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Evaluating Projects and Assessing Sustainable Development in Imperfect Economies

by

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1 Introduction

In several recent publications, it has been shown that there is a wealth-like measure that can serve as an index of intergenerational welfare. The index enables one (a) to check whether welfare will be sustained along an economic forecast, and (b) to conduct social cost-benefit analysis of policy reforms, for example, investment projects. Excepting under special circumstances, however, the index in question is not wealth itself, but an adaptation of wealth.[1]

An economy's wealth is the worth of its capital assets. As is widely recognised today, the list of assets should include not only manufactured capital, but also knowledge and skills (human capital) and natural capital. Formally, an economy's wealth is a linear combination of its capital stocks, the weights awarded to the stocks being the latter's accounting prices.

The concept of accounting prices was developed originally in the literature on economic planning (Tinbergen, 1954). The underlying presumption there was that governments are intent on maximizing social welfare. Public investment criteria were subsequently developed for economies enjoying good governance (Little and Mirrlees, 1968, 1974). In its turn, the now-extensive literature on the concept of sustainable development has been directed at societies where governments choose policies so as to maximize intergenerational welfare (e.g., Pezzey, 1992; Asheim, 1994; Hamilton and Clemens, 1999; Pezzey and Toman, 2002).

Sustainability is different from optimality. One can argue, more particularly, that that the concept of sustainable development acquires bite only when it is put to work in economies where the government, whether by design or incompetence, does not choose policies that maximise intergenerational welfare.[2] Recently the theory of intertemporal welfare indices has been extended to "imperfect economies", that is, economies suffering from weak, or even bad, governance.[3] The theory's reach therefore now extends to actual economies. The theory has also been put to use in a valuable paper by Hamilton and Clemens (1999) for judging whether in the recent past countries have invested sufficiently to expand their productive bases. Among the resources making up natural capital, only commercial forests, oil and minerals, and the atmosphere as a sink for carbon dioxide were included in the Hamilton-Clemens work. Not included were water resources, forests as agents of carbon sequestration, fisheries, air and water pollutants, soil, and biodiversity. Nor were discoveries of oil and mineral reserves taken into account. Moreover, there is a certain awkwardness in several of the steps Hamilton and Clemens took when estimating changes in the worth of an economy's capital assets. Our aim in this paper is to clarify a number of issues that arise in putting the theory of welfare indices to practical use, with the hope that the findings documented here will prove useful in future empirical work.

We are interested in three related questions: (1) How should accounting prices be estimated? (2) What index should one use to evaluate policy change? (3) Given an economic forecast, what index should one use to check whether intergenerational welfare will be sustained?

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[1] Dasgupta and Mäler (2000), Dasgupta (2001a,b), and Section 2 below.

[2] For a more extensive discussion of the distinction between optimality and sustainability, see Arrow, Daily, et al. (2002).

[3] Dasgupta and Mäler (2000) and Dasgupta (2001a,b).

In Section 2 we rehearse the basic theory. It is shown, among other things, that the same set of accounting prices should be used both for policy evaluation and for assessing whether or not intergenerational welfare along a given economic path will be sustained. In Section 3 we use the Ramsey-Solow model of national saving to illustrate the theory. The remainder of the article focuses on a number of specific assets, transacted in certain canonical institutions. The plan is as follows:

Rules for estimating accounting prices of exhaustible natural resources under both free and restricted entry are derived in Section 4. In Section 5 we show how expenditure toward the discovery of new deposits ought to be incorporated in national accounts. Section 6 develops methods for including forest depletion; and in Section 7 we show how the production of human capital could be taken into account. In Section 8 we study the production of global public goods.

If an economy were to face exogenous movements in certain variables, its dynamics would not be autonomous in time. Non-autonomy in time introduces additional problems for constructing the required welfare index. Exogenous growth in productivity, for example, is a potential reason for non-autonomous dynamics. In Section 9 we show that by suitably redefining variables it is often possible to transform a non-autonomous economic system into one that is autonomous. But such helpful transformations are not available in many other cases. In Section 10 we show how nevertheless the required welfare index can be constructed. We illustrate this by studying a small country that exports an exhaustible natural resource at a price that is time-dependent. A number of additional extensions to our basic model are discussed in the final section (Section 11).

Throughout, we assume that population remains constant. In a companion paper (Arrow, Dasgupta, and Mäler, 2002) we extend the theory to cover population change.

2. The Basic Model

2.1 Preliminaries

We assume that the economy is closed and deterministic. Time is continuous and is denoted variously by τ and t (τ, t $\geq$ 0). The horizon is taken to be infinite. For simplicity of exposition, we aggregate consumption into a single consumption good, C, and let $\underline{R}$ denote a vector of resource flows (e.g., rates of extraction of natural resources, expenditure on education and health). Labour is supplied inelastically and is normalised to be unity. Intergenerational welfare (henceforth, "social welfare") at t ($\geq$ 0) is taken to be of the Ramsey-Koopmans form,

$$W(t) = \int_t^{\infty} U(C(\tau))\exp(-\delta(\tau-t))d\tau, \qquad (\delta > 0), \tag{1}$$

where U(C) is strictly concave and monotonically increasing.

The state of the economy is represented by the vector $\underline{K}$, where $\underline{K}$ is a comprehensive list of capital assets. Let $\{C(\tau), \underline{R}(\tau), \underline{K}(\tau)\}_t^{\infty}$ be an economic programme from t to ∞. Given technological possibilities, resource availabilities, and the dynamics of the ecological-economic system, the decisions made by individual agents and consecutive governments from t onwards will determine C(τ), $\underline{R}(\tau)$, and $\underline{K}(\tau)$ --- for $\tau \geq t$ --- as functions of $\underline{K}(t)$, τ, and t. We denote these functional dependences as:

$$C(\tau) = f(\underline{K}(t), \tau, t), \tag{2}$$

$$\underline{R}(\tau) = \underline{h}(\underline{K}(t), \tau, t), \tag{3}$$

and

$$\underline{K}(\tau) = \underline{g}(\underline{K}(t), \tau, t). \tag{4}$$

Write

$$(\xi(\tau))_t^{\infty} \equiv \{C(\tau), \underline{R}(\tau), \underline{K}(\tau)\}_t^{\infty}, \text{ for } t \geq 0. \tag{5}$$

Let $\{t, \underline{K}(t)\}$ denote the set of possible t and $\underline{K}(t)$ pairs, and $\{(\xi(\tau))_t^\infty\}$ the set of economic programmes from t to infinity.

Definition 1 A "resource allocation mechanism", α, is a (many-one) mapping

$$\alpha: \{t, \underline{K}(t)\} \rightarrow \{(\xi(\tau))_t^\infty\}. \tag{6}$$

Definition 2 α is "time-autonomous" (henceforth "autonomous") if for all $\tau \geq t$, $\xi(\tau)$ is a function solely of $\underline{K}(t)$ and $(\tau\text{-}t)$.

Notice that if α is time-autonomous, then (2)-(4) can be expressed as

$$C(\tau) = f(\underline{K}(t), \tau\text{-}t), \tag{7}$$

$$\underline{R}(\tau) = \underline{h}(\underline{K}(t), \tau\text{-}t), \tag{8}$$

and $$\underline{K}(\tau) = \underline{g}(\underline{K}(t), \tau\text{-}t). \tag{9}$$

α would be non-autonomous if knowledge or the terms of trade (for a trading economy) were to change exogenously over time. In certain cases exogenous changes in population size would mean that α is not autonomous. However, by suitably redefining state variables, non-autonomous resource allocation mechanisms can sometimes be mapped into autonomous mechanisms (Section 9).

