability of health breakdown, $\pi(m)$, as a function of the body mass index, m. Source: Dasgupta (1993) p. 416.

The causes for the decreasing marginal health-benefit from food above certain levels are probably well known to everyone, but the increasing marginal health-benefit may need some explanation. The reason is that the human body uses energy to extract energy from food and if the food input is too low, there is not enough energy to make use of it in an efficient way. In this situation, a small decrease in the amount of food will not only decrease the amount of energy intake but also the amount of energy the body can extract from a given amount of food. In the western world, the problems are mainly related to the concave part of this relationship. However, among poor people in third world countries, the convex part is the more relevant one. According to the 2004 World Development Indicators,[8] 24 percent of the population in low income countries were undernourished in 1999-2001. There, the definition of undernourishment is that consumption is too low to maintain normal levels of activity, which would imply a BMI of 18.5 or less.[9] These people will be having a non-concave health-to-food relationship. As it is the poorer rather than the richer parts of the population that depend on common-pool resources for their livelihood,[10] the relevant percentage becomes even higher.

[7] See also e.g. Weir (1995) for an estimate and a discussion of the effect of income on adult mortality.

[8] The World Bank (2004).

[9] In the 20 countries with the lowest dietary energy supply level, on average 52 percent of the population were undernourished in 1990-92. In the second and third groups of countries, the percentage was 34 and 23, respectively (FAO, 1997). In low-income countries, on average 31 % of the children under the age of five suffered from malnutrition. Based on Table 6, World Development Report 1996.

[10] See e.g. Jodha (2001) Table 5.1.

The above figures make it abundantly clear that we must take the particularities of poor people into account when modelling common-pool resources in developing countries. For this purpose, I assume health to be an important component in utility. Thus, we can translate Dasgupta's food to health relationship into an S-shaped function of the utility from food, with one interval of non-decreasing positive marginal utility and one interval of non-increasing positive marginal utility from food.[11] As food in this model mainly comes from the crops grown on the farmers' fields, the implication is that the marginal utility of the harvest is largest when the BMI equivalent of the harvest is between 15 and 18.5. Based on the above discussion, I assume that the utility of consumption (or harvest) can be characterised as $U(C) \geq 0$ with $U'(C) \geq 0$, $U''(C) > 0$ for $C < \textsc{mmi}$, and $U''(C) < 0$ otherwise.[12] The inflexion point is referred to as MMI, the point of maximum marginal impact. C expresses the consumption level (or harvest size) in BMI-equivalents. Note that I implicitly assume that people will stop eating before food has a negative effect on their health. In the numerical examples, I use the following specification of the utility function, which is a good approximation of the relationship depicted in Figure 1, but with an asymptotically linear upper end.

$$U(C) = \left[1 + \frac{1}{1+C} + e^{-\gamma(C-\textsc{mmi})}\right]^{-1}, \tag{1}$$

where $\textsc{mmi} = 16.5$, and $\gamma = 1.4$.[13] The below figure shows the resulting utility function. I have assumed that the y-axis in Figure 1 is measured on a scale from 0 - 1 (for probability), and utility is assumed to be measured on the same scale. The x-axis shows the BMI-equivalent of

[11]See also Ravallion (1997) who uses a survival function that is concave above a consumption floor, below which there is simply not enough food to sustain the basic functions of the body.

[12]For readers who are not quite comfortable using an S-shaped utility function, note that I could instead use a linear utility function together with an S-shaped survival function. Let $P_A(\alpha\pi_{a_1,a_2})$ represent the probability of staying alive as a function of the size of the harvest. By letting $P_A(\alpha\pi_{a_1,a_2})\,\alpha\pi_{a_1,a_2} = U(\alpha\pi_{a_1,a_2})$, it is evident that the results will be identical.

[13]Note that for simplicity, the function is assumed to be symmetric around MMI. To make the relationship as similar as possible to that suggested by Dasgupta, I let $\textsc{mmi} = 16.5$, instead of the appropriate 18.5. This only slightly affects the illustrated results but should be kept in mind.

consumption.

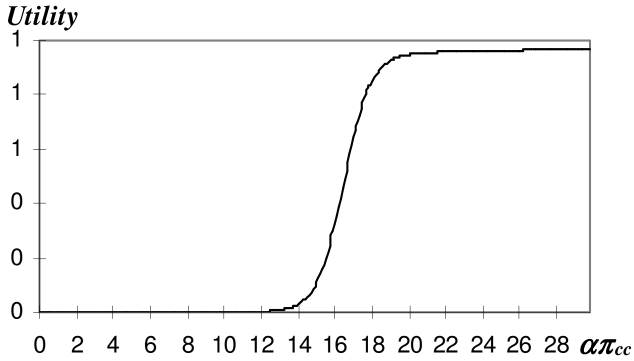

Figure 2: The utility function of equation (1).

2.2 Actions and Material Payoffs

In every period, each farmer $i \in \{1,2\}$ chooses an action $a_i \in \{c,d\}$, where c represents cooperation and d deviation. To focus the attention, the relative size of the harvests for different combinations of actions, π_{a_1,a_2}, is kept constant throughout the paper, which implies that as the level of rainfall changes, it is only the absolute productivity-level of the irrigation system that changes. The size of the harvests the farmers obtain when cooperating, relative to the size of their harvests when not cooperating, is unaffected. What we then have is a common-pool resource game with variations in the absolute size of the payoffs (with the level of rainfall), but where the relative size remains the same. We can thus express farmer 1's consumption as $C = \alpha\pi_{a_1,a_2}$, with α being the amount of water.[14] By assumption,

$$\pi_{d,c} > \pi_{c,c} > \pi_{d,d} > \pi_{c,d}. \tag{2}$$

Being a single deviator gives the largest harvest, and attempting to cooperate when the other farmer deviates results in the smallest harvest. I assume the sum of the harvests to be maximised under mutual cooperation, that is

$$2\pi_{c,c} > \pi_{d,c} + \pi_{c,d}. \tag{3}$$

Thus, the stage game will be a prisoner's dilemma with $\{d,d\}$ as the unique equilibrium.

[14] Assuming that the harvest size is a linear function of the amount of rainfall is of course a simplification of reality. It may be more correct to assume that the harvest size is also an S-shaped function of water. This could make the results even more pronounced.

