The Beijer Institute of Ecological Economics

DISCUSSION PAPER

Beijer Discussion Paper Series No. 242

Spatial Policies and Land Use Patterns: Optimal and Market Allocations

Efthymia Kyriakopoulou and Anastasios Xepapadeas. 2013.

The Beijer Institute of Ecological Economics
The Royal Swedish Academy of Sciences
Box 50005 SE-104 05 Stockholm Sweden
Phone: +46 8 673 95 00 Fax: +46 8 15 24 64
E-mail: beijer@beijer.kva.se

Spatial Policies and Land Use Patterns: Optimal and Market Allocations

Efthymia Kyriakopoulou*

University of Gothenburg and Beijer Institute of Ecological Economics

Anastasios Xepapadeas†

Athens University of Economics and Business and Beijer Fellow

Abstract

Environmental conditions and pollution levels have been proven to affect firms' and households' location decisions in various ways. In this paper, we study the optimal and equilibrium distribution of industrial and residential land in a given region. Industries produce a single good using land and labor and generate emissions of a pollutant, and households consume goods and residential land and dislike pollution. The trade-off between the agglomeration and dispersion forces, in the form of industrial pollution, environmental policy, production externalities, and commuting costs, determines the emergence of industrial and residential clusters across space. We also show that the joint implementation of a site-specific environmental tax and a site-specific labor subsidy can reproduce the optimum as an equilibrium outcome.

JEL classification: R14, R38, H23.

Keywords: Agglomeration, land use, spatial policies, pollution, environmental tax, labor subsidy.

* *Corresponding Author*, Department of Economics, University of Gothenburg, Box 640, Vasagatan 1, SE 405 30, Gothenburg, Sweden. Telephone: +46 317862641. Email: efi.kyriakopoulou@economics.gu.se

† Department of International and European Economic Studies, Athens University of Economics and Business and Beijer Fellow E-mail: xepapad@aueb.gr

1 Introduction

The formation of residential and industrial clusters in a city or region reflects the existence of forces that drive the observed spatial patterns. Agglomeration and dispersion forces have been extensively analyzed in the literature of urban economics and have played an important role in explaining the initial formation and the further development of cities. In this context, it has been proven that firms benefit from operating closer to other firms since it gives them access to a pool of knowledge and the possibility of exchanging ideas is likely to boost productivity. And this is where workers come into the picture, as firms have to compete not only with the rest of the firms when they choose their location, but also with workers. Since commuting always implies extra costs, which increase with the distance, workers prefer to locate closer to their workplaces. Thus, even though in most regions of the globe there is excess supply of cheap land, economic agents are willing to pay high land rents in order to locate in large centers.

Apart from the above forces, which are well-known from both the theoretical and empirical literature, there are additional determinants of the location decisions of economic agents that need to be studied in a formal framework. Air pollution is considered an unambiguously significant factor of concern to both industries and consumers in many ways. Industries generate emissions, and since workers are negatively affected by pollution they try to avoid locating near them. However, the spatial interdependence of industries and workers explained above makes the problem of air pollution even bigger. If industries were located in pure business areas with no residents around, then the damage from the generation of emissions would be much lower compared with the case of industries being located close to residential or mixed areas. As regards urban areas, where pollution problems are getting increasingly serious, it is easy to understand that pollution externalities should be studied in a spatial context.

The interaction between industrial pollution and residential areas has often been identified as a reason for government intervention. Mostly in developed countries, governments impose high taxes or different kinds of environmental policy on polluting activities or force industries to take abatement measures to reduce the total level of pollution. In the EU, for example, air pollution has been one of the main political concerns since the late 1970s. In this context, one of the objectives of the Sixth Environment Action Programme (2010) is to achieve air quality levels that do not give rise to unacceptable impacts on human health and the environment. Different actions need to be taken at local, national, European, and international level, which clearly points to the spatial aspect of the problem.[1] In this way, the role of environmental policy is crucial in the development of residential and industrial clusters, as strict environmental measures can discourage firms from operating in specific areas, while the reduced pollution levels that will result from this kind of policy could encourage people to again locate close to business areas.

This paper contributes to the literature by extending the general equilibrium models of land use by incorporating environmental externalities. More specifically, we study how pollution from stationary sources – which affect workers negatively and make governments impose environmental regulations – combined with other agglomeration forces such as externalities in production and commuting cost will finally determine the internal structure of a region. Once the optimal and equilibrium land uses are specified, we characterize two kinds of spatial policies that can be used in order to implement the optimal allocations as equilibrium outcomes. In particular, the derived market allocations differ from the optimal ones due to the assumed externalities in the form of positive knowledge spillovers and pollution diffusion. Thus, we use the spatial model to define site-specific policies that will improve the efficiency in the given region. We show that the joint enforcement of a site-specific pollution tax and a site-specific labor subsidy will reproduce the optimal allocation as a market outcome. Numerical experiments will illustrate the differences between the two solutions and will show that industrial areas are concentrated in smaller intervals in the optimal solution. Also, mixed areas emerge in the market allocation but not in the optimal one.

[1] Information on the environmental policy enforced by the EU can be found at http://ec.europa.eu/environment/air/quality/index.htm)

The reason we use the existence of interactions among firms as the basic agglomeration force of the model lies in the fact that they have been proven to be the basic force for the clustering of economic activity.[2] These interactions facilitate exchange of information and knowledge between firms, which means that, other things being equal, each firm has an incentive to locate closer to the other firms, forming industrial or business areas. On the other hand, the formation of a pure business center increases the average commuting cost of workers and gives rise to higher wages and land rents in the area surrounding the cluster. This process acts as a centrifugal force that impedes further agglomeration of firms.