Definition 3 α is "time-consistent" if $\underline{K}(\tau') = \underline{g}(\underline{K}(t),\tau',t)$ and $\underline{K}(\tau'') = \underline{g}(\underline{K}(\tau'),\tau'',\tau')$ implies $\underline{K}(\tau'') = \underline{g}(\underline{K}(t),\tau'',t)$, or

$$\underline{g}(\underline{K}(\tau'),\tau'',\tau') = \underline{g}(\underline{K}(t),\tau'',t), \text{ for all } \tau'', \tau', \text{ and } t. \tag{10}$$

Time-consistency implies a weak form of rationality. An autonomous resource allocation mechanism, however, has little to do with rationality; it has to do with the influence of external factors (e.g., whether trade prices are changing autonomously). In what follows, we assume that α is time-consistent.

Definition 4 The "value function" reflects social welfare (equation (1)) as a function of initial capital stocks and the resource allocation mechanism. We write this as

$$W(t) = V(\underline{K}(t), \alpha). \tag{11}$$

Let K_i be the ith capital stock. We assume that V is differentiable in $\underline{K}$.[4]

Definition 5 The "accounting price", $p_i(t)$, of the ith capital stock is defined as

$$p_i(t) = \partial V(\underline{K}(t), \alpha)/\partial K_i(t) \equiv \partial V(t)/\partial K_i(t). \tag{12}$$

Note that accounting prices are defined in terms of hypothetical perturbations to an economic forecast. Specifically, the accounting price of a capital asset is the present discounted value of the perturbations to U that would arise from a marginal increase in the quantity of the asset. Given the resource allocation mechanism, accounting prices at t are functions of $\underline{K}(t)$. The prices depend also on the extent to which various capital assets are substitutable for one another. It should be noted that accounting prices of private "goods" can be negative if property rights are dysfunctional, such as those that lead to the tragedy of the commons. Note too that if α is autonomous, accounting prices are not explicit functions of time.

2.2 Marginal Rates of Substitution vs Market Observables

Using (1) and (12), it is simple to confirm that, if α is time consistent, $p_i(t)$ satisfies the dynamical equation,

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[4] Differentiability everywhere is a strong assumption. For practical purposes, however, it would suffice to assume that V is differentiable in K_i almost everywhere. The latter would appear to be a reasonable assumption even when production possibilities (including ecological processes) are realistically non-convex. See Brock and Starrett (2000), Dasgupta and Mäler (2000) and Arrow, Daily <u>et al</u>. (2002).

$dp_i(t)/dt = \delta p_i(t) - U'(C(t))\partial C(t)/\partial K_i(t).$ (13)

(13) reduces to Pontryagin equations for co-state variables in the case where α is an optimum resource allocation mechanism. In any event, we show below that, in order to study the evolution of accounting prices under simple resource allocation mechanisms, it is often easier to work directly with (12).

From (12) it also follows that accounting price ratios ($p_i(t)/p_j(t)$, $p_i(t')/p_i(t)$, and consumption discount rates (see below)) are defined as marginal social rates of substitution between goods. In an economy where the government maximizes social welfare, marginal rates of substitution among goods and services equal their corresponding marginal rates of transformation. As the latter are observable in market economies (e.g., border prices for traded goods in an open economy), accounting prices are frequently defined in terms of marginal rates of transformation among goods and services. However, marginal rates of substitution in imperfect economies do not necessarily equal the corresponding marginal rates of transformation. A distinction therefore needs to be made between the ingredients of social welfare and "market observables". Using market observables to infer social welfare can be misleading in imperfect economies. That we may have to be explicit about welfare parameters (e.g., δ and the elasticity of U) in order to estimate marginal rates of substitution in imperfect economies is not an argument for pretending that the economies in question are not imperfect after all. In principle it could be hugely misleading to use the theory of optimum control to justify an exclusive interest in market observables.

2.3 Sustainable Development

IUCN (1980) and World Commission (1987) introduced the concept of sustainable development. The latter publication defined sustainable development to be "... development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (World Commission, 1987: 43). Several formalizations are consistent with this phrase.[5] We develop our analysis on an interpretation based on maintaining social welfare, rather than on maintaining the economy's productive base. We then show that the requirement that economic development be sustainable implies, and is implied by, the requirement that the economy's productive base be maintained (Theorem 1). The equivalence result gives intellectual support for the definition of sustainability we adopt in:

Definition 6 The economic programme $\{C(t), \underline{R}(t), \underline{K}(t)\}_0^\infty$ corresponds to a "sustainable development path" at t if $dV(t)/dt \geq 0$.

Notice that the above criterion does not attempt to identify a unique economic programme. In principle any number of technologically and ecologically feasible economic programmes could satisfy the criteria. On the other hand, if substitution possibilities among capital assets are severely limited and technological advances are unlikely to occur, it could be that there is no sustainable economic programme for an economy. The concept of sustainability helps us to better understand the character of economic programmes. As a notion it is particularly useful for judging the performance of imperfect economies.

We may now state

Theorem 1 $dV(t)/dt = \partial V(t)/\partial t + \Sigma_i[p_i(t)dK_i(t)/dt].$ (14)

Moreover, if α is autonomous, then $\partial V(t)/\partial t = 0$, and (14) reduces to,

$dV(t)/dt = \Sigma_i[p_i(t)dK_i(t)/dt].$ (15)

Call the right-hand-side (RHS) of (15) "genuine" investment at t. Equation (15) states that at each

[5] Pezzey (1992) contains an early, but thorough, account.

date the rate of change in social welfare equals genuine investment.

Integrating (15) yields

Theorem 2 For all $T \geq 0$,

$$V(T) - V(0) = \Sigma_i[p_i(T)K_i(T) - p_i(0)K_i(0)] - {}_0\!\int^T[\Sigma_i\{dp_i(\tau)/d\tau\}K_i(\tau)]d\tau. \quad (16)$$

Theorems 1 and 2 are equivalence results. They do not say whether α gives rise to an economic programme along which social welfare is sustained. For example, it can be that an economy is incapable of achieving a sustainable development path, owing to scarcity of resources, limited substitution possibilities among capital assets, or whatever. Or it can be that the economy is in principle capable of achieving a sustainable development path, but that because of bad government policies social welfare is unsustainable along the path that has been forecast.

2.4 Project Evaluation

Imagine that even though the government does not optimize, it can bring about small changes to the economy by altering the existing resource allocation mechanism in minor ways. The perturbation in question could be small adjustments to the prevailing structure of taxes for a short while, or it could be minor alterations to the existing set of property rights for a brief period, or it could be a small public investment project. Call any such perturbation a "policy reform".[6]

Consider as an example an investment project. It can be viewed as a perturbation to the resource allocation mechanism α for a "brief" period (the lifetime of the project), after which the mechanism reverts back to its earlier form. We consider projects that are small relative to the size of the economy. How should they be evaluated?

For simplicity of exposition, we suppose there is a single manufactured capital good (K) and a single extractive natural resource (S). The aggregate rate of extraction is denoted by R. Let the project's lifetime be the period [0, T]. Denote the project's output and inputs at t by the vector $(\Delta Y(t), \Delta L(t), \Delta K(t), \Delta R(t))$.[7]

The project's acceptance would perturb consumption under α. Let the perturbation at t (≥ 0) be $\Delta C(t)$. It would affect U(t) by the amount $U'\Delta C(t)$. However, because the perturbation includes all "general equilibrium effects", it would be tiresome if the project evaluator were required to estimate $\Delta C(t)$ for every project that came up for consideration. Accounting prices are useful because they enable the evaluator to estimate $\Delta C(t)$ indirectly. Now, it is most unlikely that consumption and investment have the same accounting price in an imperfect economy. So we divide $\Delta Y(t)$ into two parts: changes in consumption and in investment in manufactured capital. Denote them as $\Delta C(t)$ and $\Delta(dK/dt)$, respectively.

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[6] Over the years economic evaluation of policy reform in imperfect economies has been discussed by a number of economists (e.g., Meade, 1955; Dasgupta, Marglin, and Sen, 1972; Mäler, 1974; Blitzer, Dasgupta, and Stiglitz, 1981; Starrett, 1988; Ahmad and Stern, 1990; and Dreze and Stern, 1990), but they did not develop a formal account for intertemporal economies.