2.3 The Repeated Game

In the repeated game, I assume discrete time, t, and an infinite horizon. The size of the harvest in a certain time period determines the level of the farmers' utility during that same period. I assume that the farmers have identical discount factors, δ, which are independent of their consumption levels.[15]

A strategy is a prescription of what action to take at every stage, given the history of the game. We are interested in characterising a strategy generating the maximum amount of cooperation. From Abreu (1986, 1988), we know that in a repeated prisoner's dilemma game, a trigger strategy (where the agents choose the cooperative action in every period until, for the first time, they notice that someone has deviated, and thereafter shift to playing "deviate" forever) is optimal in this sense. If such trigger strategies cannot sustain cooperation, neither can any other strategies. Otherwise, cooperation is a possible equilibrium outcome.[16] The discounted utility of behaving cooperatively, when all players do, is then

$$\sum_{t=0}^{\infty} \delta^t U\left(\alpha \pi_{c,c}\right), \tag{4}$$

and the discounted utility of deviating is

$$U\left(\alpha \pi_{d,c}\right) + \sum_{t=1}^{\infty} \delta^t U\left(\alpha \pi_{d,d}\right). \tag{5}$$

To test whether the trigger strategy can sustain cooperation, it suffices to check whether it will be beneficial for the agents to deviate from this strategy in a single period. Thus, for cooperation to be a subgame-perfect equilibrium, expression (4) must be equal to or greater than expression (5).[17] Thus, cooperation can be sustained for all discount factors above the critical level (the critical discount factor),

$$\delta^*\left(\alpha\right) = \frac{U\left(\alpha \pi_{d,c}\right) - U\left(\alpha \pi_{c,c}\right)}{U\left(\alpha \pi_{d,c}\right) - U\left(\alpha \pi_{d,d}\right)}. \tag{6}$$

What I am interested in here is the effect on the critical discount factor of varying the the size of the common-pool resource, in this case measured

[15] However, this implies assuming that the discount rate is not affected even when survival is threatened. Thus, we should be careful when interpreting the results for the lowest consumption levels, where the survival constraint may add to the difficulty of achieving cooperation.

[16] The purpose of using such a grim strategy is to see if cooperation is at all possible under the above assumptions regarding the utility function.

[17] Note that this is one among many equilibria, and that even though a cooperative equilibrium exists, it is not necessarily the one to be chosen.

by the amount of rainfall. From Spagnolo (1998), we know that with concave utility, the more concave are the agents' utility functions, the smaller will the critical discount factor at which a certain set of material payoffs can be supported as a subgame-perfect equilibrium outcome be. The intuition behind this result is that an agent with a strictly concave utility function has a lower marginal valuation of the increased payoff gained by deviating and a higher marginal valuation of the decreased payoff when punished for it, than an agent with a linear utility function.[18] Applying this argument to my S-shaped model, the implication is that with the same relative harvest sizes, the utility gained by deviating relative to the utility lost when punished for such an action will be larger when utility is convex than when it is concave. Thus, it should take a larger discount factor to deter deviations on the convex than on the concave segment. The following proposition verifies that this will be the case. See Spagnolo (1996, 1998) for the original proposition and proof regarding the effect of concave utility.

Proposition 1 *The critical discount factor is increasing in the convexity of the utility function.*

Proof. Let δ^L be defined as the parameter that fulfills

$$\alpha\pi_{c,c} = \delta^L \alpha\pi_{d,d} + (1 - \delta^*)\, \alpha\pi_{d,c} \tag{7}$$

and let $U\left(\alpha\pi_{a_i,a_j}\right)$ be a convex transformation of $\alpha\pi_{c,c}$. By the definition of convexity, we know that, for all α

$$U(\alpha\pi_{c,c}) \leq \delta^L U(\alpha\pi_{d,d}) + \left(1 - \delta^L\right) U(\alpha\pi_{d,c}). \tag{8}$$

Solving for δ^Lresults in

$$\delta^L \leq \frac{U(\alpha\pi_{d,c}) - U(\alpha\pi_{c,c})}{U(\alpha\pi_{d,c}) - U(\alpha\pi_{d,d})}. \tag{9}$$

where the right hand side equals the definition of the critical discount factor in (6). Hence,

$$\delta^L \leq \delta^*, \tag{10}$$

i.e. the critical discount factor is increased by a convex transformation of the utility function.

[18]Note, however, that while Spagnolo keeps the absolute material payoffs constant, I keep the relative size of the payoffs constant and thus, there is an increase in both the size and the spread of the material payoffs.

■

To understand what happens in the intermediate segment, where parts of the material payoff matrix are on the concave segment and other parts of it are on the convex segment of the utility function, i.e. when $\text{MMI}/\pi_{d,c} < \alpha < \text{MMI}/\pi_{d,d}$, it helps to solve for $U(\alpha\pi_{c,c})$ in equation (6)

$$U(\alpha\pi_{c,c}) \leq \delta^* U(\alpha\pi_{d,d}) + (1-\delta^*)\, U(\alpha\pi_{d,c})\,. \tag{11}$$

First, as the payoff from deviating increases above MMI, the growth rate of this payoff will be slowing down. To keep (11) satisfied, this must be countered by a decreasing critical discount factor. When α further increases, so that $\alpha\pi_{c,c} >$MMI, the cooperative payoff also starts to grow at a slower rate, and we get a counteracting force, which slows down the decrease in the critical discount factor. Figure 3 shows the resulting shape of the critical discount factor, with the x-axis showing the BMI-equivalent of the harvest size under cooperation.

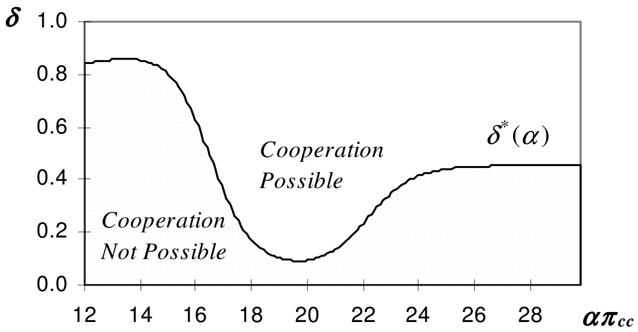

Figure 3: The critical discount factor when $\pi_{d,c} = 1.1\pi_{c,c}$ and $\pi_{d,d} = 0.9\pi_{c,c}$.

3 Empirical Implications

The results of the above analysis imply that if utility is linear, the amount of water available is irrelevant for the probability of cooperation. However, if utility is not linear, the curvature of the utility function is of crucial importance for the chances of cooperative management of the common-pool resource. With an S-shaped utility function, it is easier for a group to sustain cooperation if the amount of water is such that utility is measured on the concave segment of the utility function than on the convex segment. The most discouraging result is, of course, that the groups with the greatest need to increase the harvest size above the cooperative level are also the ones with the greatest risk of instead having it reduced.

