The Beijer Institute of Ecological Economics

DISCUSSION PAPER

Beijer Discussion Paper Series No. 208

Pricing resilience in a dynamic economy-environment system: A capital-theoretic approach

Karl-Göran Mäler,Chuan-Zhong Li and Georgia Destouni. 2007.

The Beijer Institute of Ecological Economics
The Royal Swedish Academy of Sciences
Box 50005 SE-104 05 Stockholm Sweden
Phone: +46 8 673 95 00 Fax: +46 8 15 24 64
E-mail: beijer@beijer.kva.se

Pricing resilience in a dynamic economy-environment system: A capital-theoretic approach*

by

Karl-Göran Mäler[1],Chuan-Zhong Li[2] and Georgia Destouni[3]

[1]The Beijer Institute of Ecological Economics, the Royal Swedish Academy of Sciences, P.O. Box 50005, SE-104 05, Stockholm, Sweden

[2]Department of Economics, the University of Uppsala, P.O. Box 513, SE-751 20 Uppsala, Sweden

and [3]Department of Hydrology, Stockholm University, SE-106 91 Stockholm, Sweden

*Supported by the Swedish Research Council Formas.

Abstract

This paper develops a theory for pricing ecological resilience in a dynamic economy-environment system. Following Holling (1973), we define resilience as the maximal perturbation that the system can absorb without flipping into an undesirable state. Based on a multisector growth model, we derive the shadow price of resilience with respect to the probabilities that the system will flip in the future. We also expore the implications of different stochastic processes characterizing the resilience stock. The theory is illustrated by a numerical example from southeast Australia

Key Words and Phrases: Resilience, ecosystem, valuation, and inclusive wealth

1 Introduction

The recent decades have witnessed increasing awareness of the importance of ecosystem services for sustainable development (cf Perrings, 1995; Dasgupta and Mäler, 2000; Dasgupta and Mäler, 2001; Arrow et al, 2003). One of the services that has interested both ecologists and economists is the ecosystem's resilience and its role in dynamic welfare analysis. By resilience in this paper is meant the capacity for an ecosystem to cope with disturbances without shifting from a normal into a qualitatively different state which is less undesirable (Holling, 1973; Perrings and Walker, 1997; Carpenter et al, 2001; Walkers et. al., 2004). A system with very low resilience may simply lose its stability and functioning by a small perturbation while that with higher resilience may absorb larger shocks without any dramatic changes. This implies that policies that improve the resilience of a system should promote sustainability and improve human well-being. However, since resilience is typically not traded in the marketplace, there is no price information available to indicate its value. How to cast resilience in the framework of social cost-benefit analysis and sustainability measurement is, therefore, not a trivial issue.

Nevertheless, several attempts have been made along this direction in the recent literature. Serrao et. al. (1996) study sustainability and resilience informally in the context of the Amazonian upland ecosystems. The idea is that when the state of nature undergoes a change across a threshold, which lies beyond a society's ability to respond, the current social welfare may not be supported. Gunderson (2001) discusses the loss of ecological resilience in two ecosystems in Florida, one on the species loss in a wetland area and the other on the die-offs of seagrass in a shallow Bay. With a buffer of resilience in the systems, adaptive environmental assessment and management actions can provide robust responses to the loss. A more formal model of resource management subject to resilience effects is developed by Perrings and Walker (1997). They are concerned with the optimal management of rangelands in Australia where grazing affects fire risks which in turn determines the composition of species and the functioning of the ecosystem.

Although the concept of ecological resilience is taken in to account in the above-mentioned and a number of other studies (cf Norton, 1995; Scheffer et al, 2001; Trosper, 2002; Ekins et al., 2003), it is more or less regarded as an ancillary measure along the side of the "genuine" capital forms such as natural, physical and human capital stocks. This may undermine the role of resilience in a more comprehensive dynamic welfare analysis. The objective of this paper is to fill the gap by treating resilience as an asset, a stock variable, in its own right. For a tropical ecosystem, for example, a keystone species may play a vital role for the system's functioning and stability. When its biomass undergoes below a threshold level, an undesirable structural change may occur. In this case, the excess of the species' biomass over its minimum viable population level may constitute a resilience stock. The larger the excess is, the less likely that an external shock such as a sudden climatic change would drive down the keystone species to extinction, and thereby retain the normal flow of services from the ecosystem. In other words, the probability for the system to flip from the currently preferred state to an alternative one - an undesirable state - would be smaller (ceteris parabus) for a higher resilient system. Following this reasoning, we will attempt to price ecosystem resilience according to its marginal contribution to social well-being by its role in maintaining ecosystem functioning and stabilities. When the shadow price of the resilience stock is assessed, it will be possible to integrate the value in cost-benefit analysis and sustainability assessment.