The trade-off of production externalities and commuting costs has been explained extensively in a lot of studies, such as in Lucas and Rossi-Hansberg (2002), Rossi-Hansberg (2004) and Fujita and Thisse (2002) (Chapter 6). In an earlier paper, Fujita and Ogawa (1892) presented a model of land use in a linear city, where the population was fixed and firms and households would compete for land at the different spatial points. In this paper, using a general equilibrium model of land use and following Lucas and Rossi-Hansberg (2002) in the modeling of knowledge spillovers, we examine how pollution created by emissions, which are considered to be a by-product of the production process, determines the residential and industrial location decisions and hence affects the spatial structure of a region. Accordingly, pollution affects negatively the productivity of labor, implies implementation of environmental policy in the form of a site-specific tax, and discourages workers from locating in polluted sites. An important point here is that pollution comes from a stationary source yet diffuses in space, creating uneven levels of pollution at different spatial points. As far as the policy is concerned, firms will be obliged to pay a site-specific pollution tax, the size of which will depend on the marginal damage of pollution at the site where they will decide to locate. However, the higher the number of industries that locate in a spatial interval, the more polluted this interval will be. Thus, if firms decide to locate close to each other so as to benefit from positive knowledge spillovers, they will have to pay a higher pollution tax and suffer some loss in the

[2] For empirical studies confirming the role of knowledge spillovers in the location decisions of firms, see Keller (2002), Bottazzi and Peri (2003) and Carlino et al.(2007).

form of decreased labor productivity due to pollution. Thus, pollution discourages the agglomeration of economic activity. As for the consumers, they are negatively affected by pollution and prefer to locate in "clean" areas. Yet this means that they will have to move further away from the firms, which implies higher commuting costs. The balance among these opposite forces, as well as the use of land for both production and residential purposes, will finally define the industrial and residential areas.

The first models of spatial pollution (e.g., Tietenberg, 1974, Henderson, 1977) assumed a pre-determined location for housing and industry, without giving the possibility to workers to locate in an area that is already characterized as industrial and without allowing for a change in the spatial patterns. The paper that is closest to the present one in the modeling of pollution is Arnott et al. (2008), who assume non-local pollution in order to investigate the role of space in the control of pollution externalities. More analytically, they study how the trade-off between pollution costs and commuting costs affect the location combinations of housing and industry around a circle. They show that in a spatial context, in order to achieve the global optimum, a spatially differentiated added-damage tax is needed. The difference between the present paper and Arnott et al. (2008) (apart from the methodological part, which will be explained below) is that we examine how pollution diffusion interacts with the force that has been identified to explain most of the spatial industrial concentration in clusters, i.e., the positive production externalities. This interaction determines the equilibrium and optimal land uses and help us characterize spatial policies in the form of environmental taxes and labor subsidies that reproduce the optimum as equilibrium outcome. Another form of interaction between pollution diffusion and a natural cost-advantage site, as well as its effects on the distribution of production across space, are analyzed in Kyriakopoulou and Xepapadeas (2013). Their results suggest that in the market allocation, the natural advantage site will always attract the major part of economic activity. However, when environmental policy is spatially optimal, the natural advantage sites lose their comparative advantage and do not act as attractors of economic activity.

The methodological contribution of this paper lies in the use of a novel approach that allows for endogenous determination of land use patterns through endogenization of the kernels describing the two externalities. This approach is based on a Taylor-series expansion method (Maleknejad et al., 2006) and helps us solve the model and provide an accurate solution for the level of the residential and industrial land rents, which will finally determine the spatial pattern of our region. The method also helps in the determination of the site-specific policies studied here, which can be used to reproduce the optimal structure as a market outcome. We believe that this constitutes an advance compared to the previous studies, where arbitrary values were assigned to the functions describing the spillover effects (as in Lucas and Rossi-Hansberg, 2002) or there is not an explicit endogenous solution of the externality terms (as in Arnott et al., 2008). We believe that the spatial policies derived here, which can be calculated using the novel approach described above, provide new insights and can contribute to the improvement of efficiency in the internal of a region.

The rest of the paper is organized as follows. In Section 2 we present the model and solve for the optimal and market allocations. In Section 3 we describe the spatial equilibrium conditions, while in Section 4 we derive the optimal, spatial policies which can be used to close the gap between efficient and equilibrium allocations. In Section 5 we present the numerical algorithm that is used to derive the different land use patterns, and then we show some numerical experiments. Section 6 concludes the paper.

2 The Model

2.1 The region

We consider a single region that is closed, linear, and symmetric. It constitutes a small part of a large economy. The middle of the region is normalized to $\frac{S}{2}$, while the total length is given by S and 0 and S are the left and right boundaries, respectively. The whole spatial domain is used for industrial and residential purposes. Industrial firms and households can be located anywhere inside the region. Land is owned by absent landlords.

2.2 Industrial Firms

There is a large number of industrial firms operating in the internal of our region. The location decisions of these firms are determined endogenously.

Assumption 1. Production

All firms produce a single good that is sold at a world price, and the world price is considered exogenous to the region. The production is characterized by a constant returns to scale function of land, labor $L(r)$, and emissions $E(r)$. Production per unit of land at location r is given by:

$$q(r) = g(z(r))x(A(r), L(r), E(r)), \tag{1}$$

where q is the output, L is the labor input, and E is the amount of emissions generated in the production process. Also, production is characterized by two externalities: one positive and one negative. Hence, A is the function that describes the negative externality, which is basically how pollution at spatial point r affects the productivity of labor at the same spatial point. z describes the positive production externality in the form of knowledge spillovers.

In the numerical simulations, the functions g and x are considered to be of the form:

$$\begin{aligned} g(z(r)) &= e^{\gamma z(r)} \\ x(A(r), L(r), E(r)) &= (A(r)L(r))^b E(r)^c. \end{aligned}$$

The two opposing forces that will be shown to affect the location decisions of firms are associated with the two kinds of production externalities mentioned above. More specifically, the main force of agglomeration is related to the positive knowledge spillovers, while the dispersion force comes from the negative consequences of pollution. The trade-off between these two forces defines the industrial areas in our spatial domain.

Assumption 2. Positive knowledge spillovers

Firms are positively affected by locating near other firms because of externalities in production, namely positive knowledge spillovers. The positive production externality is assumed to be linear and to decay exponentially at a rate δ with the distance between (r, s):

$$z(r) = \delta \int_0^S e^{-\delta(r-s)^2} \lambda(s) \ln L(s) ds.$$

Note that $\lambda(r)$ is the proportion of land occupied by firms at spatial point r, and $1 - \lambda(r)$ is the proportion of land occupied by households at r. The function $k(r, s) = e^{-\delta(r-s)^2}$ is called normal dispersal kernel, and it shows that the positive effect of labor employed in nearby areas decays exponentially at a rate δ between r and s.