[7] If the project has been designed efficiently, we would have:

$$\Delta Y(t) = (\partial F/\partial K)\Delta K(t) + (\partial F/\partial L)\Delta L(t) + (\partial F/\partial R)\Delta R(t),$$

where F is an aggregate production function (Y = F(K,L,R)). The analysis that follows in the text does not require the project to have been designed efficiently. As we are imagining that aggregate labour supply is fixed, $\Delta L(t)$ used in the project would be the same amount of labour displaced from elsewhere.

U is the unit of account.[8] Let w(t) denote the accounting wage rate. Next, let $\hat{q}(t)$ be the accounting price of the extractive resource input of the project and $\lambda(t)$ the social cost of borrowing capital (i.e., $\lambda(t) = \delta - [dp(t)/dt]/p(t)$).[9]

From the definition of accounting prices, it follows that:

$$\int_0^\infty [U'\Delta C(\tau)]d\tau = \int_0^T [U'\Delta C(\tau)+p(\tau)\Delta(dK(\tau)/d\tau)-w(\tau)\Delta L(\tau)-\lambda(\tau)p(\tau)\Delta K(\tau)-\hat{q}(\tau)\Delta R(\tau)]\exp[-\delta\tau]d\tau \quad (17)$$

But the RHS of (17) is the present discounted value of social profits from the project (in utility numeraire). Moreover, $\int_0^\infty [U'\Delta C(\tau)]d\tau = \Delta V(0)$, the latter being the change in social welfare if the project were accepted. We may therefore write (17) as,

$$\Delta V(0) = \int_0^T [U'\Delta C(\tau)+p(\tau)\Delta(dK(\tau)/d\tau)-w(\tau)\Delta L(\tau)-\lambda(\tau)p(\tau)\Delta K(\tau)-\hat{q}(\tau)\Delta R(\tau)]\exp[-\delta\tau]d\tau \quad (18)$$

Equation (18) leads to the well-known criterion for project evaluation:

Theorem 3 A project should be accepted if and only if the present discounted value of its social profits is positive.

2.5 Evaluating Projects and Assessing Sustainability

In order to make the connection between Theorems 1 and 3, we study how genuine investment is related to changes in future consumption brought about by it. Imagine that the capital base at t is not $\underline{K}(t)$ but $\underline{K}(t)+\Delta\underline{K}(t)$, where as before, Δ is an operator signifying a small difference. In the obvious notation,

$$V(\alpha, \underline{K}(t)+\Delta\underline{K}(t)) - V(\alpha, \underline{K}(t)) \approx \int_t^\infty U'\Delta C(\tau)\exp[-\delta(\tau-t)]d\tau. \quad (19)$$

Now suppose that at t there is a small change in α, but only for a brief moment, Δt, after which the resource allocation mechanism reverts back to α. We write the increment in the capital base at $t+\Delta t$ consequent upon the brief increase in genuine investment as $\Delta\underline{K}(t)$. So $\Delta\underline{K}(t)$ is the consequence of an increase in genuine investment at t and $\underline{K}(t+\Delta t)+\Delta\underline{K}(t)$ is the resulting capital base at $t+\Delta t$. Let Δt tend to zero. From equation (19) we obtain

Theorem 4 Genuine investment measures the present discounted value of the changes to consumption

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[8] Dasgupta, Marglin, and Sen (1972) and Little and Mirrlees (1974), respectively, developed their accounts of social cost-benefit analysis with consumption and government income as numeraire. Which numeraire one chooses is, ultimately, not a matter of principle, but one of practical convenience.

[9] The following is how $\hat{q}(t)$ could in principle be estimated: Suppose other things being the same, $\Delta R(t)$ is the chantge in resource use. Let this change cause displacements $\Delta C(t)$, $\Delta(dK(t)/dt)$, $\Delta(dS(t)/dt)$ in consumption, net capital accumulation, and net growth in the natural-resource base, respectively. Denote by q(t) the accounting price of the resource <u>in situ</u>. We then have:

$$\hat{q}(t)\Delta R(t) = U'\Delta C(t) + p(t)\Delta(dK(t)/dt) + q(t)\Delta(dS(t)/dt).$$

As an approximation, $\hat{q}(t)$ could be estimated from the willingness-to-pay for a unit of the resource in production, through the use, for example, of contingent-valuation techniques (Freeman, 1993). Notice
that if manufactured capital were to depreciate at a constant rate, say γ, the social cost of borrowing capital would be $\lambda(t) = \delta + \gamma - [dp(t)/dt]/p(t)$.

At a full-optimum, $p(t)\partial F/\partial R(t) = q(t) = \hat{q}(t)$, and $U' = p(t)$.

services brought about by it.[10]

2.6 Numeraire

So far we have taken utility to be the unit of account. In applied welfare economics, however, it has been found useful to express benefits and costs in terms of current consumption. It will pay to review the way the theory being developed here can be recast in consumption numeraire. For simplicity of exposition, assume that there is a single commodity, that is, an all-purpose durable good that can be consumed or reinvested for its own accumulation. Assume too that utility is iso-elastic and that the elasticity of marginal utility is η. Define $\overline{p}(t)$ to be the accounting price of the asset at t in terms of consumption at t; that is,

$$\overline{p}(t) = p(t)/U'(C(t)). \quad (20)$$

It follows from (20) that,

$$[d\overline{p}(t)/dt]/\overline{p}(t) = [dp(t)/dt]/p(t) + \eta[dC(t)/dt]/C(t). \quad (21)$$

Let $\rho(t)$ be the social rate of discount in consumption numeraire. $\rho(t)$ is sometimes referred to as the consumption rate of interest (Little and Mirrlees, 1974). From (1),

$$\rho(t) = \delta + \eta[dC(t)/dt]/C(t).^{11} \quad (22)$$

Using (22) in (21) we obtain the relationship between the asset's prices in the two units of account:

$$[d\overline{p}(t)/dt]/\overline{p}(t) = [dp(t)/dt]/p(t) + \rho(t) - \delta. \quad (23)$$

3 Illustration

It will prove useful to illustrate the theory by means of a simple example, based on Ramsey (1928) and Solow (1956). As in Section 2.6, imagine that there is an all-purpose durable good, whose stock at t is K(t) (≥ 0). The good can be consumed or reinvested for its own accumulation. There are no other assets. Population size is constant and labour is supplied inelastically. Write output (GNP) as Y. Technology is linear. So $Y = \mu K$, where $\mu > 0$. μ is the output-wealth ratio. GNP at t is $Y(t) = \mu K(t)$.

Imagine that a constant proportion of GNP is saved at each moment. There is no presumption though that the saving rate is optimum; rather, it is a behavioural characteristic of consumers, reflecting their response to an imperfect credit market. Other than this imperfection, the economy is assumed to function well. At each moment expectations are fulfilled and all markets other than the credit market clear. This defines the resource allocation mechanism, α. Clearly, α is autonomous in time. We now characterise α explicitly.

Let the saving ratio be s ($0 < s < 1$). Write aggregate consumption as C(t). Therefore,

$$C(t) = (1-s)Y(t) = (1-s)\mu K(t). \quad (24)$$

Capital is assumed to depreciate at a constant rate γ (> 0). Genuine investment is therefore,

$$dK(t)/dt = (s\mu-\gamma)K(t). \quad (25)$$

K(0) is the initial capital stock. The economy grows if $s\mu > \gamma$, and shrinks if $s\mu < \gamma$. To obtain a feel for orders of magnitude, suppose $\gamma = 0.05$ and $\mu = 0.25$. The economy grows if $s > 0.2$, and shrinks if $s < 0.2$.

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[10] Theorem 4 is, of course, familiar for economies where the government maximises social welfare (see Arrow and Kurz, 1970).

[11] To prove (22) notice that, by definition, $\rho(t)$ satisfies the equation

$$U'(C(t))\exp(-\delta t) = U'(C(0))\exp(-\textstyle\int_0^t[\rho(\tau)]d\tau).$$

If we differentiate both sides of the above equation with respect to t, (22) follows.