The remaining part of the paper is structured as the following. In section 2, we develop a capital-theoretical model with the resilience stock considered as a stock variable. An exact formula for pricing the resilience stock is developed and its implication for dynamic welfare measurement is also explored. Section 3 and 4 provide an essential link between the flip probabilities in space and over time. The link is based on a generalized Ito process for the resilience stock, which is required as an input in the pricing formula. Section 5 illustrates the theory by an example from Australia on the value of resilience in an ecosystem. Section 6 sums up the study.

2 The capital-theoretic model

Consider an economy-environment system with a number of conventional capital stocks, including human resources, natural and environmental assets, with initial values $\mathbf{K}(0) = \mathbf{K}_0 > \mathbf{0}$ at time $t = 0$. In addition, there is a pure resilience stock $X(t)$ following a dynamic process over time $t \geq 0$, starting from an initial stock $X(0) = X_0$. We assume two possible regimes, a normal one with the resilience stock $X(t)$ greater than its threshold value[2] $\tilde{X}$, i.e. $X(t) > \tilde{X}$, associated with a normal stream of utility flows, and a disturbed regime following a flip of the ecological system with $X(t) < \tilde{X}$. Initially, we have the normal regime such that $X_0 > \tilde{X}$. However, as time goes, there is a probability at each point in time t such that the system flips from the normal to a disturbed regime. The flip probabilities over time will depend on the properties of the underlying stochastic process $X(t)$.

For the moment, let us simply assume that the probability density for a flip at time $s > 0$, from the normal regime to a disturbed, an undesirable regime, be $\theta(X_0, s)$. The cumulative probability for a flip over a time interval $[0, t]$ becomes then $F(X_0, t) = \int_0^t \theta(X_0, s) ds$, with properties $F(X_0, 0) = 0$ and $\lim_{t \to \infty} F(X_0, t) = 1$. The corresponding probability for the normal regime to survive over the interval $[0, t]$, conditional on a normal regime at the initial date 0 can be expressed as $S(X_0, t) = 1 - F(X_0, t)$. With a given stream of flip risks, we will first derive an expression for the expected intertemporal welfare, and then study the implicit value of the resilience stock. Conditional on the normal regime, a governing rule α_1 as defined in Arrow et al. (2003) would map the initial condition $\mathbf{K}_0$ into a stream of vector-valued consumptions $\mathbf{C}_1(t)$ and capital stocks $\mathbf{K}_1(t)$ such that $\mathbf{C}_1(t) = \mathbf{F}(\mathbf{K}_0, \alpha_1, t)$ and $\mathbf{K}_1(t) = \mathbf{G}(\mathbf{K}_0, \alpha_1, t)$, $t \in [0, \infty)$, where $\mathbf{F}$ and $\mathbf{G}$ are two different vector-

[2] Without loss of generality, we can always normalize the resilience stock to be $X(t) - \tilde{X}$ such that the normalized threshold is identically 0.

valued functions. If the system flips at time $s \in [0, \infty)$ with structural changes, then consumption and capital stocks henceforth would follow an alternative path governed by an adapted rule α_2 such that $\mathbf{C}_2(t) = \mathbf{F}(\mathbf{K}(s), \alpha_2, t - s)$ and $\mathbf{K}_2(t) = \mathbf{G}(\mathbf{K}(s), \alpha_2, t - s)$, for $t \in [s, \infty)$.

Preferences are represented by a time-invariant utility function $V(\mathbf{C}(t), \mathbf{K}(t))$, which satisfies all regularity conditions. Note that we allow capital stocks to enter the utility function as some environmental assets may generate amenity values. For notational ease, we denote the instantaneous utility at time t conditional on the normal regime by $U_1(t) = V(\mathbf{C}_1(t), \mathbf{K}_1(t))$, and that conditional on the disturbed regime by $U_2(t) = V(\mathbf{C}_2(t), \mathbf{K}_2(t))$ for all $t \in [0, \infty)$. Obviously, $U_1(t) > U_2(t)$ for any point in time t.