Furthermore, increasing the difference between relative payoffs makes cooperation easier at intermediate and large resource levels but increases the critical discount factor when the resource is small, as illustrated below. The reason is that the larger the span is between the material payoffs, the larger is the span of consumption levels where the non-linearity of the utility function affects the critical discount factor.

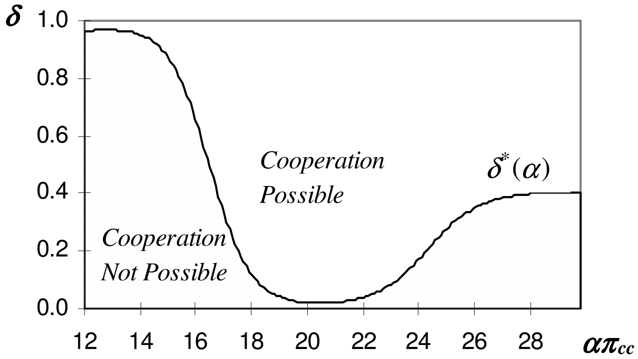

Figure 4: The critical discount factor with larger differences between relative payoffs, $\pi_{d,c} = 1.2\pi_{c,c}$ *and* $\pi_{d,d} = 0.8\pi_{c,c}$.

Given that we accept the assumption of utility being dependent on health, the model gives clear-cut numerical results. From the graphical presentations in Figures 3 and 4, it is obvious that a BMI of around 20 when cooperating, that is, where the utility function is most concave, provides the best chances of successful cooperation. If the resource is smaller, a further decrease will make cooperation more difficult. Comparing these results with the above discussion on the average BMI in poor countries, we can conclude that it is the poor but not starving who have the best chances of cooperating, that decreases in the size of the resource will make cooperation more difficult, and that this group constitutes a substantial part of the population in poor countries.

3.1 Additional Income Sources

With a slight change in the model, we can analyse the case where the agents have an additional source of income, β, for example a wage from day labour.[19] The consumption level will now be a sum of the income

[19] For an analysis of the effect of inequality in the size of the additional income sources, see Ternström (2002a).

from the two sources, $C = \beta + \alpha\pi_{a_1,a_2}$. Let the additional income source be the one depending on the exogenous variable factors, and let the size of the harvest depend only on the cooperative success of the farmers. The result is to give the exogenous variable an additive, rather than a multiplicative, effect.[20] By performing the same analysis as above, we can analyse how different sizes of the complementary income affect the cooperative efforts of the farmers. The critical discount factor will now be as follows, with the subscript *add* for additive,

$$\delta^*_{add}(\alpha) = \frac{U(\beta + \alpha\pi_{d,c}) - U(\beta + \alpha\pi_{c,c})}{U(\beta + \alpha\pi_{d,c}) - U(\beta + \alpha\pi_{d,d})}. \tag{12}$$

The figure below shows that if, instead of letting the level of rainfall differ, we give the group of farmers a complementary source of food or income and let this differ, we obtain similar results, but with a different interpretation. The effect is opposite to that of increasing the relative difference between payoffs, as we did in Figure 4. Here, keeping the absolute difference between payoffs constant, in effect results in decreased relative payoff differences as the size of the exogenous variable increases. With a smaller absolute difference, the non-linearity of the utility function has a more limited effect on the critical discount factor.

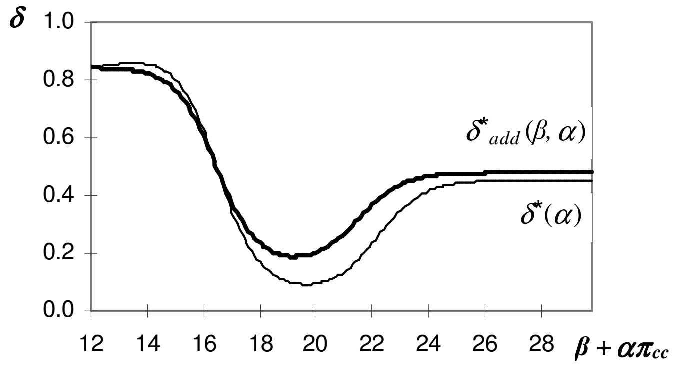

Figure 5: The critical discount factor with a complementary income source when the return from the common-pool resource is too small to survive on; $\alpha\pi_{c,c} = 12$, $\pi_{d,c} = 1.1\pi_{c,c}$ *and* $\pi_{d,d} = 0.9\pi_{c,c}$.

The implication is that the success of the cooperative management of a common-pool resource also depends on the size of the users' complementary income sources. If the complementary income is such that it minimises the critical discount factor, the system is sensitive to both increases and decreases in the size of the complementary income. It is also

[20] This also implies constant absolute, instead of constant relative, payoff sizes.

important to note that if we assumed utility to be linear, changes in a complementary income source would have no effect on the management of the common-pool resource.

By varying the size of the material payoffs relative to the size of the complementary income, we can study the effects of different degrees of dependence on the common resource. As the payoffs from using the resource become a smaller part in total consumption, the sensitivity to changes in the additional source of income decreases and the critical discount factor becomes flatter. This implies that as dependence on the common-pool resource decreases, it becomes more difficult to cooperate at intermediate income levels. Hence, the assumption of a non-linear utility function provides explanations for both Ostrom's (1990) observation regarding resource dependence and Jodha's (1988, 2001) observations regarding market integration.

Baland and Platteau's (1996) description of what happened to the fishermen in Gahavälla (see the introduction) is a good example of a change in an additional income source. The common-pool fishery had developed and improved over a long period of time, from which we may suspect that the system was operating on a scale where cooperation was easily sustained. The disappearance of their complementary source of income implied a left-ward move along the utility curve and an increased critical discount factor. Consideration for these kinds of effects should be given when choosing the location for aid projects, both when the project itself requires cooperative management, and when there are pre-existing common-pool resources.

3.2 Storing, Saving and Selling

What would happen if we were to introduce the possibility of saving part of the harvest until future periods? In theory, users with convex utility would increase their total utility by making their consumption as uneven as possible. Thus, they could gain a very high marginal utility in one period at the cost of an only slightly reduced utility in other periods. It is, however, difficult to imagine that a person close to dying from starvation would voluntarily relinquish any of his consumption today for use in a future period, since that future period may never come. Thus, I shall refrain from using my model to analyse that case. As far as credits are concerned, I shall simply assume that users on the convex segment are ineligible for loans, and thus will not be able to make use of credit markets, even if they want to.

