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Knowledge Asymmetries and Learning in Common Pools

Abstract

The role of environmental uncertainty has not been ignored in the common pool literature but underlying most of this research is an explicit or implicit assumption of symmetric uncertainty. I introduce asymmetric uncertainty by letting users have access to private information about the resource and differ with respect to informational precision. To what extent will the existence of knowledge asymmetries influence exploitation decisions and overall welfare? If actions are unobservable informational asymmetries *per se* do not influence behavior. However, if actions are observable the distribution of uncertainty can have drastic implications for exploitation decisions, the probability of resource collapse and consequently for overall welfare.

Key Words: Common pool resources, Asymmetric uncertainty, Knowledge spill-over.

1 Introduction

Many natural resources have been exploited and degraded to the point where we can no longer expect them to continue to produce a steady stream of goods and services. There are for example empirical evidence that the world's fish stocks are threatened by collapse, that more than 50 percent of grasslands are degraded, that deforestation is widespread, resulting in extensive species and habitat loss. [15]

Such resource degradation is often explained by referring to the institutional structure of the resources. Many natural resources are so called common pool resources. They are characterized by rivalry in consumption and non-excludability among the users and when combined, these two characteristics are typically associated with inefficiencies in terms of over-exploitation unless users manage to cooperate. (See seminal work by Hardin [14] and Olson [19].)

The management problems associated with common pool resources have stimulated much research and as a result there already exists an extensive theoretical, experimental and empirical common pool literature devoted to investigate how different factors influence individual choices and consequently the resource. (For overviews, see Ostrom [20, 22], Ostrom et al. [21], Baland and Platteau [2], Bromley [7], and Wade [25].)

The role of environmental uncertainty has not been ignored. Most common pool systems are associated with environmental uncertainty. Whereas uncertainty regarding the internal structure of the system (e.g. how the resource is affected by exploitation and other resources) can be reduced over time through extended use and careful observation, uncertainty stemming from external factors (e.g. timing and quantity of rainfall, temperature variations and sudden outbreak of diseases) cannot. Thus, some element of environmental uncertainty will always remain. For example, there rarely exist direct mechanisms to determine the exact size or replenishment rate of a natural resource. [20]

Empirical observations confirm that uncertainty can complicate the exercise of sustaining a common resource. (See discussions in Ostrom [20] and Agrawal [1]) The prevailing theoretical and experimental results are that an increase in the level of uncertainty leads to higher exploitation variability and a higher probability of resource collapse. (For an elaborate overview see Kopelman et al. [17].)

However, existing work has mainly focused on the case of symmetric uncertainty. In reality, people tend to have access to different sources of information and may also differ with respect to the skill of processing this information. The symmetry of information could vary from situation to situation, as it depends on how expensive it is to acquire information and on the rules for disseminating it (for a more detailed discussion see Ostrom [20], pp 33). In a rivalry setting, agents may have incentive to use their own and others' knowledge strategically, which could have consequences for individual and overall welfare.

In this paper I relax the assumption of symmetric information and analyze how asymmetric uncertainty can influence peoples' strategies and what consequences there may be for the resource. To be able to make direct comparisons with earlier results from the common pool literature I choose the probability of resource collapse as welfare measure. Moreover, a social planner whose main priority is to maintain the overall welfare of current and future generations should be more concerned about the probability of resource collapse than the distribution of payoffs.

The setup is the following. A well-defined group has exclusive access to a common resource and each individual in this group makes an exploitation decision. Users do not cooperate and make their exploitation decisions individually, which results in over-exploitation in the standard common pool setting. In a worst case scenario the exploitation can even cause a resource collapse associated with substantial losses. Exactly how much exploitation the resource can sustain without collapsing is uncertain. I introduce asymmetric information by letting each agent have access to private information of different quality about the matter, which is common knowledge.

The setup chosen opens up for several possible interpretations. The resource in question can be a non-renewable resource, in which case there is uncertainty regarding the size of the resource, or it can be a renewable resource, in which case the uncertainty could be related to replenishment rate. The resource can even represent a renewable resource characterized by non-linear dynamics and multiple stable states. If too much pressure is put on the resource there will be a flip from a "good" state to an alternate degraded state [24]. In this case the uncertainty would be concerning where this flip occurs, i.e. the location of the threshold.

Will the existence of informational asymmetries influence the inefficiencies associated with common pool management? This paper demonstrates that the answer depends on whether actions are observable or not.

When actions are unobservable the game is essentially a simultaneous moves game where users make their exploitation decisions based on their own information and expectations about their rivals' information. Under these specifications the distribution of uncertainty *per se* does not influence individual exploitation decisions or the probability of resource collapse. Consistent with conventional results from the common pool literature the probability of resource collapse increases with the overall level of uncertainty. (See for example, Budescu et al. [8], Budescu et al. [9].)