Suppose that the system flips at a *known* date s, then the intertemporal welfare at time $t = 0$ would be

$$W_0(s) = \int_0^s U_1(t) \exp(-rt))dt + \int_s^\infty U_2(t) \exp(-rt)dt \tag{1}$$

where r denotes the rate of pure time preference. For *stochastic* flipping dates, the expected intertemporal welfare at time $t = 0$ can be expressed by

$$E(W_0) = \int_0^\infty \theta(X_0,s)W_0(s)ds = \int_0^\infty W_0(s)dF(X_0,s) \tag{2}$$

By integrating (2) by parts and making use of (1), the expected intertemporal welfare at time $t = 0$ can now be written as

$$\begin{aligned} E(W_0) &= W_0(s)F(X_0,s)\,|_0^\infty - \int_0^\infty F(X_0, s)\,[U_1(s) - U_2(s)] \exp(-rs)ds \\ &= \int_0^\infty U_1(s) \exp(-rs)ds - \int_0^\infty F(X_0, s)\,[U_1(s) - U_2(s)] \exp(-rs)ds \end{aligned} \tag{3}$$

where the second equality follows from the properties of cumulative distribution function $F(X_0,s)$ and the boundedness of the utility function $V(.)$. While the first integral on the second line of (3) represents the "normal" wealth, the second integral measures the expected loss in intertemporal welfare due to the risks of future flips. An alternative expression of (3) is

$$E(W_0) = \int_0^{\infty} \left[S(X_0, s) U_1(s) + F(X_0, s) U_2(s) \right] \exp(-rs) ds \tag{4}$$

where $S(X_0, s) = 1 - F(X_0, s)$ denotes the survival probability of the normal regime from time $t = 0$ to s, conditional on an initial normal state $X_0 > \tilde{X}$. The expression (4) corresponds to the present discounted value of future expected utilities - the weighted average of two extreme utility streams, one as the perfectly normal stream and the other as the fully disturbed utility stream over an infinite future. Note that we have converted the expression involving two subsequent time periods to an expression of a linear combination of two parallel streams due to the stochastic flip dates.

Now, what is the shadow value of the resilience stock X_0 at the initial date $t = 0$? By applying the Leibniz rule, we obtain the following expression for this shadow value

$$q(0) = \frac{\partial E(W_0)}{\partial X_0} = \int_0^{\infty} \frac{\partial S(X_0, s)}{\partial X_0} \left[U_1(s) - U_2(s) \right] \exp(-rs) ds \tag{5}$$

Since the resilience stock is defined in a positive way, we expect $\frac{\partial S(X_0,s)}{\partial X_0} \geq 0$, i.e. the survival probability up to a given date s is a non-decreasing function of the initial resilience stock at the initial date. Then, by the assumption $U_1(s) - U_2(s) \geq 0$ for all $s \in [0, \infty)$, we have $q(0) \geq 0$, i.e. the marginal contribution of each unit of the resilience stock is non-negative.

For dynamic welfare analysis, it proves convenient to define the expected wealth at the initial date as a function of the capital stocks including the resilience stock such that $\hat{W}(\mathbf{K}_0, X_0) = E(W_0)$ as in (4). The change in the the value of this measure over an infinitessimal time interval $[0, dt]$ is

$$\begin{aligned} d\hat{W}(\mathbf{K}_0, X_0) &= \frac{\partial \hat{W}(\mathbf{K}_0, X_0)}{\partial \mathbf{K}_0} d\mathbf{K} + \frac{\partial \hat{W}(\mathbf{K}_0, X_0)}{\partial X_0} dX \\ &= \mathbf{P}(t) d\mathbf{K}(\mathbf{t}) \mid_{t=0} + q(t) dX(t) \mid_{t=0} \end{aligned} \tag{6}$$

where $\mathbf{P}(0) = \partial \hat{W} \mathbf{K}_0, X_0)/\partial \mathbf{K}_0$ denotes the vector of shadow prices for the conventional capital stocks. Note that this price vector does not correspond to shadow prices following the hypothetical "perfectly normal path". Thus, it is possible to decompose $\mathbf{P}(0)$ into two parts, one as the perfectly "normal" shadow prices and the other as the expected losses, but we will not pursue this issue further here. According to Arrow et al (2003), if the change in their inclusive wealth, i.e. the genuine investment, is positive, i.e. $d\hat{W}(\mathbf{K}_0, X_0) > 0$, then welfare over the short time interval $[0, dt]$ is increasing, or the initial welfare is sustainable.

3 Flip probabilities in space and time

Now, we will relate the cumulative probability $F(X_0, s)$ or equivalently the survival function $S(X_0, s)$ to the dynamic process $X(t)$. Without loss of generality, we assume that $X(t)$ follows a stochastic process and the threshold $\tilde{X}$ remains constant, though there are situations for which it may be more realistic to assume the actual paths $X(t)$ as deterministic while the threshold $\tilde{X}$ is stochastic over time. This latter case can be readily handled by normalizing the resilience stock by $R(t) = X(t) - \tilde{X}$ with a normalized threshold value 0.