Hence, I here focus on the concave part of the utility function. With concave utility, being able to reallocate the consumption of some of the additional harvest gained when deviating to one punishment period or more will increase the marginal utility of the reallocated amount and thus, the total benefit from deviating. However, unless the storage methods are perfect, there will be a loss connected with transferring the harvest in time. The smaller is this loss, the more of an obstacle to cooperation storage will be. Assuming that a share $s \in [0, 1]$ of the difference between the size of the harvest when deviating and when being punished is saved for one period, and that a fraction $r \in [0, 1]$ of the saved harvest remains after one year of storage, we can write the condition for storage facilities to be harmful to cooperation as

$$U\left[\alpha\pi_{d,c} - s\left(\alpha\pi_{d,c} - \alpha\pi_{d,d}\right)\right] + \delta U\left[\alpha\pi_{d,d} + rs\left(\alpha\pi_{d,c} - \alpha\pi_{d,d}\right)\right] \quad (13)$$

$$> U\left(\alpha\pi_{d,c}\right) + \delta U\left(\alpha\pi_{d,d}\right). \quad (14)$$

The larger is r and the more concave is the utility function, the larger is the left-hand side of equation (13), and the more of a threat will storage be to cooperation.

If we introduce credit markets on top of a perfect storage method, thus adding the possibility of earning interest on the saved amount, the effect is the same as if the fraction remaining after storage were larger than one ($r > 1$). Furthermore, credit institutions would make it possible to spread the gains from deviating over more than two periods, thereby further increasing the marginal utility of deviating.

The effect of introducing goods markets may be illustrated as an increase in the marginal benefit from deviating, since there is a market on which the additional harvest gained by deviating can be exchanged for other goods with a higher marginal utility. This implies making the concave part of the utility function steeper, that is, making utility move more quickly towards its maximum.[21] The below figure (where I have increased γ in equation (1) to 1.6 for $\alpha\pi_{a_1, a_2} \geq$ MMI), illustrates that this makes cooperation more difficult at the upper end of the utility function. Jodha (1988, 2001) suggest that one reason that the introduction of a nearby marketplace is harmful to the cooperative management of a common-pool resource, is that it reduces the social cohesion, thus making it more difficult to maintain the social norms regulating the use of the common resource. Another explanation could thus be the possibility of changing the composition of consumption and thereby get a higher

[21] Kranton (1996) and Spagnolo (1998) provide different approaches and more formal analyses of the effect of market access on reciprocal-exchange and cooperation, respectively.

marginal utility from deviating.

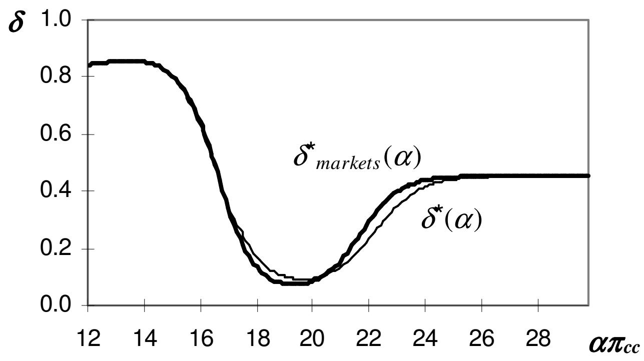

Figure 6: The critical discount factor with access to markets, when $\pi_{d,c} = 1.1\pi_{c,c}$, $\pi_{d,d} = 0.9\pi_{c,c}$.

4 Variations in the State of the Resource

In this section, I extend the base-line model to analyse the effect on cooperation of variations in the amount of rainfall. I here focus on stochastic variations, and refer to Appendix A for an analysis of seasonal variations. Note that this is no longer a repeated game, since there is variation in the state of the resource. Thus, the trigger strategy used so far may no longer be the optimal strategy. The extension in Section 5 shows a strategy that may improve the outcome.

4.1 Stochastic Variations

Suppose that weather is variable and somewhat unpredictable. The farmers know the possible levels of rainfall and their likelihood, but do not know what the actual level will be until each period begins. We thus continue to assume that farmers have full information about the level of rainfall in the present period, but now assume future levels of rainfall to be stochastic. To simplify the analysis, we assume *i.i.d.* shocks and only two possible levels of rainfall, wet (α_w) or dry (α_d), with $\alpha_w \geq \alpha_d$ always.[22]

In each period of time, we let the probability of the high level of rainfall be p_w. Define the possible levels of rainfall in future periods as

[22] In reality, there will be many possible levels of rainfall, not just two as assumed here. This should not have any major effect on the analysis, as the only difference in the equation for the critical discount factor will be that the expected loss from deviating consists of more terms.

$\alpha_\tau \in \{\alpha_w,\ \alpha_d\}$, where $\tau \in \{t > 0\}$. We write the expected utility[23] from a certain combination of actions in any future period as

$$E_\tau \left[U\left(\alpha_\tau \pi_{a_1,a_2}\right)\right] = p_w U\left(\alpha_w \pi_{a_1,a_2}\right) + (1-p_w)\, U\left(\alpha_d \pi_{a_1,a_2}\right). \tag{15}$$

To find the critical discount factor of a wet period, $\delta^*_{stoch}(\alpha_w)$, we set the discounted expected utility from cooperating equal to the discounted expected utility from deviating,

$$U\left(\alpha_w \pi_{c,c}\right) + \sum_{t=1}^{\infty} \delta^t E_\tau \left[U\left(\alpha_\tau \pi_{c,c}\right)\right] = U\left(\alpha_w \pi_{d,c}\right) + \sum_{t=1}^{\infty} \delta^t E_\tau \left[U\left(\alpha_\tau \pi_{d,d}\right)\right], \tag{16}$$

and solve for the critical discount factor,

$$\delta^*_{stoch}(\alpha_w) = \frac{U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{c,c}\right)}{U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{c,c}\right) + E_\tau\left[U\left(\alpha_\tau \pi_{c,c}\right)\right] - E_\tau\left[U\left(\alpha_\tau \pi_{d,d}\right)\right]}. \tag{17}$$

The critical discount factor of a dry period is, correspondingly,

$$\delta^*_{stoch}(\alpha_d) = \frac{U\left(\alpha_d \pi_{d,c}\right) - U\left(\alpha_d \pi_{c,c}\right)}{U\left(\alpha_d \pi_{d,c}\right) - U\left(\alpha_d \pi_{c,c}\right) + E_\tau\left[U\left(\alpha_\tau \pi_{c,c}\right)\right] - E_\tau\left[U\left(\alpha_\tau \pi_{d,d}\right)\right]}. \tag{18}$$

The sum in the numerator represents the benefit from deviating, while the sum of the two expected utilities in the denominator represents the expected loss from doing so. Since the expected punishment is the same for both outcomes, it will take a higher discount factor to sustain cooperation in the outcome giving the largest benefit from deviating. Thus, we can state the following:

Proposition 2 *When rainfall is stochastic, cooperation is easier in the wet than in the dry period if and only if* $U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{c,c}\right) < U\left(\alpha_d \pi_{d,c}\right) - U\left(\alpha_d \pi_{c,c}\right)$.