When actions are observable and sequential informational asymmetries influence individual exploitation decisions and profits. Although the probability of resource collapse increases with the overall level of uncertainty, the distribution of uncertainty matters as well. The order of moves is particularly relevant. If a social planner can influence knowledge levels, the choice of who to target becomes critical and sometimes non-trivial. These results all hinge on the information spill-over within the model. Although information is private, the information of a predecessor may spill-over to a follower through her action when actions are sequential. The predecessor then has two pieces of information and bases her decision both, which means that there are informational efficiency gains that are not present when actions are simultaneous. As a matter of fact, when there are informational asymmetries, learning can mitigate the inefficiency associated with sequential actions.

The game takes place in a one-shot environment and one might have some concerns about that. However, in a repeated environment players are assumed to face exactly the same game in each period. If some variable, as for example the information structure, changes over time, this is no longer true, as the structure of the game has changed. This implies that a one-shot game could be a better representation of reality than a repeated game. Moreover, even though the game is a one-shot game, it could very well be the case that the users face a similar (but not exactly the same) decision problem tomorrow and even if we account for reputation effects or communication effects, it will not affect the qualitative aspect of the results.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 is devoted to the analysis of the model and to the presentation of the results. Section 4 concludes the paper.

2 The model

2.1 The economics of exploitation

Consider a group of n users with exclusive access to a common resource with rivalry in consumption. The resource is question can for example be a fishing ground, a water reserve, a forest patch or a common pasture. Each user i, where $i \in I = \{1, 2, ..., n\}$ makes an exploitation decision $h_i \in \mathbb{R}$, on this resource.

Let $x \in \mathbb{R}$, be some critical value of the resource that can be exploited without causing a resource collapse. If total exploitation exceeds this critical value, $\sum_{i \in I} h_i > x$, there is a resource collapse with the implication that further consumption possibilities are seriously diminished (they may even be lost). If total exploitation does not exceed this critical value, $\sum_{i \in I} h_i \leq x$ exploitation is sustainable and the resource can continue to produce goods and services.[1]

Formally common pool resource games can be seen as a special class of cournot competition games [23]. This is also the approach adopted here. The payoff to each user i can then be represented by the following function.

$$\pi_i(.) = h_i \left(x - h_i - \sum_{j \in I, j \neq i} h_j \right)$$

[1] Note that both x and h_i can be negative. A negative value of x could for example represent the case where there already has been a resource collapse and where the objective is to minimize the losses by choosing a "negative" level of exploitation, by for example making some investment in the resource.

This payoff function, which captures rivalry and consequences of a potential resource collapse, is equivalent to a standard cournot competition payoff function assuming a linear demand function. If each user makes the exploitation decision independently to maximize own individual payoff, the negative externality imposed by one user on the other users will not be internalized. The result is over-exploitation of the resource compared to the cooperative solution, generating smaller payoffs. In a worse case scenario it can even cause a resource collapse associated with substantial losses for all users.

This payoff function also captures that exploitation decisions typically are proportional to some cost. If there is no resource collapse each user can recover her individual cost and make a positive profit proportional to her exploitation. However, in case of a resource breakdown, the cost incurred cannot be recovered and the losses will be proportional to the exploitation level.

With this setup non-cooperative users over-exploit the resource and more so when actions are sequential (see Table 1, derivations can be found in the appendix).

Table 1. Equilibrium outcomes when the resource is known.

	Simultaneous order	Sequential order k refers to order	Cooperation
Individual expl.	$h_i = \frac{x}{n+1}$	$h_i = \frac{x}{2^k}$	$h_i = \frac{x}{2n}$
Total expl.	$\sum_{i \in I} h_i = \frac{nx}{n+1}$	$\sum_{i \in I} h_i = \frac{(2^n-1)x}{2^n}$	$\sum_{i \in I} h_i = \frac{x}{2}$
Prob. of collapse	0	0	0

In a simultaneous order game actions are unobservable and give rise to a strategic uncertainty causing users to be more cautionary than under a sequential order game. In a sequential order game followers perfectly observe the actions taken by predecessors and adjust their claims accordingly. Anticipating this, the predecessors can take advantage of their positions and increase their claims.[2]

2.2 Introducing uncertainty

I introduce uncertainty by letting the critical value x be unknown. However, each user has the same uninformative (improper) prior that x is uniformly distributed on the interval $(-\infty, \infty)$ with probability density 1.[3] Each user also observes a private signal s_i, where $s_i = x + \varepsilon_i$ and ε_i is normally distributed with mean zero and variance v_i. Signals are drawn independently. The expected value of each error term is zero, implying that in expectation each agent will observe the correct resource size, x. Moreover, the probability that agent i receives the correct signal, is given by $\Pr(\varepsilon_i = 0)$, which translates to $f_i(0) = 1/\sqrt{2\pi v_i}$ for the normal distribution.