Initially, we have $X(0) > \tilde{X}$ (i.e. $R(0) > 0$) such that the ecosystem is in the normal regime. As time goes, the stochastic variable $X(t)$ fluctuates such that the probability for the system to flip over a short time period $[t, t + dt]$, conditional on no flip up to t, can be described by

$$\Pr(X(t) \leq \tilde{X}) = G_t(\tilde{X}) \tag{7}$$

where $G_t(\cdot)$ denotes the cumulative probability function for $X(t)$ at time t, valid for the infinitesimal time interval $[t, t + dt]$. Along the other dimension, this is also seen as the hazard rate at time t, $\lambda(t)$, so that

$$\Pr(X(t) \leq \tilde{X}) = G_t(\tilde{X}) = \lambda(t) \tag{8}$$

The survival probability over $[t, t + dt]$, conditional on a survival up to time t, can thus be expressed as

$$1 - \lambda(t)dt \approx \exp\left(-\lambda(t)dt\right) \tag{9}$$

for $dt \to 0$. This can be readily verified by a first order Taylor expansion of the exponential function $\exp(-\lambda(t)dt)$ at $dt = 0$. While the discrete version of the total survival probability from time 0 to η is given by $\Pi_{n=1}^{m}\left(1 - \lambda(t_i)dt\right)$ with $t_1 = 0, t_i = t_{i-1} + dt$, $t_m = \eta$, $i = 2, 3, ..., m$, the corresponding continuous time version of the survival probability function from time 0 to t, conditional on the initial resilience stock X_0, becomes

$$S(X_0, t) = \exp\left(-\int_0^t \exp\left(\lambda(s)ds\right)\right) = \exp\left(-\Lambda(t)\right) \tag{10}$$

where $\Lambda(t) = \int_0^t \exp\left(\lambda(s)ds\right)$ denotes the integrated hazard function. The cumulative distribution function for a flip up to time t is given as above by $F(X_0, t) = 1 - S(X_0, t)$.

It is worth mentioning that the cumulative distribution function $G_t(X)$ in (7) is defined over "space" (the resilience stock) for a given time t, whereas the cumulative distribution function over "time", $F(X_0, t)$, is defined over time for $[0, t]$. How are the two different cumulative distribution functions related to each other? And how are they associated with the probability density function $\theta(X_0, t)$ from the previous section? Since $1-$ $F(X_0, t) = \exp\left(-\Lambda(t)\right)$, we can take the time derivative on both hand-sides to obtain

$$\theta(X_0, t) = \lambda(t)\exp\left(-\Lambda(t)\right) \tag{11}$$

and then to integrate then densities to obtain

$$F(X_0, t) = \int_0^t \lambda(s)\exp\left(-\Lambda(s)\right)ds \tag{12}$$

To get a better feel about the relationships, let us assume that the hazard rate $\lambda(t)$ is a constant $\bar{\lambda}$, then the integrated hazard becomes $\Lambda(t) = \exp(\bar{\lambda}t)$, and therefore the flip probability follows an exponential form with $\theta(X_0, t) = \bar{\lambda}\exp(-\bar{\lambda}t)$ and the survival probability $S(X_0, t) = \exp(-\bar{\lambda}t)$. Given a positive stream of the hazard rate, the probability for survival over an infinite time horizon is zero since $\lim_{t\to\infty} S(X_0, t) = 0$. Then, what is the implicit price of the resilience stock? Well, if the resilience stock is defined in a positive way with $S_1(X_0, t) > 0$, then a larger stock X_0 would increase the survival probability $S(X_0, t)$ over time and thereby delay the expected flip date.

Now, the value of resilience can in principle be calculated by substituting (10) for $S(X_0, t)$ in (5), and the expected welfare indicator with respect to $dX(t)$ becomes

$$\begin{aligned} E(d\hat{W}(\mathbf{K}_0, X_0)) &= \frac{\partial \hat{W}(\mathbf{K}_0, X_0)}{\partial \mathbf{K}_0} d\mathbf{K} + \frac{\partial \hat{W}(\mathbf{K}_0, X_0)}{\partial X_0} E(dX) \\ &= \mathbf{P}(t) d\mathbf{K}(\mathbf{t}) \mid_{t=0} + q(t)E(dX(t)) \mid_{t=0} \end{aligned} \tag{13}$$

4 The resilience stock dynamics as a stochastic process

Depending on the type of ecosystems, the resilience stock may evolve in different ways. To fix ideas, we assume a convenient stochastic process for a stylized ecosystem, a generalized Ito process

$$dX(t) = a(X, t)dt + b(X, t)dZ(t)$$

where $dZ(t) = \varepsilon\sqrt{dt}$ with ε assumed to be normally distributed with zero mean and constant variance. Then, the expected welfare indicator depends on whether the drift rate is zero or not. If there is no drift ($a = 0$), then the value of resilience should not enter the welfare formula in (13) even if resilience itself has a value $q(t)$. If there is a drift, then we have

$$E(d\hat{W}(\mathbf{K}_0, X_0)) = \mathbf{P}(t) d\mathbf{K}(\mathbf{t}) \mid_{t=0} + q(t)a(X, t)dt \mid_{t=0} \tag{14}$$