This kind of production externality relates the production at each spatial point with the employment density in nearby areas. In this context, firms benefit from the interaction with the other firms if they locate in areas with a high density of industries. This assumption has been used extensively in urban models of spatial interactions and comprises one of the driving forces of business agglomeration.[3]

Assumption 3. Pollution

The production process generates emissions that diffuse in space and increase the total concentration of pollution in the city. This is reinforced in areas with a high concentration of economic activity, where a lot of firms operate and pollute the environment. The use of emissions in the production and the negative consequences that follow require enforcement of environmental regulation. Since emissions, as well as the concentration of pollution, differ throughout the spatial domain, environmental regulations will be site-specific. In particular, environmental policy is stricter in areas with high concentrations of pollution and laxer elsewhere. This means that it is more costly for firms to locate at spatial points with high levels of pollution. However, apart from the cost of pollution in terms of environmental policy, firms avoid locating in polluted sites since pollution affects the productivity of labor negatively. As a result, pollution works as a centrifugal force among firms.

[3] For empirical studies that confirm the significance of this force, see Footnote 2. For the theoretical modeling of knowledge spillovers, see Lucas (2001), Lucas and Rossi-Hansberg (2002), and Kyriakopoulou and Xepapadeas (2013).

As stated above, the generation of emissions during the production of the output damages the environment. The damage function per unit of land is given by

$$D(r) = X(r)^{\phi}, \tag{2}$$

where D is the damage per unit of land and $\phi \geq 1$, $D'(X) > 0$, $D''(X) \geq 0$.[4] Aggregate pollution, X, at each spatial point r is a weighted average of the emissions generated in nearby industrial locations and is given by:

$$\ln X(r) = \int_0^S e^{-\zeta(r-s)^2} \lambda(s) \ln E(s) ds,$$

with the normal dispersal kernel equal to $k(r,s) = e^{-\zeta(r-s)^2}$. Using similar interpretation with the kernel describing the production externality, emissions in nearby areas affect the total concentration of pollution at the spatial point r, while this effect declines as the distance between the different spatial points r and s increases. ζ is a parameter indicating how far pollution can travel; it depends on weather conditions and the natural landscape.

Finally, the negative effect of pollution on the productivity of labor is given by $A(r) = X(r)^{-\kappa}$, where $\kappa \in [0, \bar{\kappa}]$ determines the strength of the negative pollution effect. $\kappa = 0$ implies that there is no connection between aggregate pollution and labor productivity, while a large value of κ means that workers become unproductive due to the presence of pollution.

The negative effects of pollution on the productivity of labor are usually explained through their connection with health effects.[5] The air pollution in China can be thought of as an example of this. In 2012, the China Medical Association warned that air pollution was becoming the greatest threat to health in the country, since lung cancer and cardiovascular disease were increasing due to factory- and vehicle-generated air pollution. More precisely, a wide range of airborne particles and pollutants from combustion (e.g.,

[4]In order to model the damage function, we follow Koldstad (1986), who defines damages at a specific location as a function of aggregate emissions of the location. We do not directly relate damages to the number of people living in that location, so as to avoid the potential contradiction of assigning very low damages to a heavily polluted area that lacks high residential density.

[5]See, e.g., Williams (2002) and Bruvoll et al. (1999).

woodfires, cars, and factories), biomass burning, and industrial processes with incomplete burning create the so-called "Asian brown cloud", which is increasingly being renamed the "Atmospheric Brown Cloud" since it can be spotted in more areas than just Asia. The major impact of this brown cloud is on health, which explains the need for a positive κ parameter above.

2.3 Households

A large number of households are free to choose a location in the interval of the given region. The endogenous formation of residential clusters is determined by two forces that affect households' location decisions: commuting costs and aggregate pollution.

Assumption 4. Utility maximization.

Consumers derive positive utility from the consumption of the good produced by the industrial sector and the quantity of residential land, while they receive negative utility from pollution. Thus, a household located at the spatial point r receives utility $U(c(r), l(r), X(r))$,where c is the consumption of the produced good and l is residential land.

To obtain a closed-form solution, we assume that the utility U is expressed as

$$U(r) = c(r)^a l(r)^{1-a} - X(r)^{\phi}, \tag{3}$$

where $0 < a < 1$ and $\phi \geq 1$.

As explained above, the residential location decisions are determined by two opposing forces. The first one is related to commuting costs, which are modeled below. This is a force that impedes the formation of pure residential areas since workers have an incentive to locate close to their workplace so as not to spend much time/money commuting. As a result, commuting costs promote the formation of mixed areas where people live next to their workplaces.

The second force is a force that promotes the concentration in residential clusters and comes from the fact that the consumers receive negative utility from pollution. Accordingly, they tend to locate far from the industrial firms to avoid polluted sites. The pollution levels at each spatial point, which are determined by the location and production decisions of industrial firms, are considered as given for consumers.

Assumption 5. Commuting costs

Consumers devote one unit of time working in the industrial sector, part of which is spent commuting to work. Agents who work at spatial point r, but live at spatial point s, will finally receive $w(s) = w(r)e^{-k|r-s|}$. This equation corresponds to a spatially discounted accessibility, which has been used extensively in spatial models of interaction. Now, if a consumer lives at r and works at s, the wage function becomes $w(s) = w(r)e^{k|r-s|}$. If r is a mixed area, people who live there work there as well, and $w(r)$ denotes both a wage rate paid by firms and the net wage earned by workers.

2.4 Agglomeration forces

The centripetal and centrifugal forces explained above are summarized in the following table.

Forces promoting:	**Industrial Firms**	**Households**
Concentration in clusters	Strong knowledge spillovers	High pollution levels
Dispersion	High pollution levels	High commuting costs

To summarize the effect of the agglomeration forces assumed in this paper, industrial firms concentrate in clusters under the presence of strong knowledge spillovers, while high pollution levels work in the opposite direction since they imply a double negative effect for the same firms. Moreover, high pollution levels promote the formation of residential clusters, since residents try to avoid the industrial polluted areas. However, this tendency is moderated in the case where these agents have to pay high commuting costs. The use of land for industrial and residential purposes prevents the two parts from locating around a unique spatial point.