Users differ with respect to knowledge. The difference in knowledge levels stems from the difference in variances. One way to interpret knowledge levels is by considering them to be given before signals are observed, users then *ex ante* differ with respect to skill of processing information. The probability of finding the correct information, or put differently, the probability of interpreting the observed information correctly, is higher for a relatively knowledgeable user with a relatively low variance. An alternative interpretation is the following. At the outset users

[2] These results are consistent with stackelberg competition for the sequential order game and with cournot competition for the simultaneous order game. [11]

[3] The uninformative prior states that prior to the observed signals any value for x between $(-\infty, \infty)$ is "equally likely". It is improper because the actual density of such a prior is zero. However, the posterior probabilities will still sum (or integrate) to 1 even if the prior values do not, and so the priors only need to be specified in the correct proportion. [5]

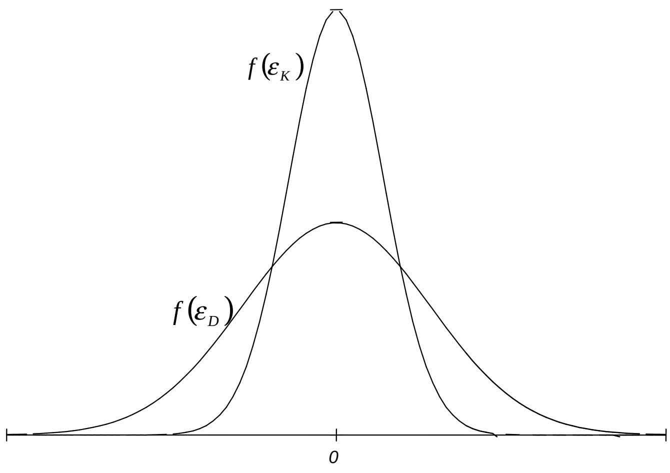

Figure 1: Knowledge asymmetries

are identical, but then, each of them receives a signals and one user receives a more informative signal (with a lower variance) than the other user and becomes more knowledgeable.

By depicting probability densities for the error terms, Figure 1 illustrates knowledge asymmetries for two users, where subscript K denotes the relatively knowledgeable user (the expert henceforth) and subscript D denotes the relatively ignorant user.

It is important to realize that although the exact content of users' respective information remains private each user do have information on the other users' informational precision.

3 Analysis

To fully capture the effect of knowledge asymmetries I compare equilibrium outcomes for a simultaneous order game with a sequential order game.

When actions are simultaneous users do not observe each others claims and base their respective decision on expectations only. This is also equivalent to a sequential game with unobservable actions and could for example represent a case of geographically distant fishermen sharing a common fishing ground.

When actions are sequential and observable the action of a user can potentially reveal some information about her information to the followers - there may be informational spill-overs, which could increase efficiency as better informed decisions can be made. When the order of moves is sequential it can represent a case when the order of moves has been regulated beforehand or when there is some "natural" ordering due to some environmental characteristic or institutional structure of the resource. This could for example be the case of down-stream and up-stream users sharing a common water reserve.

3.1 Unobservable actions - simultaneous order

If actions are unobservable each user maximizes her payoff individually given her expectation about the other users. The solution concept is Bayesian Nash Equilibrium (BNE) and the game is denoted $\Gamma_U(n)$.

Proposition 1 *In a BNE of the game $\Gamma_U(n)$ each user i, where $i \in \{1, 2, ..., n\}$, claims $h_i^* = \frac{s_i}{n+1}$. The probability of resource collapse given by*

$$1 - \frac{1}{\sqrt{2\pi v_z}} \int_{-\infty}^{x} e^{-\frac{u^2}{2v_z}} du, \qquad (1)$$

is increasing in overall level of uncertainty, v_z, where $v_z = \sum_i^n v_i$.

Unless otherwise stated, the proof of this proposition and the following are in the Appendix. From Proposition 1 we know that the overall level of uncertainty is simply the sum of individual levels of uncertainty, which leads us to the following results.

Result 1 *In a BNE of the game $\Gamma_U(n)$,the marginal contribution to the probability of resource collapse is equal for all users irrespective of knowledge levels.*

Each user takes her decision independently given her expectation about the other users. Signals are unbiased estimators of the size of the resource which implies that each user expects the other users to observe the same signal, which further implies that the claims made will be independent of knowledge levels.

Result 2 *In a BNE of the game $\Gamma_U(n)$, knowledge asymmetries per se do not influence exploitation decisions, individual or overall welfare.*

However, we know that the realized profit to each agent will be truly maximized only if all users observe the correct signal, which is more likely the more precise the signals are. Even if a user observes the true value of x, her utility is still decreasing in the errors of the other users.

3.2 Observable actions - sequential order

For illustrative purposes I here analyze and present the case with two users. However, Section 3.4 is devoted to analyzing to what extent the results obtained for two users can be generalized to a case with n appropriators.