We can work out for an example, with the mean reverting, the Ornstein Ulhlenbecl model (cf Dixit and Pindyck, 1994)

$$dX(t) = \alpha(\bar{X} - X(t))dt + \sigma dZ(t) \tag{15}$$

where $\bar{X}$ is the long-run mean value of $X(t)$, $\alpha > 0$ an adjustment parameter on the speed of convergence, and σ the standard deviation of the change. The larger the α value is, the quicker the process adjust back to its long-run normalized value $\bar{X}$. Following this process, the cumulative distribution function over "space", $G_t(X)$, has a mean value

$$E(X(t)) = \bar{X} + (X - \bar{X})\exp(-\alpha t) \tag{16}$$

and the variance as

$$Var(X(t)) = \frac{\sigma^2}{2\alpha}(1 - \exp(-2\alpha t) \tag{17}$$

which can be derived from the Kolmogorov equations and the moment generating functions. Equation (16) describes the evolution of the expected resilience stock, and (17) the variance at each point in time t. By linking these central tendency measures over space to the expressions in (7) and (10), we will be able to assess the stream of flip probabilities over time.

5 A numerical example from Southeast Australia

The data used in the section is taken from our recent joint paper by Walker et al (2006) describing the situation of the Goulburn-Broken Catchment in southeast Australia. About 300,000 hectares in the lower part of the catchment are used for dairy pastures, agricultural production and nature conservation. The problem is that the intensive land use has resulted in biodiversity losses and rising groundwater tables, which increases the risk of soil salinization. When the water table reaches 2 meters or less below the surface, the water with dissolved salt would be drawn to the surface by capillary actions, and the process can in practice be regarded irreversible. Irrigation or rain may flush the salt down through the soil profile, but they also add

Table 1: Stocks and values under the two different regimes

	Area (1000 ha)	Price \$/ ha Normal regime	Price \$/ ha Alternative regime
Dairy land non-salinizable	48	385.85	385.85
Dairy land subject to salinity	192	385.85	38.59
Horticultural land non-salinizable	4.8	677.16	677.16
Horticultural land subject to salinity	19.2	677.16	6.77

to the height of the water table pushing up the salt. Since the saline soil is less fertile, the value of the land should depend on the water table and thereby the risk of soil salinization. The areas and unit prices (in present value) of four different land use categories for the year 2001 are shown in table 1, both for the normal regime (water table below 2 meters below surface) and the alternative one (water table less than 2 meters below surface). It can be seen that some land areas are non-salinizable even if the water goes up close to the surface such that the land prices remain unchanged. In addition, a salinization will reduce the price of dairy land by 90% and that of the horticultural land by 99%.

The water table typically fluctuates due to instantaneous changes in temperature and precipitation as well as the overall trend of the climate, and thus it can be best characterized as a stochastic process. According to a scenario with continued dry climate condition in the future (Walker et al, 2006), the water table is expected to fall from its "initial" 3.5 meters in 2001 below surface to an average of 5 meters in the year of 2030. In spite of this trend, the probability for the water table to rise close to surface to salinity the soil is close to 1.0, if no preventive actions taken.

Based on these facts, we describe the water table dynamics by

$$dX(t) = 0.09(5 - X(t))dt + 0.5dZ,\ X(0) = 3.5 \tag{18}$$

A random realization of the process is depicted in Figure 1 with fluctuations in green color. The curve in red represents the trend for the water table to fall from 3.5 to

5.0 meters over the 30 years period, and the lower and upper dotted curves in blue denote the 95% confidence interval.