Integrating (25), we obtain,

$$K(\tau) = K(t)\exp[(s\mu-\gamma)(\tau-t)], \qquad \text{for all } \tau \text{ and } t,\ \tau \geq t \geq 0, \tag{26}$$

from which it follows that,

$$C(\tau) = (1-s)\mu K(\tau) = (1-s)\mu K(t)\exp[(s\mu-\gamma)(\tau-t)], \text{ for all } \tau \text{ and } t,\ \tau \geq t \geq 0. \tag{27}$$

If the capital stock was chosen as numeraire, wealth would be K(t), and NNP would be $(\mu-\gamma)K(t)$. Each of wealth, GNP, NNP, consumption and genuine investment expands at the exponential rate $(s\mu - \gamma)$ if $s\mu > \gamma$; they all contract at the exponential rate $(\gamma - s\mu)$ if $s\mu < \gamma$. We have introduced capital depreciation into the example so as to provide a whiff (albeit an artificial whiff) of a key idea, that even if consumption is less than GNP, wealth declines when genuine investment is negative. Wealth declines when consumption exceeds NNP.

Current utility is U(C(t)). Consider the iso-elastic form:

$$U(C) = -C^{-(\eta-1)}, \quad \text{where } \eta > 1. \tag{28}$$

δ is the social rate of discount if utility is numeraire. Let $\rho(t)$ be the social rate of discount if consumption is the unit of account. It follows that

$$\rho(t) = \delta + \eta[dC(t)/dt]/C(t) = \delta + \eta(s\mu - \gamma). \tag{29}$$

The sign of $\rho(t)$ depends upon the resource allocation mechanism α. In particular, $\rho(t)$ can be negative. To see why, suppose the unit of time is a year, $\delta = 0.03$, $\gamma = 0.04$, $s = 0.10$, $\eta = 2$, and $\mu = 0.20$. Then (29) says that $\rho(t) = -0.01$ per year.[12]

Social welfare at t is,

$$V(t) = \int_t^\infty U(C(\tau))\exp[-\delta(\tau-t)]d\tau. \tag{30}$$

Using (28) and (29) in (30), we have:

$$V(t) = -[(1-s)\mu K(t)]^{-(\eta-1)}\int_t^\infty \exp\{-[(\eta-1)(s\mu-\gamma)+\delta](\tau-t)\}d\tau,$$

or, assuming that $[(\eta-1)(s\mu-\gamma)+\delta] > 0$,

$$V(t) = -[(1-s)\mu K(t)]^{-(\eta-1)}/[(\eta-1)(s\mu-\gamma)+\delta]. \tag{31}$$

V is differentiable in K everywhere. Moreover, $\partial V(t)/\partial t = 0$. Equations (26) and (31) confirm Proposition 1.[13]

We turn now to accounting prices.

(i) Utility Numeraire

Begin by taking utility to be numeraire. Let p(t) be the accounting price of capital. Now

$$p(t) \equiv \partial V(t)/\partial K(t) \equiv \int_t^\infty U'(C(\tau))[\partial C(\tau)/\partial K(t)]\exp[-\delta(\tau-t)]d\tau. \tag{32}$$

Using (31) in (32) we have,

$$p(t) = (\eta-1)[(1-s)\mu]^{-(\eta-1)}K(t)^{-\eta}/[(\eta-1)(s\mu-\gamma)+\delta]. \tag{33}$$

Using equations (26), (27), (31), and (33) it is simple to check that $p(t) \neq U'(C(t))$, except when $s = (\mu+(\eta-1)\gamma-\delta)/\mu\eta$. Let s* be the optimum saving rate. From equation (31) we have,

$$s^* = (\mu+(\eta-1)\gamma-\delta)/\mu\eta. \tag{34}$$

Note that $p(t) < U'(C(t))$ if $s > s^*$, which means there is excessive saving. Conversely, $p(t) > U'(C(t))$ if $s < s^*$, which means there is excessive consumption.

(ii) Consumption Numeraire

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[12] These are not fanciful figures. Per capita income in a number of countries in sub-Saharan Africa declined over the past three decades at as high a rate as 1 percent per year.

[13] As the economy has a single asset, Proposition 3 is trivially true.

Write $\bar{p}(t) = p(t)/U'(C(t))$. (35)

Using (32) in (35) yields

$\bar{p}(t) = \int_t^{\infty}[U'(C(\tau))/U'(C(t))][\partial C(\tau)/\partial K(t)]\exp[-\delta(\tau-t)]d\tau$. (36)

Now use (27), (28) and (36) to obtain

$\bar{p}(t) = \int_t^{\infty}(1-s)\mu\exp(-\rho(\tau-t))\exp(s\mu-\gamma)d\tau$, (37)

where $\rho = \delta + \eta(s\mu-\gamma)$.

(37) simplifies to:

$\bar{p}(t) = (1-s)\mu/[\rho-(s\mu-\gamma)]$. (38)

Observe that $\bar{p}(t) > 1$ (resp. < 1) if $s < s^*$ (resp. $> s^*$).[14]

In order to obtain a sense of orders of magnitude, suppose $\eta = 2$, $\mu = 0.20$, $\gamma = 0.05$, and $\delta = 0$. From (34) we have $s^* = 0.625$. Now imagine that $s = 0.40$ (by Ramsey's criterion, this is undersaving!). Using (29) we have $\rho = 0.06$ per unit of time. So (38) reduces to $\bar{p}(t) = 4$. In other words, a saving rate that is approximately 30 percent short of the optimum corresponds to a high figure for the accounting price of investment: investment should be valued four times consumption.

Although intergenerational equity is nearly always discussed in terms of the rate at which future well-being is discounted (see, e.g. Portney and Bryant, 1998), equity would be more appropriately discussed in terms of the curvature of U. Let the unit of time be a year. Suppose $\gamma = 0$, $\delta = 0.02$, and $\mu = 0.32$. Consider two alternative values of η: 25 and 50. It is simple to confirm that $s^* = 0.038$ if $\eta = 25$ and $s^* = 0.019$ if $\eta = 50$. Intergenerational equity in both consumption and welfare (the latter is a concave function of the former) can be increased indefinitely by making η larger and larger: C(t) becomes "flatter" as η is increased. In the limit, as η goes to infinity, s^* tends to γ (equation (34)), which reflects the Rawlsian maxi-min consumption as applied to the intergenerational context.[15]

Having illustrated the theory by means of an example, we now proceed to study extensions of the basic model by focussing on particular capital assets and a few well known types of institutional imperfections.

4 Exhaustible Resources: the closed economy

Accounting prices of exhaustible resources when depletion rates are optimal have been much studied (e.g., Dasgupta and Heal, 1979; see below). What is the structure of their accounting prices when resources are instead common pools?

Two property-rights regimes suggest themselves: open access and restricted entry. They in turn need to be compared to an optimum regime. It is simplest if we avoid a complete capital model. So we resort to a partial equilibrium world: income effects are assumed to be negligible. Let R(t) be the quantity extracted at t. Income is the numeraire. Let U(R) be the area under the demand curve below R. So U′(R) is taken to be the market demand function. U is taken to be an increasing and strictly concave function of R for positive values of R. In order to have a notation that is consistent with the one in the foregoing example, we take the social rate of interest to be an exogenously given constant, ρ.

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[14] A special case of formula (38) appears in Dasgupta, Marglin and Sen (1972). However, unlike our present work, the earlier publication did not provide a rigorous welfare economic theory for imperfect economies.

[15] Solow (1974) and Hartwick (1977) are the key articles on this limiting case.