Proof. Assume that $\delta^*_{stoch}(\alpha_w) < \delta^*_{stoch}(\alpha_d)$. Substituting from equations (17) and (18) and simplifying yields

$$U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{c,c}\right) < U\left(\alpha_d \pi_{d,c}\right) - U\left(\alpha_d \pi_{c,c}\right). \tag{19}$$

■

On the one hand, the size-effect of more rain in the wet period will always work in the direction of making cooperation more difficult, since

[23] Note that I assume the von Neumann-Morgenstern expected utility function to be identical to the S-shaped utility function used so far.

it increases the absolute difference between material payoffs. On the other hand, the non-linearity of the utility function creates a utility-effect that may work in the other direction. A necessary condition for the dry period being the more difficult for cooperation is thus that the gain from deviating falls on a steeper segment of the utility function in the dry than in the wet period. Thus, as Figure 7 shows, it is mainly with intermediate levels of rainfall in the dry period that it will be significantly more difficult to cooperate in the drier year.[24]

Combining this result with the main result of the analysis in Section 3, we can conclude that when the chances for cooperation are the highest, the relatively poorer period is the greatest challenge to cooperation. This is exactly what was reported in some of the empirical studies referred to in the introduction. In functioning common-pool resources, deviations mainly occur in the less productive period.

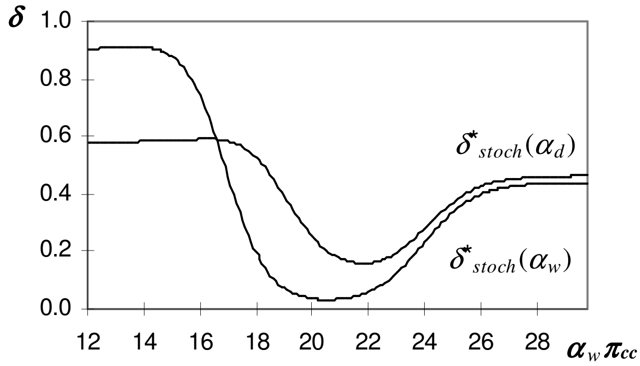

Figure 7: Stochastic variations in the amount of rainfall when $\pi_{d,c} = 1.1\pi_{c,c}$,
$\pi_{d,d} = 0.9\pi_{c,c}$, $\alpha_d = 0.9\alpha_w$, *and* $p_w = 0.5$. *Note that the x-axis gives the cooperative payoff in the wet period.*

Let us look at some implications of these results. From Figure 7, it is obvious that there is one point at which cooperation is particularly sensitive to changes in the levels of rainfall. At the intersection of the two curves, a very slight change in α can cause a regime shift in terms of in which period it is easier to cooperate.

A change in the difference between the possible levels of rainfall also have drastic effects. According to the IPCC,[25] one likely effect of global

[24]The main difference between this case and that with seasonal variations lies in the calculation of the sizes of future harvests, where the probability in the stochastic case, and the timing in the seasonal case, affect the outcome. When I let the relative probability of the levels of rainfall in the stochastic case equal the relative length of the seasons in the seasonal case, the results are very similar.

[25]See e.g. Houghton, Callander and Varney (1992).

warming is an increased variability in the climate. A simple numerical example illustrates how this could affect the management of common-pool resources. In Figure 8, I have increased the difference between the two possible levels of rainfall by letting $\alpha_d = 0.8\alpha_w$. When $\alpha_w\pi_{c,c}$= 20 in Figure 7 and $\alpha_w\pi_{c,c}$= 21.11 in Figure 8, we have the same average amount of rainfall (and thus, the same average consumption level) in the two cases. Although the variability in rainfall is only slightly increased, the effect is dramatic, especially for the critical discount factor of the dry period. Unless the farmers' discount factor is very high, this will lead to a collapse of cooperation. If, at the same time, there is a change in the average amount of rainfall, this will also alter the critical discount factors. The results of course depend on the exact specification of the model, and on how much the variability in rainfall changes. The main lesson to be learned is that the management of common-pool resources could be very sensitive to disturbances. Thus, global warming may have a hidden side-effect, which has the potential of causing substantial costs to society, both in terms of conflicts and a less efficient use of local natural resources.

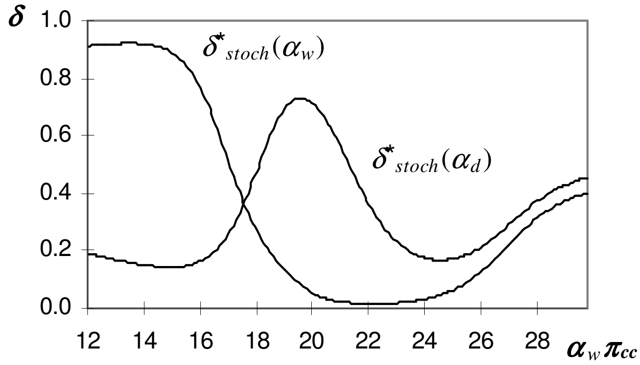

Figure 8: The effects of an increased variability in rainfall, $\pi_{d,c} = 1.1\pi_{c,c}$,
$\pi_{d,d} = 0.9\pi_{c,c}$, $\alpha_d = 0.8\alpha_w$, *and* $p_w = 0.5$.