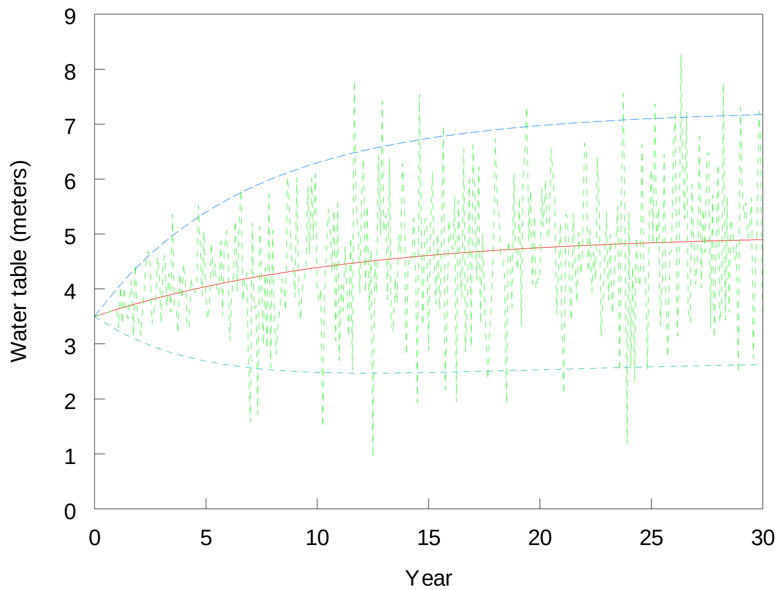

Figure 1: Simulated stochastic water tables with trend

What is our resilience stock here? We may define it by the distance from the critical water table $\tilde{X} = 2$. For example, if $X(t) = 3.5$ meters, then the resilience stock is $R(t) = X(t) - \tilde{X} = 1.5$ meters; and if $X(t) = 10$ meters, the resilience stock is $R(t) = 8$ meters. Salinization would certainly take place when the resilience stock lies below 0. However, since $X(t)$ is random for $t > 0$, we can only state the salinity risk in probability terms.

Using (8) through (10), we have calculated the survival probabilities over time for the stochastic resilience stock process (18), as depicted in Figure 2. While the solid curve in blue describes the probability that the system will remain in its normal state starting from an actual initial water table $X_0 = 3.5$, the dotted curve in green depicts the survival probability over time for an hypothetical initial state with $X_0 = 4.5$.

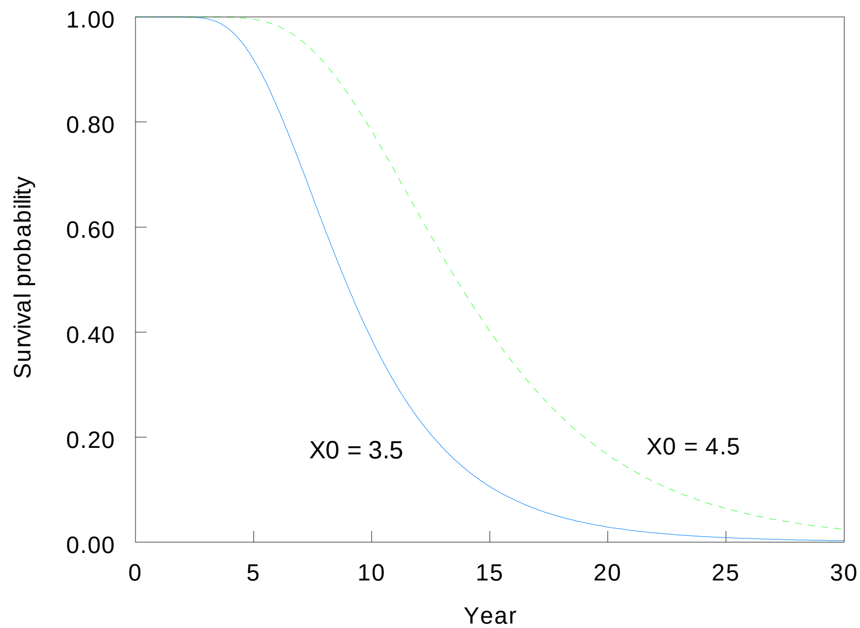

Figure 2: Survival probabilities for different initial water tables

To calculate the shadow price of the resilience stock, we also need to assess the instantaneous loss[3] caused by salinization, $\Delta U(t) = U_1(t) - U_2(t)$, at each point in time t. As in Walker et al (2006), we calculate the loss based on the market prices of the land under different regimes. For this example, there is no change in the areas of the land use categories, i.e. the ordinary capital stocks. Then, by assuming that the annual interest rate as $r = 2.5\%$, we obtain the average monthly loss measured in real prices as

$$\Delta U(t) = \frac{0.025}{12}\,(192 \cdot 385.85 \cdot 0.9 + 19.2 \cdot 677.16 \cdot 0.99) = 165.72 \qquad (19)$$

[3]In this paper, we consider the lowering of the (salted) water table as an economic good that contributes positively to production and human welfare. This is true for the GBC in southeast Australia. However, we are fully aware that in other regions with (clean) water supply shortages, a water table decrease would affect human welfare negatively.