The objective of this paper is in examining the optimal and equilibrium patterns of land use under the above agglomeration forces and in designing optimal policies. The trade-off between the above forces will define residential, industrial, or mixed areas in the internal of the region under study.

2.5 The Endogenous Formation of the Optimal Land Use

We assume the existence of a regulator who makes all the industrial and residential location decisions across the spatial interval $[0, S]$. The objective of the regulator is to maximize the sum of the consumers' and producers' surplus less environmental damages in the whole region.

The optimal problem is solved in two stages. In the first stage, we derive the optimal industrial land rent, which is the rent that firms are willing to pay at each spatial point in order to locate there.

Thus, if we denote by $p = P(q)$ the inverse demand function, the optimal problem becomes:

$$\max_{L,K,E} \int_0^S \left[\int_0^{q(r)} P(v)dv - w(r)L(r) - D(r) \right] dr.$$

The FONC for the optimum are:

$$P(q)\frac{\partial q(r)}{\partial L(r)} = w(r)$$

$$P(q)\frac{\partial q(r)}{\partial E(r)} = \frac{\partial D(r)}{\partial E(r)}$$

or

$$pbe^{\gamma z(r)}X(r)^{-b\kappa}L(r)^{b-1}E(r)^c + \int_0^S pe^{\gamma z(s)}X(s)^{-b\kappa}L(s)^b E(s)^c \gamma \frac{\partial z(s)}{\partial L(r)} ds = w(r) \tag{4}$$

$$pce^{\gamma z(r)}X(r)^{-b\kappa}L(r)^b E(r)^{c-1} -$$

$$- \int_0^S \left[pb\kappa e^{\gamma z(s)} X(s)^{-b\kappa-1} L(s)^b E(s)^c + \phi X(s)^{\phi-1} \right] \frac{\partial X(s)}{\partial E(r)} ds = 0. \tag{5}$$

After making some transformations that are described in detail in Appendix A, we get the following system of second kind Fredholm linear integral equations with symmetric kernels:

$$\phi \int_0^S e^{-\zeta(r-s)^2} \varepsilon(s) ds + g_1^*(r) = y(r) \tag{6}$$

$$-\tfrac{\gamma\delta}{c} \int_0^S e^{-\delta(r-s)^2} y(s) ds + \frac{(1-b)\phi + bk}{c} \int_0^S e^{-\zeta(r-s)^2} \varepsilon(s) ds + g_2^*(r) = \varepsilon(r), \tag{7}$$

where $y(r) = \ln L(r)$ and $\varepsilon(r) = \ln E(r)$, while $g_1^*(r)$ and $g_2^*(r)$ are some known functions. In order to determine the solution of the system (6) - (7), we use a Taylor-series expansion method (Maleknejaket et al., 2006), which provides accurate, approximate solutions of systems of second kind Fredholm integral equations. Following this technique, we get the optimal amount of inputs $L^*(r)$ and $E^*(r)$, which can determine the optimal level of production at each spatial point, $q^*(r)$. The optimal emission level will finally define the total concentration of pollution at each spatial point r, $X^*(r)$, as well as the damage, $D^*(r)$.

Using the above optimal values, we can define the optimal industrial land-rent as follows:

$$R_I^*(r) = pq^*(r) - w(r)L^*(r) - D^*(r). \tag{8}$$

Equation (8) describes the maximum amount of money that firms locating at the spatial point r are willing to pay in order to settle there.

In the second stage, we derive the optimal residential land-rent function, i.e., the maximum amount of money that agents are willing to spend in order to locate at a specific spatial point. Thus, total revenues, $w(r)$, are spent on the land they rent at a price $R_H(r)$ per unit of land and on the consumption of the good, $c(r)$, which can be bought at a price p.

So, consumers minimize their expenditures:

$$w(r) = R_H(r)l(r) + pc(r) = \min_{l,c} [R_H(r)l + pc] \tag{9}$$

subject to

$$U(c, l, X) \geq \bar{u} \tag{10}$$

so that no household will have an incentive to move to another spatial point inside or outside the region. To solve for the equilibrium, we assume that a consumer living at site r considers the amount of aggregate pollution $X(r)$ at the same spatial point as given. This is actually derived by the first stage of the problem, so here we use the optimal value $X^*(r)$.

Using equation (3), we form the Lagrangian of the problem as follows,

$$L = R_H(r)l(r) + pc(r) + \varpi[\bar{u} - c^a l^{1-a} + D^*(r)], \tag{11}$$

and obtain the following first order conditions (FONC):

$$R_H(r) = (1-a)\varpi l^{-a} c^a \tag{12}$$

$$p = a\varpi c^{a-1} l^{1-a}. \tag{13}$$

Solving the FOC and making some substitutions, we get the optimal residential land rent at each spatial point:

$$R_H^*(r) = \left[\frac{w(r)}{(\overline{u} + D^*(r))(\frac{1-\alpha}{\alpha})^{\alpha} \frac{1}{1-\alpha}} \right]^{\frac{1}{1-\alpha}},$$

where $w(r) = w(s)e^{-k|r-s|}$ is the net wage of a worker living at r and working at s. Also, $R_H^*(r)$ is the rent per unit of land that a worker bids at location r while working at s and enjoying the utility level $\bar{u}$. We observe that $\frac{\vartheta R_H^*(r)}{\vartheta X^*(r)} < 0$. This means that residential land rents are lower in areas with high pollution concentrations. In other words, people are willing to spend more money on areas with better environmental amenities. This is supported by the fact that the highest residential rents in the real world are observed in purely residential areas in the suburbs of the cities, far from the polluted business centers.

Finally, assuming that the land density is 1, we can define the optimal population density N at each spatial point r,

$$N\ (r)l(r) = 1 \Longrightarrow N(r) = \frac{1}{l(r)^{*}}$$

$$N^{*}(r) = \frac{(w(r))^{\frac{a}{1-a}}}{(\bar{u} + D^{*}(r))^{\frac{1}{1-a}} (\frac{1-a}{a})^{\frac{a}{1-a}} (\frac{1}{1-a})^{\frac{a}{1-a}}}.$$

It is obvious that the population distribution moves upward when the net wage increases and when the concentration of pollution at the same spatial point decreases. The comparison between the $R_I^{*}(r)$ and the $R_H^{*}(r)$ at each spatial point provides the optimal land uses.