The game played when actions are observable and taken in a sequential order with observable actions is denoted $\Gamma_O(2)$. Users still differ with respect to knowledge and know I let subscript L denote the leader and subscript F denote the follower. Because users do not know each other's signals, the start of the game does not form a proper subgame until posterior beliefs have been specified and therefore we cannot test whether the continuation strategies constitute a Nash equilibrium. This implies that players' beliefs about other players' actions must be specified as part of the equilibrium. The solution concept employed is Perfect Bayesian Equilibrium (PBE). A strategy profile $h = \{h_L, h_F\}$ and a belief system $\mu = \{\mu_L(x), \mu_F(x)\}$ constitute a PBE if each agents payoff is maximized given the belief system and the other agents strategy, and if the belief system is consistent with Bayes updating.[4]

I only consider fully separating pure strategy equilibria. Thus each exploitation decision has an inverse function $h_i^{-1}(.)$ that maps each claim to a unique signal s_i.

Proposition 2 *In a PBE of the game $\Gamma_O(2)$, the leader claims $h_L^* = s_L/2$ and the follower claims $h_F^* = (\mu_F(x) - h_L)/2$, where $\mu_F(x) = (v_F/(v_F + v_L))\, s_L + (v_L/(v_F + v_L))\, s_F$. The probability of resource collapse given by*

$$1 - \frac{1}{\sqrt{2\pi v_Q}} \int_{-\infty}^{x} e^{-\frac{u^2}{2v_Q}} du, \tag{2}$$

increases in overall level of uncertainty, v_Q where $v_Q = v_L\,(9v_F + v_L)\,/\,(v_F + v_L)$.

Because signals are unbiased, each user expects that the other user observe the

[4] For a more formal definition see Fudenberg and Tirole [12].

same signal. The leader will therefore follow the own signal and does not take the knowledge levels into account when making her exploitation decision.

However, the action of the leader perfectly reveals her signal so the equilibrium is fully separating, which means that there are no informational inefficiencies.[5] The follower has access to both signals and the weight attached to each signal is determined by the variances according to Bayesian updating in a Gaussian framework[6], meaning that more weight will be attached to the signal of the expert. If there were no asymmetries, equal weight would be attached to the signals.

Result 3 *In a PBE of the game $\Gamma_O(2)$, the probability of resource collapse is lower if the expert is the leader.*

Proof. We know from Proposition 1 that the overall level of ignorance, v_Q is given by $v_Q = v_L \left(9v_F + v_L\right) / \left(v_F + v_L\right)$. It is then easy to verify that the overall ignorance level is higher when the expert is second mover;

$$v_K \frac{9v_D + v_K}{v_D + v_K} < v_D \frac{9v_K + v_D}{v_K + v_D}. \tag{3}$$

■

This result is intuitive. Regardless of ordering, the second mover has access to the two pieces of information. However, the first mover only has one piece of information. The most precise decision of a leader (and hence of the group) is therefore obtained if the leader is also the expert.

[5] In informational economics, information inefficiencies often translates to one of two extremes. One extreme - imitating, so called herd behavior, implies that people will be doing what others are doing rather than using their own information. The other extreme is no learning, where users base their decisions on own information only. Banerjee [3] and Bikhchandani et al. [6] separately and independently introduced social learning as rational behavior and with them the concept of herding.

[6] See Chamley [10] for more details about Bayesian updating and social learning.

3.3 Policy implications

Profits of the users will be truly maximized only of all users observe the correct value of x. Moreover, the probability of resource collapse increases in overall level of uncertainty. It may therefore be in the interest of a social planner to reduce the overall level of uncertainty if possible. If actions are unobservable and/or simultaneous we know that the choice of who to target does not matter. When actions are observable and sequential it does however. Because order matters for the probability of resource collapse, the marginal contribution of users' variances to the probability of resource collapse differs. Suppose it is possible for a policy maker to influence knowledge levels.[7] Which knowledge level should be targeted? A qualified guess is that one should target the leader. By doing so one would not only increase the precision of the leader's decision but through the informational externalities one would also influence the knowledge level of the follower. This is true if users do not differ with respect to knowledge, but not necessarily if there are knowledge asymmetries.

Result 4 *In a PBE of the game $\Gamma_F(2)$, reducing the variance of an expert leader minimizes the probability of resource collapse. If the ignorant user is the leader the variance of the ignorant leader should be reduced only if $v_D < \frac{9}{7}v_K$.*

Proof. In order to reduce overall variance and thereby reducing probability of resource collapse the knowledge level of the user with the highest marginal contribution to overall ignorance should be targeted. One should therefore reduce the

[7] A policy maker could for example provide more accurate information to some of or all of the users.

variance of the leader if

$$\frac{dv_Q}{dv_L} > \frac{dv_Q}{dv_F} \tag{4}$$

$$\frac{9v_F^2 + 2v_F v_L + v_L^2}{(v_F + v_L)^2} > \frac{8v_L^2}{(v_F + v_L)^2} \tag{5}$$

which is true as long as $v_L < \frac{9}{7}v_F$. ■

Thus a social planner should always target an expert leader. A social planner should also target an ignorant leader if the knowledge asymmetries are not big enough. However, if the leader is ignorant and ignorant enough a social planner should target the follower/expert. The intuition goes as follows. When the leader is very ignorant and may exploit to the point where a resource collapse is likely it is vital that the follower can make the most precise decision possible so that a resource collapse can be avoided.