measured in thousand dollars. Then, we calculate the marginal value of the resilience stock by using a discrete-time version of (5), i.e. the present discounted value of the expected gains over 360 months (30 years) accrued from a marginal increase in the initial resilience stock:

$$\begin{aligned} q(0) &= \sum_{i=1}^{360} \Delta S(i) \cdot \Delta U(t) \cdot \left(1 + \frac{r}{12}\right)^{-i} \\ &= \sum_{i=1}^{360} \Delta S(i) \cdot (165.72) \cdot \left(1 + \frac{0.025}{12}\right)^{-i} \\ &= 10858 \end{aligned}$$

where $\Delta S(i)$ measures the change in survival probability for month i, ($i = 1, 2, ..., 360$), caused by a hypothetical perturbation of the initial water table from 3.5 to 4.5 meters from the surface - the vertical distance between the two curves in figure 2, and the third term is the discount factor. The interpretation of the number is that, if the resilience stock increases by one meter, i.e. the water table falls from 3.5 to 4.5, then the reduced future flip risks caused by the change in water table is worth 10858 thousand dollars. We also calculate the shadow prices of the the resilience stock for some hypothetical initial water tables, e.g. $X_0 = 4.0$, 4.5 and 5.0, and obtain the shadow prices as 8629, 7674 and 6735, respectively. It is seen that the lower the water table, or the higher the resilience stock, the lower the shadow price per unit change in the water table, as expected. These number may be useful for economic analysis at least in two ways. One is social cost benefit analysis. Suppose that the cost to pump up the salted water to lower the water table from 3.5 to 4.5 below the surface is less than 10858, then it is socially profitable to pump, otherwise not. Another application is to incorporate the value in dynamic welfare analysis within the inclusive wealth framework (Arrow et. al. 2003). For a local-in time change, say, from year 2001 to 2002, the expected decrease in the water table is $E\left(dX\right) = 0.09 \cdot 1.5 = 0.135$ meters according to (16). The corresponding change in the inclusive wealth over the year becomes

$$
\begin{aligned}
E(d\hat{W}(\mathbf{K}_0, X_0)) &= \mathbf{P}(t)d\mathbf{K}(\mathbf{t}) \mid_{t=0} + q(t)E(d(X(t)) \mid_{t=0} \\
&= 0 + 10858 \cdot 0.135 \\
&= 1466
\end{aligned}
$$

where the contribution of the ordinary capital stocks (the areas of the four land use categories) is zero as they are assumed to be constant over time. The increase in inclusive wealth here is therefore purely due to the improved resilience by the natural trend of falling water tables. It is worth mentioning that the trend may also be forced by pumping activities or other measures to enhance the inclusive wealth, though the cost os installing and running the pumps should be deducted. How should we evaluate the dynamic welfare effect over a longer time period? Following Arrow et al (2003) and Dasgupta and Mäler (2001), we calculate the change in the expected inclusive wealth over the whole 30-year-period by

$$
\begin{aligned}
\Delta W &= q(t)\Delta X(t) \mid_{t=0}^{30} - \int_0^{30} E(X(t))\dot{q}(t)dt \\
&= 13598
\end{aligned}
$$

in which the capital gains over time have been removed. With this stylized example, we can conclude that the intertemporal welfare as of the year 2001 can be sustained in the future[4].

6 Concluding remarks

In this paper, we have developed a capital theoretical model for pricing ecological resilience. In additional to the conventional capital stocks such as natural, physical and

[4]It is worth mentiong that we have not included the contribution of other services of the land, regional infrastructural and educational investments etc in the calculation exercise here. The issue will be explored further in the next stage of the project. After all relevant value components are aggregated in an inclusive wealth framwork, we can say more about the overall sustainability prospect about the region.

human capital, we also consider the variable that affects the resilience of ecosystem functions and stability as a capital stock. Thus, the resilience stock may have a value in its own right even if it may not be directly involved in the production process. In comparison to the other capital forms, this resilience asset enters the model in a different way. Under the normal condition, it does matter for the ecosystem services and human-well being. However, when the resilience stock variable crosses a threshold, then the ecosystem will flip into a qualitatively different state, presumably an undesirable one. After the flip, the production potential and other intangible services of the system will be rather different and the governing rules for managing the system should also be adapted. If we look at the ex post outcome, the inclusive wealth (Arrow et al., 2003) will undergo an abrupt, a non-continuos fall, at the time when the flip takes place. However, since the flip date in the future is not known with certainty, we will take advantage of the (ex anti) expected inclusive wealth as a welfare measure, which is smooth over time. The pattern of future flip risks depends on how the resilience stock evolves over time.

When the resilience stock is high, e.g. when the biomass of a keystone species is far above its minimum viable population level, we would expect a high probability for the system to remain stable for a given future time horizon. On the contrary, if the system resilience is close to zero, then a small external shock may drive the species into extinction. Thus, the resilience stock has a value per se for its role in retaining ecosystem functioning and stability. We consider the dynamics of the resilience stock as a stochastic process, and developed a link between the flip probability over "space", e.g. biodiversity, vegetation connectivity, ground water table, and that over time through the use of hazard rates. The shadow price of resilience is formally defined as the present discounted value of future improvements in welfare accrued from the reduced risks of flip due a unit increase in the initial resilience stock.