5 Partial Cooperation

Now, if farmers know that cooperation will fail because it cannot be sustained in some periods, is there any alternative strategy which could improve their situation? There are empirical examples of common-pool resources where the users forgive fellow users for breaking the rules, if this is due to bad times. In their study of land relations in Rwanda, André and Platteau (1998) found that there was a more lenient attitude towards *voleurs par faim* (thieves out of hunger), than towards *voleurs par défaut* (vicious thieves). McKean (1986) describes how rule-breaking in a Japanese village forest was ignored if it took place in particularly bad years. Below, I examine whether introducing a more forgiving attitude will increase the chances of cooperation in the model presented above.[26]

First of all, define easy periods as periods when the critical discount factor, δ^*, is not larger than the farmers' discount factor, δ, and difficult periods as those when it is. Thus, cooperation is by definition possible in easy periods only. Let the variable θ describe whether cooperation could have been credibly sustained in period t ($\theta_t = 1$), or not ($\theta_t = 0$):

$$\theta_t = \begin{cases} 1 & if \quad \delta \geq \delta_t^*; \\ 0 & if \quad \delta < \delta_t^*. \end{cases} \tag{20}$$

Let the actions taken by each of the farmers in period t be represented by

$$a_t = \{a_{1,t},\ a_{2,t}\}. \tag{21}$$

We can now describe the history of the game at date T,

$$h_T = \{a_t,\ \theta_t\}_{t=0}^{T-1}. \tag{22}$$

Let the forgiving trigger strategy prescribe cooperation in easy periods, $\theta_t = 1$, until history for the first time contains any time period when it would have been possible to cooperate, but some farmer did not, that is, until

$$h_T = \{a_{i,t} = d,\ \theta_t = 1\} \qquad some\, t < T,\ some\, i, \tag{23}$$

and then deviate for ever. Deviations in difficult periods, $\theta_t = 0$, will be forgiven and are not punished, which means that we must adjust the expected cost of deviating to the removal of the punishment in the difficult period.[27] Substituting from equation (15), if farmers are allowed to ignore deviations in dry years because these are difficult periods, the two expected utilities in the denominator of equation (17) become

$$\begin{aligned} & E_\tau \left[U\left(\alpha_\tau \pi_{c,c}\right)\right] - E_\tau \left[U\left(\alpha_\tau \pi_{d,d}\right)\right] \\ & = p_w U\left(\alpha_w \pi_{c,c}\right) + \left(1 - p_w\right) U\left(\alpha_d \pi_{d,d}\right) \\ & - p_w U\left(\alpha_w \pi_{d,d}\right) + \left(1 - p_w\right) U\left(\alpha_d \pi_{d,d}\right) \\ & = p_w \left[U\left(\alpha_w \pi_{c,c}\right) - U\left(\alpha_w \pi_{d,d}\right)\right]. \end{aligned} \tag{24}$$

[26] As above, I will here focus on the stochastic case, and refer the analysis of the seasonal case to Appendix B.

[27] Note that we here assume a full reversal to non-cooperative behaviour in the difficult periods. Another, and perhaps more realistic, assumption would be that partial deviations are allowed. We also assume that agents can effortlessly return to cooperation after the difficult period.

The equation giving the critical discount factor changes accordingly. If the wet period is the easy period, we have (with superscript f for extended)

$$\delta_{stoch}^{f}(\alpha_w) = \frac{U(\alpha_w \pi_{d,c}) - U(\alpha_w \pi_{c,c})}{U(\alpha_w \pi_{d,c}) - U(\alpha_w \pi_{c,c}) + p_w [U(\alpha_w \pi_{c,c}) - U(\alpha_w \pi_{d,d})]}, \tag{25}$$

and the equivalent for the dry period when that is the easy period. I want to compare the lower of these with

$$\delta_{max}^{*} = max\{\delta_{stoch}^{*}(\alpha_w), \delta_{stoch}^{*}(\alpha_d)\}. \tag{26}$$

When δ_{stoch}^{f} is below δ_{max}^{*}, as for values of $\alpha_w \pi_{c,c}$ between 17 and 20.5 in Figure 9, the forgiving trigger strategy can improve the chances of cooperation. If the farmers' discount factor is between these two, they will be able to sustain cooperation by following the forgiving trigger strategy, although cooperation was not possible without forgiveness.

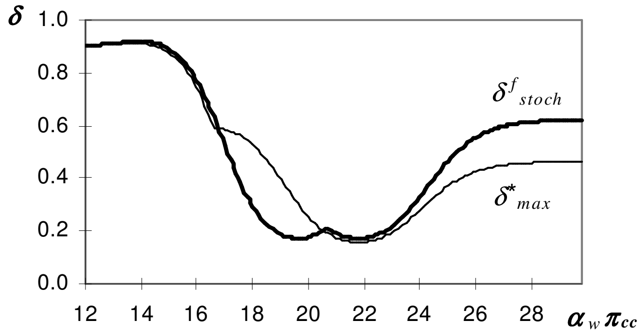

Figure 9: The effect of forgiveness with stochastic variations in the level of rainfall, $\pi_{d,c} = 1.1\pi_{c,c}$, $\pi_{d,d} = 0.9\pi_{c,c}$, $\alpha_d = 0.9\alpha_w$, *and* $p_w = 0.5$.

Note the similarity with Rotemberg and Saloner's (1986) result that price wars in oligopolies should be observed in periods of high demand. The intuition behind their result is that deviation gives a higher gain when demand is high. To avoid a total breakdown of cooperation, the oligopoly allows for a lower price in such periods, in other words partial deviations are forgiven to a certain extent. In my case, a total breakdown of cooperation can also be avoided by forgiving deviations in periods when the net gain from deviating is particularly high. The difference is that I measure the gain in terms of utility instead of material payoffs, and that I do not allow for partial deviations.

6 Final Remarks

In this paper, I have shown that when utility is non-linear, the consumption level of the users of a local common-pool resource affects the chances of cooperative management of the resource. The results correspond well with empirical studies of common-pool resource management. In particular, we saw that groups of users with an intermediate consumption level, here meaning poor but not starving, will have the best chances of cooperative management. If there is variation in the size of the resource, the relatively poorer period will constitute the largest threat to cooperation for this group.

The model builds on some assumptions that may seem rather restrictive. I confine myself to groups of users whose utility is closely related to their state of health which, in turn, is closely related to their consumption level. Furthermore, I assume mainly that they have no access to goods or credit markets. Nevertheless, when we place the local common-pool resource in one of the least developed countries, neither of these assumptions is at all implausible.