When there is no uncertainty the sequential order protocol is worse than the simultaneous order protocol from a welfare perspective because the leader anticipates the follower's best response and can take advantage of her position. When there is uncertainty with respect to the resource this may not longer be true due to informational efficiency gains from learning.

Result 5 *If the magnitude of the knowledge asymmetries is big enough,* $v_K < \frac{1}{7}v_D$*, and if the expert user is the leader, the probability of resource collapse in the game is lower under a sequential protocol* $\Gamma_F(2)$ *than under a simultaneous order protocol* $\Gamma_O(2)$*.*

Proof. I compare the probability of resource collapse for the simultaneous order protocol with the sequential order protocol. From Propositions 1 and 2 we know that the probability of resource collapse increases in total variance so we can restrict our comparison to variances. First assume that the leader is the expert, $v_L = v_K, v_F = v_D$. The total variance for the simultaneous order protocol is higher if

$$v_K + v_D > v_K \frac{9v_D + v_K}{v_D + v_K}, \tag{6}$$

which holds if $v_K < \frac{1}{7} v_D$. Next assume that the ignorant agent is the leader, $v_L = v_D, v_F = v_K$. The total variance for the simultaneous order protocol is higher if

$$v_K + v_D > v_D \frac{9v_K + v_D}{v_K + v_D}, \tag{7}$$

which never holds. ■

Thus, learning combined with knowledge asymmetries mitigates and may even reduce the inefficiency associated with sequential order, but only if the knowledgeable is first mover. Thus, it may be in the interest of a social planner to be a carrier of information if actions are unobservable.

3.4 Increasing the number of appropriators

This section is devoted to analyze and discuss if and how some of the results obtained for the sequential order case can be generalized to an extended case where there are more than two users. However, I will not fully characterize equilibrium outcomes, as this is not necessary for the purpose of this section.

Now suppose that there are n users making exploitation decisions in a predetermined order. The users differ with respect to knowledge. I let subscript 1 denote the first player, subscript 2 the second and so on, $i \in \{1, 2, ...k, ...n.\}$. The last appropriator maximizes the following function:

$$\max_{h_n} E_n[h_n(x - h_n - \sum_{j=1}^{n-1} h_j)] \tag{8}$$

where $\sum_{j=1}^{n-1} h_j$ is the sum of observed claims by the predecessors. The first order condition reads:

$$E_n(x - \sum_{j=1}^{n-1} h_j - 2h_n) = 0 \tag{9}$$

Note that $E_n(x) = \mu_n(x)$. This implies that the best response of user n is given by:

$$h_n = \frac{\mu_n(x) - \sum_{j=1}^{n-1} h_j}{2} \tag{10}$$

where

$$\mu_n(x) = \frac{\sum_{i=1}^{n} \left(\frac{1}{v_i}\right) s_i}{\sum_{j=1}^{n} v_j} \tag{11}$$

(See appendix for derivation). Following the procedure employed for the sequential game with complete information we can derive a general best response function and a belief for the $k:th$ user;

$$h_k = \frac{\mu_k(x) - \sum_{j=1}^{k-1} h_j}{2}, \tag{12}$$

$$\mu_k(x) = \frac{\sum_{i=1}^{k} \frac{s_i}{v_i}}{\sum_{j=1}^{k} \frac{1}{v_j}} \tag{13}$$

From the best response function we can note a few things. First, user $k\prime s$ expectation about x is given by the belief derived from Bayes rule. We can also note that the first user only base her decision on the own signal, $\mu_1(x) = s_1$, whereas the follower has two signals to base her decision on, the third user three signal and so on. This means that followers will be able to make more precise decisions than predecessors.

From an overall welfare perspective it is then best is the first user is the most knowledgeable, that would ensure the most precise decision for a first user. The second user has two signals, the own signal and the signal of the leader. The most precise decision of a second user can then only be guaranteed if the second user is the second most knowledgeable user and so on.

The probability of resource collapse is given by $\Pr\left(\sum_{i\in I} h_i < x\right)$. First note that

$$\sum_{i=1}^{n} h_i = \sum_{k=1}^{n} \frac{\left(\mu_k(x) - \sum_{j=1}^{i-1} h_k\right)}{2} \tag{14}$$

Substitute $\mu_k(x)$ with the expression above (11) and make use of the fact that $s_i = x + \varepsilon_i$. We can note that this expression is a linear combination of the error terms meaning that the probability of resource collapse can be written as follows.

$$\Pr\left(\sum_{i\in I} \phi_i \varepsilon_i < x\right) \tag{15}$$

where ϕ_i is a weight attached to each error term. The exact value of each weight will depend on the exact distribution of knowledge. Probability of resource collapse is given by

$$1 - \frac{1}{\sqrt{2\pi v_\omega}} \int_{-\infty}^{x} e^{-\frac{u^2}{2v_\omega}} du \tag{16}$$

where $\omega = \sum_{i\in I} \phi_i^2 v_i$. We can then see that the probability of resource collapse is increasing in overall level of uncertainty (see proof of Proposition 1).