Social welfare at t is,

$$V(t) = {}_t\int^{\infty} U(R(\tau))\exp[-\rho(\tau-t)]d\tau. \qquad (39)$$

Let S(t) be the stock. Then,

$$dS(t)/dt = -R(t). \qquad (40)$$

4.1 The Optimum Regime

Consider first an optimizing economy. Assume that extraction is costless (constant unit extraction cost can be introduced easily). Let p*(t) denote the accounting price of the resource underground (equivalently, the Hotelling rent, or the optimum depletion charge per unit extracted). We know that

$$dp^*(t)/dt = \rho p^*(t). \qquad (41)$$

This is the Hotelling Rule. Moreover, optimum extraction, R*(t), must satisfy the condition,

$$U'(R(t)) = p^*(t). \qquad (42)$$

Assume that U′(R) is iso-elastic:

$$U(R) = -R^{-(\eta-1)}, \quad \text{where } \eta > 1. \qquad (28a)$$

Then

$$R^*(t) = (\rho/\eta)S(0)\exp(-\rho t/\eta). \qquad (43)$$

We next consider the two imperfect regimes.

4.2 Restricted Entry

For vividness, assume that there are N identical farmers (i, j = 1,2,...,N), drawing from an aquifer. Extraction is costless. We model the situation in the following way:[16]

At t, farmer i owns a pool of size $S_i(t)$. Each pool is separated from every other pool by a porous barrier. Water percolates from the pool which is larger to the one which is smaller. Let λ_{ij} (> 0), be the rate at which water diffuses from pool i to pool j. We assume that $\lambda_{ij} = \lambda_{ji}$. Denote by $R_i(t)$ the rate at which i draws from his pool. There are then N depletion equations:

$$dS_i(t)/dt = \Sigma_{N\backslash i}[\lambda_{ji}\{S_j(t) - S_i(t)\}] - R_i(t), \qquad (44)$$

where "$\Sigma_{N\backslash i}$" denotes summation over all j other than i.

The payoff function for farmer i at time t is

$$ {}_t\int^{\infty} U(R_i(\tau))\exp[-\rho(\tau-t)]d\tau. \qquad (45)$$

Farmers play non-cooperatively. For tractablity, we study an open loop solution: Farmers are assumed to be naive (when computing his own optimum extraction rates, each takes the others' extraction rates as given).

Let $p_i(t)$ be the (spot) personal accounting price of a unit of i's own resource pool. The present value Hamiltonian for i's optimization problem would then be,

$$H(t) = U(R_i(t))\exp(-\rho t) + [\Sigma_{N\backslash i}(\lambda_{ji}\{S_j(t) - S_i(t)\}) - R_i(t)]p_i(t)\exp(-\rho t). \qquad (46)$$

It follows from (46) that $p_i(t)$ obeys the equation,

$$dp_i(t)/dt = [\rho + \Sigma_{N\backslash i}(\lambda_{ji})]p_i(t). \qquad (47)$$

For notational simplicity, assume that $\lambda_{ij} = \lambda$ for all i, j. Then (47) reduces to

$$dp_i(t)/dt = [\rho + (N-1)\lambda]p_i(t). \qquad (48)$$

Write $[\rho + (N-1)\lambda] = \beta$. We conclude that the rush to extract because of insecure property rights amounts

[16] Mckelvey (1980) has studied a special case of the model of diffusion developed below.

to each extractor using an implicit discount rate, β, which is in excess of the social discount rate ρ.[17] Assume now that the elasticity of demand is a constant, η (> 1). The optimum rate of extraction is therefore,

$$R(\tau) = (\beta/\eta)S(t)\exp[-\beta(\tau-t)/\eta)], \text{ for all } \tau \geq t. \quad (49)$$

In order to have a meaningful problem, we take it that $\beta/\eta > \beta-\rho$ (see below).

Let p(t) be the resource's (social) accounting price. We know that $p(t) = \partial V(t)/\partial S(t)$. Using (43), it follows that,

$$p(t) = \int_t^\infty U'(R(\tau))[\partial R(\tau)/\partial S(t)]\exp[-\rho(\tau-t)]d\tau. \quad (50)$$

Write $\bar{p}(t) = p(t)/U'(R(t))$. Then (48) and (50) imply,

$$\bar{p}(t) = (\beta/\eta)\int_t^\infty \exp[-(\rho-\beta(\eta-1)\eta)](\tau-t)]d\tau, \quad (51)$$

or,

$$\bar{p}(t) = \beta/[\beta - \eta(\beta-\rho)] > 1. \quad (52)$$

(Notice that $\bar{p}(t) = 1$ if $\beta = \rho$.)

As a numerical illustration, consider the case where $\rho = 0.06$, $\beta = 0.10$, and $\eta = 2$. In this case, $\bar{p}(t) = 5$, which reflects a considerable imperfection in the resource allocation mechanism in question: the resource's accounting price is five times its market price.

4.3 Open Access

We next study an open-access pool. To have a meaningful problem, we now assume that extraction is costly. For simplicity, let the unit extraction cost be a constant k (> 0). The equilibrium extraction rate, R(t), is then the solution of the equation,

$$U'(R(t)) = k. \quad (53)$$

Equation (53) confirms that, for any given level of reserves, there is excessive extraction: the equation implies that Hotelling rents are dissipated entirely. Let $\bar{R}$ be the solution of (53). We then have,

$$dS(t)/dt = -\bar{R}. \quad (54)$$

Reserves remain positive for a period $T = S/\bar{R}$. Let us normalize utility by setting $U(0) = 0$. It follows that,

$$V(t) = \int_t^{(t+S(t)/\bar{R})}[U(\bar{R}) - k\bar{R}]\exp(-\rho(\tau-t))d\tau. \quad (55)$$

Let p(t) be the accounting price of the unextracted resource. Then,

$$p(t) = \partial V(t)/\partial S(t) = [(U(\bar{R}) - k\bar{R})/\bar{R}]\exp(-\rho S(t)/\bar{R}) > 0. \quad (56)$$

Write $\bar{p}(t) = p(t)/U'(\bar{R})$, which is the ratio of the resource's shadow price to its unit extraction cost. Then, from (53) and (56),

$$\bar{p}(t) = [(U(\bar{R}) - k\bar{R})/k\bar{R}]\exp(-\rho S(t)/\bar{R}) > 0. \quad (57)$$

(57) resembles a formula proposed by El Sarafy (1989) for estimating depletion charges.[18] The charge is positive because an extra unit of water in the aquifer would extend the period of extraction. Notice that $\bar{p}(t)$ is bounded above by the ratio of the Marshallian consumer surplus to total extraction cost; furthermore, it increases as the aquifer is depleted and attains its upper bound at the date at which the pool is exhausted. If reserves are large, $\bar{p}(t)$ is small, and free access involves no great loss - a familiar result.

What are plausible orders of magnitude? Consider the linear demand function. Assume therefore that

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[17] In the limit, as λ tends to infinity, β tends to infinity, implying that depletion is instantaneous.

[18] Hartwick and Hageman (1993) have a fine discussion that links El Sarafy's formula to Hicks' formulation of the concept of national income (Hicks, 1942).

$$U(R) = aR - bR^2, \qquad \text{where } a > k \text{ and } b > 0. \tag{58}$$

From (53) and (58),

$$\overline{R} = (a - k)/2b. \tag{59}$$

Substituting (58) and (59) in (57),

$$\overline{p}(t) = (\overline{R}/k)\exp[-\rho S(t)/\overline{R}], \tag{60}$$

or, $$\overline{p}(t) = [(a-k)/2k]\exp[-2b\rho S(t)/(a-k)]. \tag{61}$$

Equation (61) says that

$$\overline{p} \geq 1 \text{ iff } \rho S \leq [(a-k)/2b]\ln[(a-k)/2k]. \tag{62}$$

(60) (alternatively, (61)) expresses the magnitude of $\overline{p}$ in terms of the parameters of the model. Suppose, for example, that $\rho = 0.02$ per year, $S/\overline{R} = 100$ years (i.e. at the current rate of extraction, the aquifer will be exhausted in 100 years), $(a-k)/2k = 20$ (e.g., k = \$0.50 and (a-k) = \$20). Then

$$\overline{p} = 20\exp(-2) \approx 7. \tag{63}$$

We should conclude that the value to be attributed to water at the margin is high (about 7 times extraction cost). As the date of exhaustion gets nearer, the accounting price rises to its upper bound, 20.