2.6 The Endogenous Formation of the Equilibrium Land Use

Equilibrium and optimal land uses will differ because of the existence of externalities. On the one hand, the decisions about the amount of emissions generated by each firm affect the total concentration of pollution in the internal of our region. However, in equilibrium, when firms choose the amount of emissions that will be used in the production process, they do not realize or do not take into account that their own decisions affect aggregate pollution, which actually describes their myopic behavior. When, for instance, a firm increases the amount of generated emissions at site r, aggregate pollution is increased not only at r, but also in nearby places through the diffusion of pollution. These higher levels of aggregate pollution affect firms in two ways: first, they increase the cost of environmental policy. Second, they make the negative pollution effect on the productivity of labor stronger. Finally, firms in equilibrium do not consider the fact that their own location decisions affect the productivity of the rest of the firms through knowledge spillovers. For instance, they do not realize the fact that employing one extra worker will not only increase their productivity but also the productivity of nearby firms. Therefore, equilibrium location decisions do not internalize fully the above effects, which distorts the optimal land uses studied above and makes them differ from the equilibrium ones.

To derive the equilibrium solution, we assume that a firm located at spatial point r chooses labor and emissions to maximize profits:

$$R_I(r) = \max_{L,E}\{pe^{\gamma z(r)}(A(r)L(r))^b E(r)^c - w(r)L(r) - \tau(r)E(r)\},$$

where $\tau(r)$ is the environmental tax enforced by the government. The tax here is assumed to be a site-specific environmental policy instrument, which is equal to the marginal damage of emissions, i.e., $\tau(r) = MD^*(r)$. The solution will be a function of (z, A, τ, p, w): $L = \hat{L}(z, A, \tau, p, w)$ and $E = \hat{E}(z, A, \tau, p, w)$. The maximized profits at each spatial point $\hat{R}_I(z, A, \tau, p, w)$ can also be interpreted as the business land rent, which is the land rent that a firm is willing to pay so as to operate at this spatial point.

Following the discussion at the beginning of this section, a firm located at site r treats the concentration of pollution $X(r)$, the negative pollution effect on the productivity of labor $A(r)$, and the effect of knowledge spillovers in the production process $z(r)$ as exogenous parameter X^e, A^e, and z^e respectively. This assumption implies that the tax $\tau(r)$ is also treated as a parameter at each spatial point.

The first order necessary conditions (FONC) for profit maximization are:

$$pbe^{\gamma z(r)}X(r)^{-bk}L(r)^{b-1}E(r)^c = w(r) \tag{14}$$

$$pce^{\gamma z(r)}X(r)^{-bk}L(r)^{b}E(r)^{c-1} = \tau(r). \tag{15}$$

So, we solve explicitly for:

$$\hat{L}(z, w, \tau) = \left(\frac{c^c b^{1-c} A e^{\gamma z}}{\tau^c w^{1-c}}\right)^{\frac{1}{1-b-c}} \tag{16}$$

$$\hat{E}(z, w, \tau) = \left(\frac{c^{1-b} b^b A e^{\gamma z}}{\tau^{1-b} w^b}\right)^{\frac{1}{1-b-c}}. \tag{17}$$

Substituting (16) and (17) into the maximized profit function, we solve explicitly for the industrial land rents:

$$\hat{R}_I(z,w,\tau) = \left(\frac{e^{\gamma z} A b^b c^c}{\tau^c w^b}\right)^{\frac{1}{1-b-c}} (1-b-c). \tag{18}$$

In the explicit solution for L, E, and R_I presented above, there are two integral equations: one describing knowledge spillovers and the other describing the concentration of pollution at each spatial point.[6] Most authors who have studied knowledge spillovers of this form use simplifying assumptions about the values that the kernels take at each spatial point. However, this approach forces firms to locate around the sites that correspond to the highest assumed arbitrary values of knowledge spillovers, and hence we do not take into account that $L(s)$ and $E(s)$, $s \in S$, appear in the right-hand side of (16)-(17) and therefore these equations have to be solved as a system of simultaneous integral equations. Instead of following this approach, we choose to use a novel method of solving systems of integral equations, which was also implemented in Kyriakopoulou and Xepapadeas (2013). More specifically, if we take logs on both sides of equations (14)-(15) and do some transformations that are described in Appendix B, the FONC result in a system of second kind Fredholm integral equations with symmetric kernels:

$$\frac{\gamma\delta}{1-b-c}\int_0^S e^{-\delta(r-s)^2} y(s)ds + \frac{c(1-\phi)-bk}{1-b-c}\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_1(r) = y(r) \tag{19}$$

$$\frac{\gamma\delta}{1-b-c}\int_0^S e^{-\delta(r-s)^2} y(s)ds + \frac{(1-b)(1-\phi)-bk}{1-b-c}\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_2(r) = \varepsilon(r), \tag{20}$$

where $y(r) = \ln L(r)$, $\varepsilon(r) = \ln E(r)$ and $g_1(r), g_2(r)$ are some known functions.

Proposition 1 *Assume that: (i) the kernel $k(r,s)$ defined on $[0,S]\times[0,S]$ is an L_2-kernel that generates the compact operator W, defined as $(W\phi)(r) = \int_0^S k(r,s)\phi(s)\,ds$, $0 \le s \le S$; (ii) $1-b-c$ is not an eigenvalue of W; and (iii) G is a square integrable function. Then a unique solution determining the optimal and equilibrium distributions of inputs and output exists.*

[6] There are kernels in the right-hand side of equations 16-18 (see the definition of $z(r)$, $A(r)$, and $\tau(r)$ above).

The proof of existence and uniqueness of both the optimum and the equilibrium is presented in the following steps:[7]

- A function $k(r,s)$ defined on $[0,S]\times[0,S]$ is an L_2-kernel if it has the property that $\int_0^S \int_0^S |k(r,s)|^2\, drds < \infty$.

  The kernels of our model have the formulation $e^{-\xi\ (r-s)^2}$ with $\xi = \delta,\ \zeta$ (positive numbers) and are defined on $[0,10]\times[0,10]$.