Throughout the paper, I have implicitly assumed that there is no time dependency, neither in the users' health nor in their use of the resource. Introducing time dependence, for example in the form of a stock-variable for health, or by letting the farmers' actions in one period have an effect on the productivity of the resource in future periods, would naturally affect the results. Even more restrictive is perhaps the implicit assumption that monitoring is perfect and costless, and that the only punishment is to revert to a total lack of cooperation. Judging from the empirical studies reported by, for example, Ostrom (1990), assuming that monitoring is costly, imperfect and of varying intensity, and that there are other forms of punishment of varying severity, would be more realistic. Finally, I have assumed the discount rate to be independent of the consumption level. If it were not, cooperation would become even more difficult at the lowest consumption levels, but may become easier at the highest consumption levels. Despite these restrictions, the model gives some very interesting insights into the effects of relaxing the assumption of linear utility.

It would be interesting to extend the model to more than two users, so that the larger is the amount of farmers sharing the water of the irrigation system, the less water there is for each of them. The harvest on each farmer's field would then partly depend on how the irrigation system is managed, partly on the annual rainfall and also partly on how many farmers share the water. Herein lies a possibility for an endogenous size of the user group decided by, for example, in- and out-migration of users to accommodate for seasonal or stochastic changes in the weather.

Exogenous changes in the number of users due to, for example, population growth within or outside the group, would have similar effects as the changes in the size of the resource that I have discussed in this paper.

References

Abreu, D., 1986, "Extremal Equilibria of Oligopolistic Supergames", *Journal of Economic Theory*, vol. 39, pp. 191-228.

Abreu, D., 1988, "Towards a Theory of Discounted Repeated Games", *Econometrica*, vol. 56, pp. 383-396.

André, C. and Platteau, J.-P., 1998, "Land Relations Under Unbearable Stress: Rwanda Caught in the Malthusian Trap", *Journal of Economic Behavior and Organization*, Vol. 34, pp. 1-47.

Baland, J.-M. and Platteau, J.-P., 1996, *Halting Degradation of Natural Resources: Is there a Role for Rural Communities?*, Oxford University Press, New York.

Berkes, F. and Folke, C. (eds.), 1998, *Linking Social and Ecological Systems: Management Practises and Social Mechanisms for Building Resilience*, Cambridge University Press.

Cardenas, J.-C., 2003, "Real Wealth and Experimental Cooperation: Experiments in the Field Lab", *Journal of Development Economics*, Vol. 70 (2003), pp. 263-289

Dasgupta, P., 1993, *An Inquiry into Well-Being and Destitution*, Oxford University Press, New York.

Dasgupta, P. and Mäler, K.-G. (eds.), 1997, *The Environment and Emerging Development Issues*, Vol. 1 and 2, Clarendon Press, Oxford.

Dayton-Johnson, J., 2003, "Small-holders and Water Resources: A Review Essay on the Economics of Locally-managed Irrigation", *Oxford Development Studies*, Vol. 31 (2003), pp. 315-339.

FAO, 1997, *The State of Food and Agriculture 1997*, Food and Agriculture Organization of the United Nations, Rome.

Green, E. and Porter, R., 1984, "Non-cooperative Collusion Under Imperfect Price Information", *Econometrica*, vol. 52, pp. 87-100.

Houghton, J.T., Callander, B.A. and Varney, S.K. (eds.), 1992, *Climate Change 1992, The Supplementary Report to the IPCC Scientific Assessment*, Intergovernmental Panel on Climate Change.

Jodha, N.S., 2001, *Life on the Edge: Sustaining Agriculture and Community Resources in Fragile Environments*, Oxford University Press, New Delhi.

Jodha, N.S., 1988, "Population Growth and Common Property Resources: Micro Level Evidence from India", presented at: *The Expert Group Meeting on Consequences of Rapid Population Growth in Less Developed Countries*, Population Division, United Nations, New York, August 23-26 1988.

Kranton, R.E., 1996, "Reciprocal Exchange: A Self-Sustaining System", *The American Economic Review*, Vol. 86 No 4, pp. 830-851.

McKean, M.A., 1986, "Management of Traditional Common Lands (Iriaichi) in Japan", in: *Proceedings of the Conference on Common Property Resource Management*, National Academy Press, pp. 533-89, Washington DC.

Ostrom, E., 1990, *Governing the Commons: The Evolution of Institutions for Collective Action*, Cambridge University Press.

Ostrom, E., Gardner, R. and Walker, J., 1994, *Rules, Games and Common-Pool Resources*, The University of Michigan Press.

Ravallion, M., 1997, "Famines and Economics", *Journal of Economic Literature*, Vol. XXXV, pp. 1205-1242.

Rotemberg, J. and Saloner, G., 1986, "A Supergame-Theoretic Model of Business Cycles and Price Wars during Booms", *American Economic Review*, vol. 76, pp. 390-407.

Spagnolo, G., 1996, "Intertemporal Substitutability, Risk Aversion, and Cooperation in Repeated Games", manuscript, Churchill College, Cambridge, and Stockholm School of Economics.

Spagnolo, G., 1998, "Markets and Cooperation", *Working Paper Series in Economics and Finance, Stockholm School of Economics*, No. 257.

Ternström, I., 2002a, "Income Inequality and Coordination in Common Pools", *The Management of Common-Pool Resources, Theoretical Essays and Empirical Evidence, Ph.D. Thesis in Economics, Stockholm School of Economics.*

Ternström, I., 2002b, "Incentives or Coordination? Cooperation in Irrigation Systems in Nepal", *The Management of Common-Pool Resources, Theoretical Essays and Empirical Evidence, Ph.D. Thesis in Economics, Stockholm School of Economics.*

Wade, R., 1988, *Village Republics: Economic Conditions for Collective Action in South India*, Cambridge University Press.

Wade, R., 1987, "The Management of Common Property Resources: Collective Action as an Alternative to Privatisation or State Regulation", *Cambridge Journal of Economics*, vol. 11, pp. 95-106.

Weir, D.R., 1995, "Family Income, Mortality, and Fertility on the Eve of the Demographic Transition: A Case Study of Rosny-Sous-Bois", *The Journal of Economic History*, vol. 55, pp. 1-26.

The World Bank, 1996, *World Development Report 1996*, Oxford University Press.

The World Bank, 2004, *2004 World Development Indicators,* The World Bank.

Appendix A: Seasonal Variations

We here analyse seasonal variations in the amount of water in the irrigation system. Many areas have one rainy and one dry season. As seasons vary, so does the level of rainfall and, by now, we know enough to expect this to have repercussions on the farmers' ability to cooperate. Once more, I assume that there are only two possible outcomes, wet and dry, but here, I let every other period be wet and every other period dry. Thus, there is the same variability as with stochastic variations, but none of the uncertainty of that case. This implies that it is the timing rather than the probabilities of the two possible outcomes that determines the expected loss from deviating. Thus, there will be a difference between the two seasons' costs of deviating, affecting their critical discount factors.