4 Concluding remarks

The purpose of this paper was to analyze how knowledge asymmetries influence individual decisions and consequently the overall welfare in a typical common pool resource setting. Our main finding is that knowledge asymmetries matter and one should not only be concerned with the overall level of uncertainty when analyzing and recommending remedies concerning problems as these.

Studies from the lab and the field both highlight the importance of reducing environmental uncertainty in common pools. This is not in any way contradictory to the conclusions one should draw from this paper. The conclusion from this paper is certainly not to enhance environmental uncertainty. Instead, this paper demonstrates that when analyzing such settings and/or suggesting policies for reducing the inefficiencies associated with them, one should not only be concerned about the level of environmental uncertainty, but also the distribution of uncertainty, as it matters too. For example, while reducing environmental uncertainty, keeping knowledge heterogeneity, "let the experts be experts", might be a good strategy compared to an approach aiming at equalizing knowledge.

However, the results presented here need to be studied empirically. As far as I understand there are no empirical studies devoted to this and I only know of one related experimental study. Lindahl and Johannesson [16] study a one-shot sequential game of fixed ordering between two players, where players make claims on a common resource. They introduce asymmetric uncertainty by letting the first mover be privately informed about the resource size. The second mover only knows the probability distribution, which is common knowledge. They find that asymmetric uncertainty does not increase the probability of over-exploitation.

However, to be able to draw more rigorous conclusions, one also needs to relax the assumption of full information for the first mover, and one should probably also include more than two appropriators and test a simultaneous order protocol as well as a sequential order protocol.

Users do not cooperate, which here also means that they do not share information through communication (it is only through actions that information spill over). In a setting where users share information the knowledge levels of all users would naturally be the same, and within this setting it would be the knowledge level implied by Bayesian updating. From the results obtained here it is fairly easy to deduce that whereas communication would decrease the probability of resource collapse in a simultaneous order game, it is not necessarily so when actions are sequential.

A natural extension of the model would be to let knowledge levels be endogenous by allowing agents to acquire additional knowledge at some specific cost. Depending on whether the new knowledge levels are observed by the rival, and on the cost associated with knowledge acquisition, it could be in the interest of at least one of the agents to acquire knowledge. It could for example be worthwhile for the leader to improve on her informational precision, and by extension also the precision of the follower, because the probability of resource collapse would decrease. Note that there would be too little knowledge acquisition because the leader would equate marginal cost of knowledge acquisition with private marginal benefit.

Throughout the paper, I have considered the case of agents sharing a common pool resource. However, given the simplicity and generality of the setup, the results should also hold for other types of resource dilemmas. Consider for example the mirror image of the common pool resource problem, public good provision where the provision threshold is uncertain.

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Appendix

Preliminaries

The optimal resource exploitation of a common pool resource with n users is obtained by choosing exploitation levels, $\{h_1, h_{2,...}, h_n\}$ in order to maximize the sum of payoffs,

$$\max_{\{h_1,h_{2,...},h_n\}} (\textstyle\sum_i^n h_i(x - \sum_i^n h_i)). \tag{A1}$$

We derive the first order conditions;

$$\text{for all } i \in I : x - 2\textstyle\sum_{j\in I, j\neq i} h_j - 2h_i = 0, \tag{A2}$$

and then obtain best response functions

$$\text{for all } i \in I : h_i = \frac{x - 2\sum_{j\in I, j\neq i} h_j}{2}. \tag{A3}$$

By symmetry we obtain optimal individual exploitation levels;

$$\text{for all } i \in I : h_i^* = \frac{x}{2n}. \tag{A4}$$

Total exploitation is then

$$\textstyle\sum_i^n h_i^* = \frac{x}{2}. \tag{A5}$$

In a simultaneous order common pool resource game each user choose exploitation level h_i in order to maximize individual payoff.

$$\max_{h_i} (h_i(x - h_i - \textstyle\sum_{j\in I, j\neq i} h_j)) \tag{A6}$$

The best response function for user i is

$$h_i = \frac{x - \sum_{j\in I, j\neq i} h_j}{2}. \tag{A7}$$

By symmetry we obtain equilibrium individual exploitation levels;

$$h_i^* = \frac{x}{n+1}. \tag{A8}$$

Total exploitation is

$$\sum_i^n h_i^* = \frac{xn}{n+1}. \tag{A9}$$

In a sequential order common pool resource game the last appropriator maximizes her individual payoff function by choosing exploitation level h_n given observed exploitation levels by predecessors $\sum_{j=1}^{n-1} h_j$.

$$\max_{h_n}(h_n(x - h_n - \sum_{j=1}^{n-1} h_j)) \tag{A10}$$

The best response function is given by;

$$h_n = \frac{x - \sum_{i=1}^{n-1} h_j}{2}. \tag{A11}$$

Anticipating this the next to last appropriator solve the following problem;

$$\max_{h_{n-1}}(h_{n-1}(x - h_{n-1} - \frac{x - \sum_{i=1}^{n-1} h_j}{2} - \sum_{j=1}^{n-2} h_j)). \tag{A12}$$