5 Exploration and Discoveries

How should one account for expenditure on explorations of new deposits of exhaustible resources? We imagine that the rate at which new reserves are discovered, N, is an increasing function of (1) current expenditure on explorations, E, (2) the accumulated expenditure on explorations, M, and (3) accumulated extraction, Z(t). Denote the discovery function be N(E(t),M(t),Z(t)), where

$$dM(t)/dt = E(t), \tag{64}$$

and $$dZ(t)/dt = R(t). \tag{65}$$

We revert to the model containing one manufactured capital good, K, and an exhaustible natural resource, S. In the familiar notation, Y = F(K, R) is taken to be the aggregate production function. The remaining equations of motion are,

$$dK(t)/dt = F(K(t), R(t)) - C(t) - E(t). \tag{66}$$

$$dS(t)/dt = N(E(t),M(t),Z(t)) - R(t). \tag{67}$$

The model has four capital assets K, S, M, and Z. Their accounting prices are denoted by p_K, p_S, p_M, and p_Z, respectively. Social welfare is given by (1). From Theorem 1, we have

$$dV(t)/dt = p_K(t)[F(K(t),R(t))-C(t)-E(t)] + p_S(t)[N(E(t),M(t),Z(t))-R(t)] + p_M(t)E(t) + p_Z(t)R(t). \tag{68}$$

There are two cases to consider:

(A) Assume that $\partial N/\partial M = 0$, which implies that $p_M=0$. In this case genuine investment means the sum of investment in manufactured capital and changes in proven reserves (N(t)-R(t)). Thus, exploration costs should not be regarded as investment. Note though that if $\partial N/\partial Z > 0$, we would expect that $p_Z(t) > 0$, which means that discoveries of new reserves should be valued differently from existing reserves. In particular, (68) implies that if $p_Z(t) > p_S(t)$, depletion has positive indirect value.

Consider now the special case where the mining industry optimizes. Then $p_K(t)=p_S(t)\partial N/\partial E$. If, in addition, $p_S(t)N(t)$ can be approximated by $p_K(t)E(t)$, one could exclude discoveries of new reserves from genuine investment, but regard instead exploration costs as part of that investment.

(B) Suppose $\partial N/\partial M > 0$. If the industry optimizes, we have

$$p_K(t) = p_M(t) + p_S(t)\partial N/\partial E, \tag{69}$$

and so $p_K(t) > p_M(t)$. It follows that genuine investment should now include not only new discoveries and investment in manufactured capital (as in Case A), but also exploration costs, using an accounting price that is less than that of manufactured capital.

6 Forests and Trees

As stocks, forests offer a multitude of services. Here we focus on forests as a source of timber. Hamilton and Clemens (1999) regard the accounting value of forest depletion to be the stumpage value (price minus logging costs) of the quantity of commercial timber and fuelwood harvested in excess of natural regeneration rates. This is an awkward move, since the authors do not say what is intended to happen to the land being deforested. For example, if the deforested land is converted into an urban sprawl, the new investment in the sprawl would be accounted for in conventional accounting statistics.[19] But if it is intended to be transformed into farmland, matters would be different: the social worth of the land as a farm should be included as an addition to the economy's stock of capital assets. In what follows, we consider the simple case where the area is predicted to remain a forest.

Let the price of timber, in consumption numeraire, be unity and let ρ (assumed constant) be the social rate of discount. Holding all other assets constant, if B(t) is aggregate forest land at, we may express social welfare as V(B(t)). The accounting price of forest land is then $\partial V(t)/\partial B(t)$, which we write as s(t).

Consider a unit of land capable of supporting a single tree and its possible successors. If the land is virgin, if a seed is planted at t=0, if F(T) is the timber yield of a tree aged T, and if T is the rotation cycle, then the present discounted value of the land as a tree-bearer is,

$$s(0) = F(T)\exp(-\rho T)/[1-\exp(-\rho T)]. \qquad (70)$$

Suppose instead that at t=0 the piece of land in question houses a tree aged τ. What is the value of the land?

If the cycle is expected to be maintained, we have

$$\bar{s}(0) = F(T)\exp[-\rho(T-\tau)]/\{1-\exp[-\rho(T-\tau)]\}. \qquad (71)$$

If instead the tree is logged now, but the cycle is expected to be maintained, the value of the land, after the tree has been felled, is given by (70). Depreciation of the forest, as a capital asset, is the difference between (71) and (70).

7 Human Capital

To develop an accounting framework for knowledge acquisition and skill formation, consider a modified version of the basic model of Section 2. In particular, the underlying resource allocation mechanism is assumed to be autonomous. As labour supply is assumed to be inelastic and population is constant, we may as well then normalize by regarding the labour-hours supplied to be unity.

Production of the consumption good involves physical capital, $K_1(t)$, and human capital, $H_1(t)$. Here, $H_1(t)$ is to be interpreted to be the human capital embodied in those who work in the sector producing the consumption good. Thus, if Y(t) is output of the consumption good,

$$Y(t) = F(K_1(t), H_1(t), \qquad (72)$$

where F is an increasing function of its arguments.

Assume that human capital is produced with the help of physical capital, $K_2(t)$, and human capital,

[19] It should be noted though that the value of urban land would more than just the new investment: there is a contribution to the value (which could be of either sign) arising from changes in population density - both in the newly developed property and in places of origin of those who migrate to the property.

$H_2(t)$, and that, owing to mortality, it depreciates at a constant rate, γ. Output of human capital is given by the technology

$$G(K_2(t), H_2(t)), \qquad (73)$$

where G is an increasing function of its arguments.

By assumption, all individuals at a given moment of time have the same amount of human capital. Therefore, $H_1(t)/[H_1(t)+H_2(t)]$ is the proportion of people employed in the sector producing the consumption good. Let the total quantity of human capital be H. It follows that

$$H_1(t) + H_2(t) = H(t). \qquad (74)$$

Write

$$K_1(t) + K_2(t) = K(t). \qquad (75)$$

For simplicity of exposition, we assume that physical capital does not depreciate. Accumulation of physical capital can be expressed as

$$dK(t)/dt = F(K_1(t), H_1(t)) - C(t), \qquad (76)$$

and the accumulation of human capital as

$$dH(t)/dt = G(K_2(t), H_2(t)) - \gamma H(t). \qquad (77)$$

Since the resource allocation mechanism, α, is assumed to be autonomous, we have

$$V(t) = V(\alpha, K_1(t), K_2(t), H_1(t), H_2(t). \qquad (78)$$

Let $p_1(t)$ and $p_2(t)$ be the accounting prices of physical capital and $q_1(t)$ and $q_2(t)$ the accounting prices of human capital, in the two sectors, respectively (i.e., $p_1(t) = \partial V(t)/\partial K_1(t)$, $q_2(t) = \partial V(t)/\partial H_2(t)$, and so forth). Therefore, wealth can be expressed as,

$$Z(t) = p_1(t)K_1(t) + p_2(t)K_2(t) + q_1(t)H_1(t) + q_2(t)H_2(t),$$

and genuine investment by

$$I(t) = p_1(t)dK_1(t)/dt + p_2(t)dK_2(t)/dt + q_1(t)dH_1(t)/dt + q_2(t)dH_2(t)/dt. \qquad (79)$$

If α were an optimum resource allocation mechanism, we would have $p_1(t) = p_2(t) = p(t)$, say, and $q_1(t) = q_2(t) = q(t)$, say. These prices would be related by the optimality conditions

$$U'(C(t)) = p(t); \quad p(t)\partial F/\partial K_1 = q(t)\partial G/\partial K_2;$$

and $$p(t)\partial F/\partial H_1 = q(t)\partial G/\partial H_2.$$

Estimating $q_1(t)$ and $q_2(t)$ poses difficult problems in practice. It has been customary to identify human capital with education and to estimate its accounting price in terms of the market return on education (i.e., salaries over and above raw labour). But this supposes, as we have assumed in the above model, that education offers no direct utility. If education does offer direct utility (and it is widely acknowledged to do so), the market return on education is an underestimate of what we should ideally be after. Furthermore, human capital includes health, which too is both a durable consumption good and capital good.