  We need to prove that $\int_0^{10}\int_0^{10} \left|e^{-\xi\ (r-s)^2}\right|^2 drds < \infty$.

  Rewriting the left part of inequality, we get $\int_0^{10}\int_0^{10} \left|\frac{1}{e^{\xi\ (r-s)^2}}\right|^2 drds$.

  The term $\frac{1}{e^{\xi\ (r-s)^2}}$ takes its highest value when $e^{\xi\ (r-s)^2}$ is very small. Yet the lowest value of $e^{\xi\ (r-s)^2}$ is obtained when either $\xi = 0$ or $r = s$ and in that case $e^0 = 1$. So, $0 < \left|\frac{1}{e^{\xi\ (r-s)^2}}\right| < 1$. When $\left|\frac{1}{e^{\xi\ (r-s)^2}}\right| = 1$ and $S = 10$, then $\int_0^{10}\int_0^{10}\left|\frac{1}{e^{\xi\ (r-s)^2}}\right|^2 drds = 100 < \infty$. Thus, the kernels of our system are L_2-kernels.

- If $k(r,s)$ is an L_2-kernel, the integral operator

  $$(W\phi)(r) = \int_0^S k(r,s)\,\phi(s)\,ds\ ,0 \leq s \leq S$$

  that it generates is bounded and

  $$\|W\| \leq \left\{\int_0^S \int_0^S |k(r,s)|^2\, drds\right\}^{\frac{1}{2}}.$$

  So, in our model the upper bound of the norm of the operator generated by the L_2-kernel is $\|W\| \leq \left\{\int_0^S \int_0^S |k(r,s)|^2\, drds\right\}^{\frac{1}{2}} = \left\{\int_0^{10}\int_0^{10}\left|\frac{1}{e^{i\ (r-s)^2}}\right|^2 drds\right\}^{\frac{1}{2}} \leq 10.$

- If $k(r,s)$ is an L_2-kernel and W is a bounded operator generated by k, then W is a compact operator.

- If $k(r,s)$ is an L_2-kernel and generates a compact operator W, then the integral equation

  $$Y - \left(\tfrac{1}{1-a-b-c}\right) W\ Y = G \tag{21}$$

[7] See Moiseiwitsch (2005) for more detailed definitions.

has a *unique* solution for all square integrable functions G if $(1-b-c)$ is not an eigenvalue of W (Moiseiwitsch [**?**]). If $(1-b-c)$ is not an eigenvalue of W, then $\left(I - \frac{1}{1-b-c}W\right)$ is invertible.

- As we show in Appendix C, both systems (6)-(7) and (19)-(20) can be transformed into a second kind Fredholm Integral equation of the form (21). Thus, a unique optimal and equilibrium distribution of inputs and output exists.□

To solve systems (6)-(7) and (19-20) numerically, we use a modified Taylor-series expansion method (Maleknejad et al., 2006). More precisely, a Taylor-series expansion can be made for the solutions $y(s)$ and $\varepsilon(s)$ in the integrals of systems (6)-(7) and (19-20). We use the first two terms of the Taylor-series expansion (as an approximation for $y(s)$ and $\varepsilon(s)$) and substitute them into the integrals of (6)-(7) and (19-20). After some substitutions, we end up with a linear system of ordinary differential equations. In order to solve the linear system, we need an appropriate number of boundary conditions. We construct them and then obtain a linear system of three algebraic equations that can be solved numerically. The analytical solution of the optimal and equilibrium model is provided in Appendices A and B.

3 Land Use Structures

Having studied the optimal and equilibrium problems, we are able to define the different land uses in each case. The region under study is strictly defined in the spatial domain $[0, S]$ and firms and households cannot locate anywhere else. Thus, a spatial equilibrium is reached when all firms receive zero profits, all households receive the same utility level $\bar{u}$, land is allocated to its highest values, and rents and wages clear the land and labor markets.

Consumers dislike pollution, which means that they have an incentive to locate far from industrial areas. On the other hand, consumers work at the firms and if they locate far from them, they will suffer higher commuting costs, which promotes the formation of mixed areas. The trade-off between these two forces will define the residential location decisions.

Firms have a strong incentive to locate close to each other in order to benefit from the positive knowledge spillovers. However, if all firms locate around a specific site, this site will become very polluted, which will increase both the cost of environmental policy and the negative productivity effect. Thus, if all firms decide to locate in one spatial interval, then they will be obliged to pay a higher environmental tax and suffer from the negative pollution effects. In other words, high pollution levels impede the concentration of economic activity. The trade-off between these forces will define the size of the industrial areas.

The conditions determining the land use at each spatial point are described in the following steps:

1. Firms receive zero profits.
2. Households receive the same level of utility $U(c, l, X) = \bar{u}$.
3. Land rents equilibrium: at each spatial point $r \in S$,

$$\begin{aligned} R(r) &= \max\{R_I(r), R_H(r), 0\} && (22) \\ R_I(r) &= R(r) \quad \text{if } \lambda(r) > 0 \text{ and } R_I(r) > R_H(r) && (23) \\ R_H(r) &= R(r) \quad \text{if } \lambda(r) < 1 \text{ and } R_H(r) > R_I(r). && (24) \end{aligned}$$

4. Commuting equilibrium: at each spatial point $r \in S$,

$$w(r) = w(s)e^{-k|r-s|} = \max_{s \in S}[w(s)e^{-k|r-s|}]. \tag{25}$$

As people choose s to maximize their net wage, this means that in equilibrium

$$w(s)e^{-k|r-s|} \leq w(r) \leq w(s)e^{k|r-s|} \tag{26}$$

This is the wage arbitrage condition that implies that no one can gain by changing his job location.

5. Labor market equilibrium: for every spatial point $r \in S$,

$$\int_0^S (1-\lambda(s))N(s)ds = \int_0^S \lambda(s)L(s)ds. \tag{27}$$

6. Industries' and households' population constraints:

$$\int_0^S (1-\lambda(s))N(s)ds = \bar{N} \tag{28}$$

$$\int_0^S \lambda(s)L(s)ds = \bar{L}, \tag{29}$$

where $\bar{N}$ is the total number of residents and $\bar{L}$ the total number of workers.