The first order condition is given by;

$$x - 2h_{n-1} - \frac{x - \sum_{i=1}^{n-2} h_j - 2h_{n-1}}{2} - \sum_{j=1}^{n-2} h_j = 0, \tag{A13}$$

The best response of the next to last user is then given by;

$$h_{n-1} = \frac{x - \sum_{j=1}^{n-2} h_j}{2}. \tag{A14}$$

Applying the same argument we can derive the general best response for the k^{th} appropriator;

$$h_k = \frac{x - \sum_{j=1}^{k-1} h_j}{2}. \tag{A15}$$

This implies that we can derive an equilibrium exploitation level of the first user; $h_1 = \frac{x}{2}$, and moving forward for the second user; $h_2 = \frac{x}{4}$ and so on. The equilibrium individual exploitation level for the k^{th} appropriator is thus given by

$$h_k^* = \frac{x}{2^k}. \tag{A16}$$

Total exploitation is given by

$$\sum_i^n h_i^* = \frac{x\left(2^n - 1\right)}{2^n}. \tag{A17}$$

Simultaneous order - unobservable actions

Proof of Proposition 1. Each user i observes the signal s_i and then choose the exploitation level which maximizes her expected payoff given the exploitation decision by all other users, $\sum_{j\in I,j\neq i} h_j$.

$$\max_{h_i} E_i \left[h_i \left(x - h_i - \sum_{j\in I,j\neq i} h_j\right)\right] \tag{A18}$$

The first order condition is;

$$E_i \left[x - 2h_i - \sum_{j\in I,j\neq i} h_j\right] = 0. \tag{A19}$$

Note that $E_i(x) = s_i$. From equation (A19) we obtain a best response for agent i,

$$h_i = \left(s_i - \sum_{j\in I,j\neq i} h_j\right)/2. \tag{A20}$$

Signals are unbiased and each agent expects the other agents to observe the same signal so "by symmetry" we obtain an equivalent best response for each agent, $i \in I$. We can then solve for equilibrium claims which is given by $h_i^* = s_i/(n+1)$ for all $i \in I$.

The probability of a resource collapse is given by $\Pr(\sum_i^n h_i > x)$. Substituting for the equilibrium strategies and make use of the fact that $s_i = x + \varepsilon_i$, the probability of resource collapse can be rewritten as $\Pr\left(\sum_i^n \varepsilon_i > x\right)$. The sum of normally distributed variables is normal. For example if $\varepsilon_i \sim N(0, v_i)$ and $\varepsilon_j \sim N(0, v_j)$ then $\varepsilon_i + \varepsilon_j \sim N(0, v_i + v_j)$ (Green, 2000). The probability of resource collapse is therefore given by

$$1 - F_Z(x) = 1 - \frac{1}{\sqrt{2\pi v_z}} \int_{-\infty}^{x} e^{-\frac{u^2}{2v_z}} du \tag{A21}$$

where $v_z = \sum_i^n v_i$. The derivative of the probability of resource collapse with respect to total variance, v_z is given by:

$$\frac{1}{4}\frac{\sqrt{2}}{\sqrt{\pi}v_z^{\frac{5}{2}}}\left(v_z\int_{-\infty}^{x} e^{-\frac{1}{2}\frac{u^2}{v_z}}\,du - \int_{-\infty}^{x} u^2 e^{-\frac{1}{2}\frac{u^2}{v_z}}\,du\right) \tag{A22}$$

Because $e^{-\frac{1}{2}\frac{u^2}{v_z}}$ and $u^2e^{-\frac{1}{2}\frac{u^2}{v_z}}$ are both continuous and differentiable functions we can use the constant rule and make use of the fact that the sum of two integrals are the integral of two sums (Berck and Sydsæter, 1993).

$$\lim_{t\to-\infty}\left(v_z\int_{t}^{x} e^{-\frac{1}{2}\frac{u^2}{v_z}}\,du - \int_{t}^{x} u^2 e^{-\frac{1}{2}\frac{u^2}{v_z}}\,du\right) = \lim_{t\to-\infty}\int_{t}^{x}\left(e^{-\frac{1}{2}\frac{u^2}{v_z}}\left(v_z - u^2\right)\right)du \tag{A23}$$