Assume that each farming year consists of two seasons of equal length, with α_w and α_d denoting the levels of rainfall in the wet and dry seasons, with $\alpha_w \geq \alpha_d$. In a wet season, the discounted utility of cooperating will be

$$\sum_{t=0}^{\infty} \delta^{2t} \left[U\left(\alpha_w \pi_{c,c}\right) + \delta U\left(\alpha_d \pi_{c,c}\right) \right], \tag{27}$$

and the discounted utility of deviating

$$U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{d,d}\right) + \sum_{t=0}^{\infty} \delta^{2t} \left[U\left(\alpha_w \pi_{d,d}\right) + \delta U\left(\alpha_d \pi_{d,d}\right) \right]. \tag{28}$$

To avoid deviation, it must be true that

$$\begin{aligned} & \delta^2 \left[U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{d,d}\right) \right] \\ & + \delta \left[U\left(\alpha_d \pi_{c,c}\right) - \delta U\left(\alpha_d \pi_{d,d}\right) \right] \\ & + U\left(\alpha_w \pi_{c,c}\right) - U\left(\alpha_w \pi_{d,c}\right) \\ & \geq 0. \end{aligned} \tag{29}$$

We can express the critical discount factor of a wet season[28] as

$$\begin{aligned} \delta^*_{seas}\left(\alpha_w\right) = & \frac{U\left(\alpha_d \pi_{c,c}\right) - \delta U\left(\alpha_d \pi_{d,d}\right)}{2\left(U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{d,d}\right)\right)} \\ & + \left\{ \left[\frac{U\left(\alpha_d \pi_{c,c}\right) - \delta U\left(\alpha_d \pi_{d,d}\right)}{2\left(U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{d,d}\right)\right)} \right]^2 \right. \\ & \left. + \frac{U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{c,c}\right)}{U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{d,d}\right)} \right\}^{1/2} \end{aligned} \tag{30}$$

[28] Note that we disregard the negative root.

and the critical discount factor of a dry season as

$$\delta^*_{seas}(\alpha_d) = -\frac{U(\alpha_w \pi_{c,c}) - \delta U(\alpha_w \pi_{d,d})}{2(U(\alpha_d \pi_{d,c}) - U(\alpha_d \pi_{d,d}))} + \left\{ \left[\frac{U(\alpha_w \pi_{c,c}) - \delta U(\alpha_w \pi_{d,d})}{2(U(\alpha_d \pi_{d,c}) - U(\alpha_d \pi_{d,d}))} \right]^2 + \frac{U(\alpha_d \pi_{d,c}) - U(\alpha_d \pi_{c,c})}{U(\alpha_d \pi_{d,c}) - U(\alpha_d \pi_{d,d})} \right\}^{1/2}. \quad (31)$$

A comparison of Figures 7 and 10 shows that despite these rather messy expressions, the result is very similar to the case with stochastic variations and an equal probability of the two outcomes. The slight difference is caused by the difference in the expected cost of deviating.

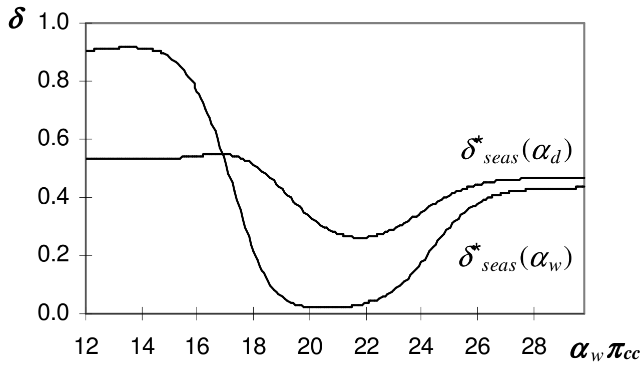

Figure 10: Seasonal variations in the level of rainfall when $\pi_{d,c} = 1.1\pi_{c,c}$, $\pi_{d,d} = 0.9\pi_{c,c}$, and $\alpha_d = 0.9\alpha_w$.

Appendix B: Partial Cooperation with Seasonal Variations

In this appendix, I confirm that a forgiving attitude can increase the chances of cooperation in the seasonal as well as in the stochastic case. When the easy season is a wet season, the forgiving trigger strategy results in the following discounted utility from cooperating,

$$\sum_{t=0}^{\infty} \delta^{2t} \left[U\left(\alpha_w \pi_{c,c}\right) + \delta U\left(\alpha_d \pi_{d,d}\right)\right], \tag{32}$$

and from deviating

$$U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_d \pi_{d,d}\right) + \sum_{t=0}^{\infty} \delta^{2t} \left[U\left(\alpha_w \pi_{d,d}\right) + \delta U\left(\alpha_d \pi_{d,d}\right)\right]. \tag{33}$$

From this, I get the critical discount factor,

$$\delta^{x}_{seas}\left(\alpha_w\right) = \frac{U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{c,c}\right)}{U\left(\alpha_w \pi_{d,c}\right) - U\left(\alpha_w \pi_{d,d}\right)}, \tag{34}$$

that is, the square root of the critical discount factor for the same amount of rainfall in the base-line case. Figure 11 shows that, as in the stochastic case, there is an intermediate size of the resource where cooperation will be facilitated if farmers are more forgiving.

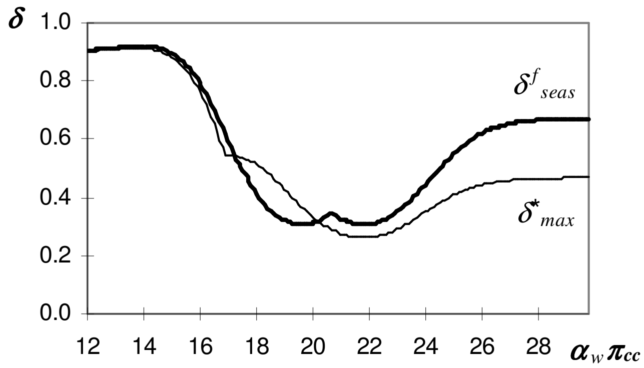

Figure 11: The effect of forgiveness with seasonal variations in the level of rainfall, when $\pi_{d,c} = 1.1\pi_{c,c}$*,* $\pi_{d,d} = 0.9\pi_{c,c}$*,* $\alpha_d = 0.9\alpha_w$*.*