7. Land use equilibrium: at each spatial point $r \in S$,

$$\begin{aligned} 0 &\leq \lambda(r) \leq 1 \qquad (30) \\ \lambda(r) &= 1 \text{ if } r \text{ is a pure industrial area} \\ \lambda(r) &= 0 \text{ if } r \text{ is a pure residential area} \\ 0 &< \lambda(r) < 1 \text{ if } r \text{ is a mixed area.} \end{aligned}$$

Equations (22)-(24) mean that each location is occupied by the agents who offer the highest bid rent. Condition (25) implies that a worker living at r will choose her working location s so as to maximize her net wage. Condition (27) ensures the equality of labor supply and demand in the whole spatial domain. This condition will determine the equilibrium wage rate at each spatial point, $w^*(r)$. Finally, conditions (28)-(29) mean that the sum of residents in all residential areas is equal to the total number of residents in the city and that aggregate labor in all industrial areas equals the total number of workers in the city.

4 Optimal Policies: Labor Subsidies and Environmental Taxation

Using the optimal values for $L^*, E^*, z^*, A^*, X^*, N^*$, and λ^*, we can determine the wages and the level of the tax that would make firms and households in the equilibrium to make the same decisions as in the optimum. Thus, we would be able to implement the optimum as an equilibrium outcome.

From the first-order conditions for the optimum (for $\lambda^*(r) = 1$),

$$w(r) = pbe^{\gamma z(r)}X(r)^{-b\kappa}L(r)^{b-1}E(r)^c + \underbrace{\int_0^S \mathbf{pe}^{\gamma z(s)}\mathbf{X(s)}^{-b\kappa}\mathbf{L(s)}^b\mathbf{E(s)}^c\boldsymbol{\gamma}\frac{\partial\mathbf{z(s)}}{\partial\mathbf{L(r)}}\mathbf{ds}}_{\text{knowledge spillover effect}} \tag{31}$$

and

$$pce^{\gamma z(r)}X(r)^{-b\kappa}L(r)^bE(r)^{c-1} -$$

$$-\underbrace{\int_0^S} \left[\underbrace{\mathbf{pb}\boldsymbol{\kappa}\mathbf{e}^{\gamma z(s)}\mathbf{X(s)}^{-b\kappa-1}\mathbf{L(s)}^b\mathbf{E(s)}^c}_{\text{labor productivity effect}} + \phi X(s)^{\phi-1}\right] \underbrace{\frac{\partial\mathbf{X(s)}}{\partial\mathbf{E(r)}}\mathbf{ds}}_{\text{spatial pollution effect}} = 0. \tag{32}$$

If the environmental tax enforced by the government is a site-specific environmental policy equal to the marginal damage of emissions, $\tau(r) = MD^*(r) = \phi X^*(s)^{\phi-1}$, then the differences between the optimum and the equilibrium are shown by the bold terms above.

Let us analyze the first-order condition with respect to labor input. Firms here seem to internalize the externality that is related to the knowledge spillover effect taking into account the positive effect of their own decisions on the productivity of the rest of the firms, located in nearby areas. Since the difference between the optimal and equilibrium FOC comes from the *knowledge spillover effect* in equation (31), the policy instrument that would partly lead the equilibrium to reproduce the optimal distributions would be a subsidy of the form $v^*(r) = \int_0^S \mathbf{pe}^{\gamma z(s)}\mathbf{X(s)}^{-b\kappa}\mathbf{L(s)}^b\mathbf{E(s)}^c\boldsymbol{\gamma}\frac{\partial\mathbf{z(s)}}{\partial\mathbf{L(r)}}\mathbf{ds}$. Thus, firms would have to pay a lower labor cost, $w(r) - v^*(r)$, employ more labor, benefit from the stronger knowledge spillovers, and produce more output.

As far as the second FOC wrt emissions is concerned, given that firms in equilibrium pay a tax equal to the marginal damage, as stated above, the difference between the two cases is presented by the *labor productivity effect* and the *spatial pollution effect* in equation (32). Thus, an optimal tax, instead of imposing $\tau(r) = MD^*(r) = \phi X^*(s)^{\phi-1}$, should be of the form $\tau^*(r) = \int_0^S \left[\mathbf{pb\kappa e}^{\gamma z(s)}\mathbf{X(s)}^{-b\kappa-1}\mathbf{L(s)}^b\mathbf{E(s)}^c + \tau(r)\right]\frac{\partial \mathbf{X(s)}}{\partial \mathbf{E(r)}}\mathbf{ds}$. It is obvious that the optimal taxation, $\tau^*(r)$, is higher than the equilibrium one, $\tau(r)$, at each spatial point in the internal of our city or region. The reason is that, first, the optimal taxation takes into account the extra damage caused in the whole region by emissions generated at r (*spatial pollution effect*). However, apart from this effect, the optimal taxation captures the fact that increased emissions in r mean lower productivity for firms locating in r and in nearby areas (*labor productivity effect* $\times$ *spatial pollution effect*). This negative productivity effect is now added to the cost of taxation, and the full damage caused by the generation of emissions during the production process is internalized.

Theorem 2 *A labor subsidy of the form*

$$v^*(r) = \int_0^S \mathbf{pe}^{\gamma z(s)}\mathbf{X(s)}^{-b\kappa}\mathbf{L(s)}^b\mathbf{E(s)}^c\gamma\frac{\partial \mathbf{z(s)}}{\partial \mathbf{L(r)}}\mathbf{ds}$$

and an environmental tax of the form

$$\tau^*(r) = \int_0^S \left[\mathbf{pb\kappa e}^{\gamma z(s)}\mathbf{X(s)}^{-b\kappa-1}\mathbf{L(s)}^b\mathbf{E(s)}^c + \phi X(s)^{\phi-1}\right]\frac{\partial \mathbf{X(s)}}{\partial \mathbf{E(r)}}\mathbf{ds}$$

will implement the optimal distributions as equilibrium ones.