The capital-theoretic part of the paper resembles to a some extent the theory of catastrophes (cf Dasgupta and Heal, 1974; Cropper, 1976; Tsur and Zemel, 2006) but with a different highlight. Rather than examining the adaptation strategies, we focus on deriving the shadow price of resilience in order to be integrated in social cost-benefit analysis and inclusive wealth evaluations. To illustrate our theory, we also provide a numerical example from southeast Australia. The resilience stock in this case is defined as the distance between the underground water table and a threshold value of it above which the dissolved salt will cause severe soil salinization. Using a mean-reversing stochastic process for the water tables, we have calculated the shadow prices of the resilience stock, and applied results for making welfare comparison using the Arrow et al (2003) inclusive wealth model.

It is worth mentioning that we have in this paper only considered a single resilience stock. In reality, however, there may be multiple variables with simultaneous and interdependent threshold effects. In this case, the model should be expanded to a vector of resilience variables with the interdependence between them taken into account. In addition, the resilience stock considered in the present paper is assumed to affect the flip probability only with no other functions. In case that the stock also has other functions such as amenity services etc., the contribution of the stock should also be included in the utility function. Finally, we have assumed that the stochastic trend of the resilience stock is determined by external forces such climate changes. In future research, it will prove useful to "internalize" the trends as an endogenous function of purposive policies and management actions.

7 References

Arrow, K. J., P. Dasgupta, and K.G. Mäler. (2003). "Evaluating Projects and Assessing Sustainable Development in Imperfect Economies." Environmental and Resource Economics 26 4: 647-85.

Carpenter, S., Walker, B., Anderies, J.M., and Abel, N. (2001). "From metaphor to measurement: resilience of what to what?" Ecosystems 4: 765-781.

Cropper, M.L. (1976), 'Regulating activities with catastrophic effects', Journal of Environmental Economics and Management, 3: 1–15.

Dasgupta, P. and K.-G. Mäler (2000). "Net National Product, Wealth, and Social Well-Being." Environment and Development Economics 5 1-2: 69-93.

Dasgupta, P. and G.M. Heal (1974), 'The optimal depletion of exhaustible resources', Review of Economic Studies, Symposium, 3–28.

Dasgupta, P. and K.-G. Mäler, "Wealth as a Criterion for Sustainable Development", World Economics, 2001, 2(3), 19-44.

Dixit, A. K. and R. S. Pindyck (1994). "Investment under uncertainty." xiv, 468.

Ekins P., S. Simon, L. Deutsch, C. Folke, and R. S. de Groot. (2003). "A framework for the practical application of the concepts of critical natural capital and strong sustainability". Ecological Economics, 44:165-185.

Gunderson, L. H. (2001). "Managing Surprising Ecosystems in Southern Florida." Ecological Economics 37 3: 371-78.

Holling, C.S. (1973). "Resilience and stability of ecological systems". Annual Review of Ecology and Systematics 4: 1-23.

Norton, B. (1995), "Resilience and options", Ecological Economics, 15, p.133–136.

Perrings, C. (1995), 'Ecological resilience in the sustainability of economic development', Economie Appliquee, 48: 121–141.

Perrings, C. and Walker, B. (1997), “Biodiversity, resilience and the control of ecological-economic systems: the case of fire-driven rangelands”, Ecological Economics, 22, p.73–83.

Perrings, C. and Stern, D. (2000), “Modelling loss of resilience in agroecosystems: rangelands in Botswana”, Environmental and Resource Economics, 16, p.185–210.

Scheffer, M., S. R. Carpenter, J. A. Foley, C. Folke, and B. Walker. (2001). "Catastrophic shifts in ecosystems". Nature 413:591-596.

Serrao, E. A. S., D. Nepstad, et al. (1996). "Upland Agricultural and Forestry Development in the Amazon: Sustainability, Criticality and Resilience." Ecological Economics 18 1: 3-13.

Trosper, R. L. (2002), “Northwest coast indigenous institutions that supported resilience and sustainability”, Ecological Economics, 41, p.329–344.

Tsur, Y. and A. Zemel (2006). "Welfare Measurement under Threats of Environmental Catastrophes." Journal of Environmental Economics and Management 52 1: 421-29.

Walker, B., H., C. S. Holling, S. R. Carpenter, and A. P. Kinzig. (2004). "Resilience, adaptability, and transformability". Ecology and Society 9(2): 5.

Walker, B., Pearson, L., Harris, M., Mäler K.G., Li, C.Z. and Biggse, R. (2006). "Incorporating Resilience in the assessment of Inclusive Wealth: An example from South East Australia". Memo.