An alternative is to use estimates of expenditures on health and education for the purpose in hand. Such a procedure may be be a reasonable approximation for poor societies, but it is in all probability far off the mark for rich societies.

8 Global Public Goods

Countries interact with one another not only through trade in international markets, but also via transnational externalities. We ignore trade and formulate a problem of the global commons. Let G(t) be the stock of a global common (the atmosphere) at t. We imagine that G is measured in terms of a "quality" index (carbon dioxide concentration). Being a global common, G is an argument in the value function V of every country. For simplicity of notation, we assume that there is a single private capital good. Let $K_j(t)$ be the stock of the private asset owned by citizens of country j and let α_j be j's (autonomous) resource allocation mechanism and $\underline{\alpha}$ the vector of resource allocation mechanisms. If V_j is j's value function, we have

$$V_j(t) = V_j(\underline{\alpha}, K_j(t), G(t)). \quad (80)$$

Let $p_j(t) = \partial V_j(t)/\partial K_j(t)$ and $g_j(t) = \partial V_j(t)/\partial G(t)$. Genuine investment can be expressed as

$$I(t) = dV_j(t)/dt = p_j(t)dK_j(t)/dt + g_j(t)dG(t)/dt. \quad (81)$$

The point of (81) is this: the expression for genuine investment is the same whether or not $\underline{\alpha}$ is based on international cooperation. On the other hand, $dK_j(t)/dt$ and $dG(t)/dt$ do depend on how the international resource allocation mechanisms are arrived at; and they affect the accounting prices, $p_j(t)$ and $g_j(t)$.

9 Exogenous Productivity Growth

To assume exogenous growth in total factor productivity (the residual) over the indefinite future is imprudent. It is hard to believe that serendipity, unbacked by R&D effort and investment, can be a continual source of productivity growth. Moreover, many environmental resources go unrecorded in growth accounting. If the use of natural capital in an economy has in fact been increasing, estimates of the residual could be presumed to be biased upward. On the other hand, if a poor country were able to make free use of the R&D successes of rich countries, it would enjoy a positive residual.

The residual can have short bursts in imperfect economies. Imagine that a government reduces economic inefficiencies by improving the enforcement of property rights, or reducing centralized regulations (import quotas, price controls, and so forth). We would expect the factors of production to find better uses. As factors realign in a more productive fashion, total factor productivity would increase.

In the opposite vein, the residual could become negative for a period. Increased government corruption could be a cause; the cause could also be civil strife, which destroys capital assets and damages a country's institutions. When institutions deteriorate, assets are used even more inefficiently than before and the residual declines. This would appear to have happened in sub-Saharan Africa during the past forty years (Collins and Bosworth, 1996).

We now study sustainability in the context of two models of exogenous productivity growth.

9.1 Labour-augmenting Technical Progress

Consider an adaptation of the model explored in Section 3. Physical capital and a constant labour force together produce a non-deteriorating all purpose commodity. The economy enjoys labour augmenting technological progress at a constant rate n. If K is capital and A is knowledge, we have in the usual notation,

$$Y(t) = F(K(t), A(t)), \quad (82)$$

$$dK(t)/dt = F(K(t), A(t)) - C(t), \quad (83)$$

and

$$dA(t)/dt = nA(t). \quad (84)$$

There are two capital goods, K and A. Let $p_K(t)$ and $p_A(t)$, respectively, be their accounting prices in utility numeraire. The sustainability criterion is then $p_K(t)dK(t)/dt + p_A(t)dA(t)/dt \geq 0$, or, equivalently,

$$dK(t)/dt + q(t)dA(t)/dt \geq 0, \text{ where } q(t) \equiv p_A(t)/p_K(t). \quad (85)$$

It is instructive to study the case where the resource allocation mechanism is optimal. The equations of motion for p_K and p_A are,

$$dp_K(t)/dt = \delta p_K(t) - p_K(t)\partial F/\partial K, \quad (86)$$

and

$$dp_A(t)/dt = \delta p_A(t) - p_K(t)\partial F/\partial A - np_A(t). \quad (87)$$

Using (85)-(87) yields,

$$dq(t)/dt = (\partial F/\partial K - n) - \partial F/\partial A. \quad (88)$$

Suppose F is constant returns to scale. Define k = K/A and c = C/A. Write f(k) ≡ F(k, 1). From (83) and (84) we have

$$dk(t)/dt = f(k(t)) - nk(t) - c(t), \quad (89)$$

or $$dk(t)/dt = (\partial F/\partial K)k(t) + \partial F/\partial A - nk(t) - c(t). \quad (90)$$

Adding (88) and (90) yields

$$d(q(t) + k(t))/dt = (\partial F/\partial K - n)(q(t)+ k(t)) - c(t). \quad (91)$$

It is simple to confirm that q+k is the present value of future consumption (discounted at the rate $\partial F/\partial K$) divided by A (the current state of knowledge). It follows that the sustainability criterion at t (condition (85)), divided by A(t), is

$$dk(t)/dt + n(k(t) + q(t)) \geq 0. \quad (92)$$

9.2 Resource Augmenting Technical Progress

Consider an alternative world, where output, Y, is a function of manufactured capital (K) and the flow of an exhaustible natural resource (R). Let A(t)R(t) be the effective supply of the resource in production at t and S(t) the resource stock at t. Then we may write,

$$Y(t) = F(K(t), A(t)R(t)), \quad (93)$$

$$dK(t)/dt = F(K(t), A(t)R(t)) - C(t), \quad (94)$$

$$dA(t)/dt = n, \quad (95)$$

$$dS(t)/dt = -R(t). \quad (96)$$

There are three state variables. But we can reduce the model to one with two state variables. Thus, write Q(t) ≡ A(t)R(t) and X(t) = A(t)S(t). Then (94) and (96) become,

$$dK(t)/dt = F(K(t), Q(t)) - C(t), \quad (97)$$

and $$dX(t)/dt = nX(t) - Q(t). \quad (98)$$

This is equivalent to a renewable resource problem, and the steady state is the Green Golden Rule, with

$$nX = Q. \quad (99)$$

Let $p_K(t)$ and $p_X(t)$ be the accounting prices of K(t) and X(t), respectively. Then the sustainability condition is,

$$p_K(t)dK(t)/dt + p_X(t)dX(t)/dt \geq 0. \quad (100)$$

It is instructive to study the case where the resource allocation mechanism is optimal. Suppose also that F is constant returns to scale. Following the approach of the previous example, let $q(t) = p_X(t)/p_K(t)$. Then it is easy to confirm that

$$(dq(t)/dt)/q(t) = \partial F/\partial K - n. \quad (101)$$

Moreover, the optimal use of the productivity adjusted natural resource, Q(t), is determined by the condition,

$$\partial F/\partial Q = q(t). \quad (102)$$

Along the optimal programme, the sustainability condition (100) is,

$$F(K(t), Q(t)) - C(t) + q(t)[nX(t) - Q(t)] \geq 0, \quad (103)$$

or $$(\partial F/\partial K)K(t) + (\partial F/\partial Q)Q(t) - C(t) + q(t)[nX(t) - Q(t)] \geq 0, \quad (104)$$

or $$(\partial F/\partial K)K(t) - C(t) + nq(t)X(t) \geq 0. \quad (105)$$

Inequality (105) says that consumption must not exceed the sum of capital income and the sustainable yield.

10 Exhaustible Resources: the exporting economy

The export of natural resources at given world prices raises issues similar to those we have just encountered in our analysis of exogenous productivity change. The exogenous "drift" term, $\partial V(t)/\partial t$, in equation (14) has to be estimated.

Assume that extraction is costless. Suppose that at time τ the world market price of an exhaustible resource is $q(\tau)$. If $R(\tau)$ is the volume of export, revenue is $q(\tau)R(\tau)$.