Through integration we obtain

$$\left[uv_z e^{-\frac{1}{2}\frac{u^2}{v_z}}\right]_t^x = \lim_{t\to-\infty}\left[xv_z e^{-\frac{1}{2}\frac{x^2}{v_z}} - tv_z e^{-\frac{1}{2}\frac{(t)^2}{v_z}}\right], \tag{A24}$$

which is equal to

$$v_z \lim_{t\to\infty} te^{\frac{1}{2}\frac{t^2}{v_z}} + v_z x e^{-\frac{1}{2v_z}x^2} = v_z x e^{-\frac{1}{2v_z}x^2}, \tag{A25}$$

which is positive for positive values of v_z and x. ■

Observable actions - sequential ordering

Proof of Proposition 2. Given the exploitation level h_L by the leader, the follower solves the following problem;

$$\max_{r_F} E_F[h_F(x - h_L - h_F)]. \tag{A26}$$

The first order condition is

$$E_F[(x - h_L - 2h_F)] = 0. \tag{A27}$$

Note that $E_F(x) = \mu_F(x)$. This implies that the best response of the follower is given by;

$$h_F = (\mu_F(x) - h_L)/2, \tag{A28}$$

where

$$\mu_F(x) = \frac{v_F}{v_F + v_L} s_L + \frac{v_L}{v_F + v_L} s_F, \tag{A29}$$

according to Bayes updating. The leader anticipates the best response of the follower and solves the following problem;

$$\max_{h_L} E_L \left[h_L \left(x - h_L - \left(\frac{\mu_F(x) - h_L}{2} \right) \right) \right]. \tag{A30}$$

For ease of exposition let $v_F/(v_F + v_L) = V_F$ and $v_L/(v_F + v_L) = V_L$, the first order condition is then;

$$E_L \left[\left(x - 2h_L - \frac{s_F V_F + s_L V_L - 2h_L}{2} \right) \right] = 0. \tag{A31}$$

We know that because signals are both unbiased estimators of the true state, $E_L(s_F \mid s_L) = s_L$, moreover $E_L(x) = s_L$. The first order condition then reads

$$s_L - 2h_L - \frac{s_L - 2h_L}{2} = 0. \tag{A32}$$

Solving for h_L we obtain the equilibrium strategy of the leader,

$$h_L^* = \frac{s_L}{2}. \tag{A33}$$

By substitution we obtain the equilibrium strategy of the follower,

$$h_F^* = \frac{v_L}{v_F + v_L}\frac{s_F}{2} + \frac{v_F}{v_F + v_L}\frac{s_L}{2} - \frac{s_L}{4}. \tag{A34}$$

The Probability of resource collapse is given by

$$\Pr\left(\frac{x+\varepsilon_L}{2} + \frac{x+\varepsilon_L}{2}\frac{v_F}{(v_F+v_L)} + \frac{x+\varepsilon_F}{2}\frac{v_L}{(v_F+v_L)} - \frac{x+\varepsilon_L}{4} > x\right), \tag{A35}$$

which is equal to

$$\Pr\left(\varepsilon_L\frac{3v_F+v_L}{v_F+v_L} + \varepsilon_F\frac{2v_L}{v_F+v_L} > x\right). \tag{A36}$$

We know that if $\varepsilon \sim N(0, v_i)$ and a is a real number, then $\varepsilon_i a \sim N(0, v_i a^2)$, .(Green, 2000). This implies that

$$\varepsilon_L\frac{3v_F+v_L}{v_F+v_L} + \varepsilon_F\frac{2v_L}{v_F+v_L} \sim N\left(0, v_L\frac{9v_F+v_L}{v_F+v_L}\right). \tag{A37}$$

We let $v_Q = v_L\frac{9v_F+v_L}{v_F+v_L}$. The probability of resource collapse can then be written as follows;

$$1 - F_Q(x) = 1 - \frac{1}{\sqrt{2\pi v_Q}}\int_{-\infty}^{x} e^{-\frac{u^2}{2v_Q}}du. \tag{A38}$$

We know from the proof of Proposition 1 that probability of resource collapse is increasing in overall ignorance, v_Q. ■

Increasing the number of appropriators

Consider the following belief system consistent with Bayes updating.

$$\mu = \{\mu_2, \mu_1, ..., \mu_n\} = \begin{cases} \mu_1 = s_1 \\ \mu_2 = \frac{s_1 v_2 + s_2 v_1}{v_1 + v_2} \\ \mu_3 = \frac{\mu_2 v_3 + s_3 v_{\mu_2}}{v_{\mu_2} + v_3} \\ = \frac{s_1 v_2 v_3 + s_2 v_1 v_3 + s_3 v_1 v_2}{v_1 v_2 + v_1 v_3 + v_2 v_3} \\ ... \\ \mu_k = \frac{\mu_{k-1} v_{\mu_k} + \mu_k v_{\mu_{k-1}}}{v_{\mu_k} + v_{\mu_{k-1}}} \\ ... \\ \mu_n \end{cases} \tag{A39}$$

The variance of the *k:th* belief is given by:

$$v_{\mu_k} = \frac{v_{\mu_{k-1}} v_{\mu_k}}{v_{\mu_{k-1}} + v_{\mu_k}} \tag{A40}$$

Dividing the nominator and denominator with the product of variances, we obtain that

$$\mu_k = \frac{\sum_{i=1}^k \frac{s_i}{v_i}}{\sum_{j=1}^k \frac{1}{v_j}} \tag{A41}$$

$$v_{\mu_k} = \frac{1}{\sum_{j=1}^k \frac{1}{v_j}} \tag{A42}$$