Write $C(\tau) = q(\tau)R(\tau)$. (106)

It follows that

$$\partial C(\tau)/\partial t = q(\tau)\partial R(\tau)/\partial t, \quad (107)$$

and $$\partial C(\tau)/\partial \tau = q(\tau)\partial R(\tau)/\partial \tau + R(\tau)\partial q(\tau)/\partial \tau, \quad (108)$$

As in Section 4, we take it that social welfare at t is of the form,

$$V(t) = \int_t^\infty U(C(\tau))\exp[-\rho(\tau-t)]d\tau. \quad (109)$$

Moreover, the stock declines according to (40), or,

$$dS(t)/dt = -R(t).$$

Let p(t) denote the resources' accounting price. Since the criterion for sustainable welfare is $dV(t)/dt$, we differentiate both sides of (109) with respect to time, which implies,

$$dV(t)/dt = -U(C(t)) + \rho V(t) + p(t)dS(t)/dt + \int_t^\infty [U'(C(\tau))\exp[-\rho(\tau-t)](\partial C(\tau)/\partial t)]d\tau. \quad (110)$$

Define $\mu(\tau,t) = \partial C(\tau)/\partial \tau + \partial C(\tau)/\partial t$. (111)

μ can be regarded as an index of the extent to which the resource allocation mechanism is non-autonomous. Partially integrating the last term in (110), we have

$$dV(t)/dt = p(t)dS(t)/dt + \int_t^\infty \{U'(C(\tau))\exp[-\rho(\tau-t)]\mu(\tau,t)\}d\tau. \quad (112)$$

The integral on the RHS of (112) is the "drift" term. As (112) shows, the index of sustainable welfare is the algebraic sum of genuine investment and the drift term. We now proceed to obtain simple rules for estimating the index in the case of two special non-optimum resource allocation mechanisms.[20]

Suppose C is constant.[21] In this case,

$$\partial C(\tau)/\partial \tau = \partial C(\tau)/\partial t = 0,$$

and $\mu(\tau,t) = 0$ in (112) is zero, and genuine investment measures changes in social welfare.

Suppose instead R is constant. It follows that

$$\partial R(\tau)/\partial \tau + \partial R(\tau)/\partial t = 0, \quad (113)$$

and $$\mu = R(\tau)\partial q(\tau)/\partial \tau = q(\tau)R(\tau)[\partial q(\tau)/\partial \tau]/q(\tau). \quad (114)$$

Using (113) and (114), we may write,

$$\int_t^\infty \{U'(C(\tau)\exp[-\rho(\tau-t)]\mu(\tau,t)\}d\tau = \bar{\mu}(t)/\rho, \quad (115)$$

where $\bar{\mu}(t)$ can be interpreted as the average capital gains on the world market, as viewed from time t. Formally, (112) can be re-written as,

$$dV(t)/dt = p(t)dS(t)/dt + \bar{\mu}(t)/\rho. \quad (116)$$

11 Further Extensions

[20] We leave it to the reader to compute the drift term in (112) when the resource allocation mechanism is optimum.

[21] In this case the resource will be exhausted in finite time. For notational simplicity, we continue to present matters as though the horizon is infinite.

A number of important features of actual economies were missing in the economic models developed so far. We comment on a few of them and show how they can be included in the theory.

1. Intragenerational distribution. The distribution of well-being within a generation has been ignored so far. Theoretically it is not difficult to include this. If there are N people in each generation and person j consumes C_j, her welfare would be $U(C_j)$.[22] A simple way to express intragenerational welfare would be to "concavify" U. Let G be a strictly concave, increasing function of real numbers. We may then express intragenerational welfare as $\Sigma_j[G(U(C_j))]$.[23] Some people would be well-off, others badly-off. The formulation ensures that at the margin, the well-being of someone who is badly off is awarded greater weight than that of someone well-off.

The social worth of consumption services (C) depends on who gets what. To accommodate this idea, we have to enlarge the set of commodities so as to distinguish, at the margin, a good consumed or supplied by one person from that same good consumed or supplied by another. This is the idea of "named goods".[24] It means, for example, that a piece of clothing worn by a poor person should be regarded as a different commodity from that same type of clothing worn by someone who is rich. Accounting prices of named goods would depend on the names attached to them. With this re-interpretation of goods and services, the results we have obtained continue to hold.

2. Stock Effects. Some natural resources have value as a stock, qua stock, either because the stock provides a flow of direct consumption services (ecosystems) or because it has intrinsic value (redwood forests). The way to accomodate such value would be to enlarge the domain on which utility is defined, by including S. Write U(C, S), where $\partial U/\partial S > 0$. The resource's accounting price would reflect this "stock effect".[25]

Stock pollutants can be introduced in a similar manner. Suppose pollution (carbon emission) is a byproduct of production. Imagine that it is a constant proportion (γ) of Y. Let P be the stock of pollutants. Assume that it depreciates at a rate π. Then the dynamics of the pollutant would be given by

$$dP(t)/dt = \gamma Y(t) - \pi P(t). \qquad (117)$$

We may write utility as U(C, P), where $\partial U/\partial P < 0$ and retrace the formal arguments in earlier sections. Accounting prices include the stock effect of pollutants. Moreover, the economy would be interpreted as harbouring two capital stocks, S and P.

3. Environmental externalities. Global public goods were introduced earlier. More general environmental externalities can be incorporated by a device identical to the one devised to incorporate distributional considerations. To describe who is affected, in which manner, and by whose actions involves

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[22] Person-specific factors (e.g., age, health status, gender) can be included in the welfare function. This is routinely done in applied economics.

[23] A more general formulation would have us define a symmetric, strictly concave function from M-vectors into the reals: $G(U(C_1), ..., U(C_M))$.

[24] Hahn (1971).

[25] Kurz (1968) and Uzawa (1974a,b) were the first to analyse the intertemporal consequences of stock effects in social welfare.

the use of named goods and services. Accounting prices should be "named", so as to distinguish private costs from social costs and private benefits from social benefits. They are a generalization of Lindahl prices in first-best economies.[26] Pigovian taxes and subsidies on externalities can be computed on the basis of named accounting prices. The accounting prices of the capital assets are functions of such taxes and subsidies.

4. Defensive expenditure. We may generalize the ideas developed in the previous point to include defensive expenditure against pollution. Denote by Q(t) the stock of defensive capital and X(t) investment in its accumulation. Equation (93) can then be re-written as,

$$dP(t)/dt = G(Y(t), Q(t)) - \pi P(t), \text{ where } G(Y(t), Q(t)) \geq 0, \partial G/\partial Y > 0 \text{ and } \partial G/\partial Q < 0. \quad (118)$$

Moreover,

$$dQ(t) = X(t) - \xi Q(t), \qquad \text{where } \xi > 0. \quad (119)$$

In the usual notation, the accumulation equation is expressed as,

$$dK(t)/dt = F(K(t)) - C(t) - X(t). \quad (120)$$

Denote by p(t) the accounting price of K, m(t) that of defensive capital and r(t) (< 0) the accounting price of the pollutant. Wealth can then be expressed as,

$$p(t)K(t) + m(t)Q(t) + r(t)P(t),$$

and genuine investment at t as,

$$I(t) = p(t)dK(t)/dt + m(t)dQ(t)/dt + r(t)dP(t)/dt. \quad (121)$$

Equation (121) says that defensive expenditure against pollution ought to be included in the estimation of genuine investment (m(t)dQ(t)/dt), but, then, so should changes in the quality of the environment be included (r(t)dP(t)/dt). To include the former, but not the latter, is a mistake.

5. Uncertainty. The economy has so far been assumed to be deterministic. Let intertemporal welfare at t = 0 be the expected value of the present discounted flow of U. Now define contingent goods, which are goods produced or consumed contingent on identifiable events. From this one can define contingent accounting prices. It is not presumed that there is a complete set of markets for contingent goods. Our account of the welfare economics of imperfect economies, nevertheless, goes through.

[26] Lindahl (1958) and Arrow (1971).

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