Proof. In equilibrium, firms will maximize their profits, households will minimize their expenditures given a reservation utility, land is allocated to its highest value, the wage no arbitrage condition is satisfied, and all workers are housed in the internal of our region. Since all the above are in line with the optimal problem as well, the only thing we need to do in order to impose the optimal allocation as an equilibrium one is to use the optimal policy instrument described in Theorem 2. Thus, the joint enforcement of a labor subsidy, which will decrease the labor cost for the firms, and a higher environmental tax will close the gap between the equilibrium and optimal allocations. ■

Proposition 3 *Efficiency in a market economy can be achieved by using the site-specific policy instruments described in Theorem 2. Uniform taxes or subsidies, which produce the same revenues or expenses, do not lead to optimal allocations.*

Proof. An industry, paying $\tau^*(r)$ for generating $E^*(r)$ emissions, receiving $v^*(r)$ for employing $L^*(r)$ workers and paying $w(r)$ wages for the same number of workers and $R_I^*(r)$ as land rents, will receive zero profits in equilibrium. Having proved the uniqueness of the equilibrium, any other level of taxes or subsidies will not satisfy the zero profit condition for the same amount of emissions and labor, and will not constitute an equilibrium outcome. ■

Site-specific taxes should be enforced in every industrial location and must equal the added damages caused by the emissions generated from this unit of land. Site-specific subsidies should be given in every industrial location and must equal the positive effects caused by the diffusion of knowledge coming from this industrial location and affecting the rest of the industries.

5 Numerical Experiments

Numerical simulations will help us obtain different maps explaining the residential and the industrial clusters formed in our city. To put it differently, the optimal and equilibrium spatial distributions of residential and industrial land rents will determine the location of firms and households in our domain. The numerical method of Taylor-series expansion, described above, will give us the optimal and equilibrium values of land rents. We solve the system of integral equations using Mathematica.

The numerical algorithm to characterize the optimal and equilibrium land use patterns consists of the following steps:

Step 1. We give numerical values to the parameters of the model.

Step 2. We solve for the optimal (and equilibrium) distributions $L^*, E^*, q^*, N^*, c^*, z^*, X^*$ $(\hat{L}, \hat{E}, \hat{q}, \hat{N}, \hat{c}, \hat{z}, \hat{X})$ at every spatial point as a function of λ.

Step 3. We derive the optimal (and equilibrium) distributions of residential and industrial land rents R_I^*, R_H^* $(\hat{R}_I, \hat{R}_H)$ and plot them in graphs so as to characterize the areas as residential, industrial, or mixed. Then, we determine the λ value (see below).

Step 4. We calculate the total number of residents and workers in the region. The aim is to have equal numbers of residents and workers, which will satisfy the condition that all workers should be housed inside the region.

Step 5. If the number of residents does not equal the number of workers, then the level of the wage changes and we start solving the problem again (back to Step 2). We follow this process until we obtain equal numbers of residents and workers. An iterative approach is used since a change in the wage level will also change the demand for the second input (emissions), which in turn will affect the aggregate pollution. However, aggregate levels of pollution change the level of environmental tax and affect both the productivity of labor and the residential location decisions.

Step 6. The λ value for each spatial point is finally determined. If an interval is purely residential or industrial, which means that one of the land rents is always higher than the other, then λ is either 0 or 1, respectively. When land rents are equal in a specific interval, we calculate a value of $0 < \lambda < 1$ such that the numbers of residents and workers are equal.

The ex-post calculation of λ allows the explicit endogenous solution of the externalities of the model, and we consider this to be an advantage of this approach over previous solutions where the spatial kernels were arbitrarily chosen.

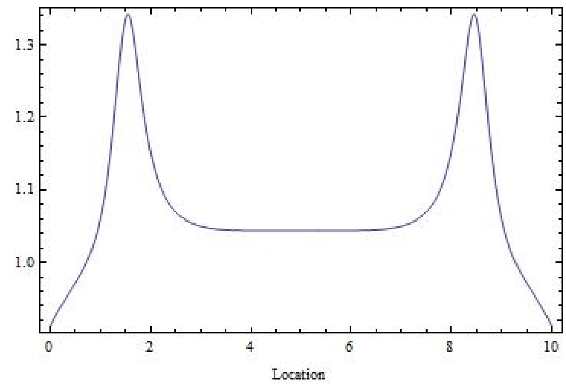
The results of this numerical algorithm are presented below. Figure 1 shows the optimal distributions of labor, emissions, output, and land rents, assuming the following values for the parameters: $\delta = 2$, $\zeta = 0.5$, $\kappa = 0.01$ and $k = 0.001$.[8] The distribution of workers, emissions, and output is higher around two spatial points ($r = 1.6,\ 8.4$). This happens because at the optimum all the externality effects are internalized by the

[8] For a discussion on these parameter values, see Kyriakopoulou and Xepapadeas (2013) and Lucas and Rossi-Hansberg (2002).

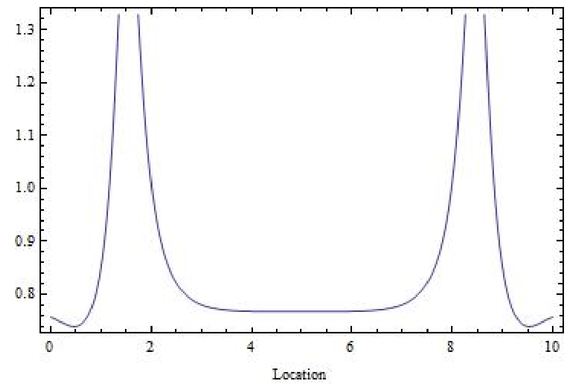
regulator. Thus, high levels of pollution that come from the production process increase the per unit damage of emissions at polluted sites, as well as the negative effect on the productivity of labor. This prevents industrial concentration around one spatial point, as it is predicted by models considering only the positive spillover effects. In other words, the first reason industrial activity at the optimum concentrates around two spatial points is that it captures benefits from the positive knowledge spillovers, which are higher in areas with high employment density. The second one is that by avoiding creating highly polluted areas, it keeps the productivity loss associated with aggregate pollution at a lower level.

Studying households' location decisions, we can observe in the last part of Figure 1 (d) that residents are willing to pay higher land rents in less polluted areas, i.e., in the center of our region and close to the two boundaries. It is also very obvious that in the spatial intervals preferred by the industries, the residential land rents are very low. Note that the gap between the levels of the two land rents is represented by the black areas. As a result, we could argue that the optimal land use structure includes two industrial areas and three residential areas in between.

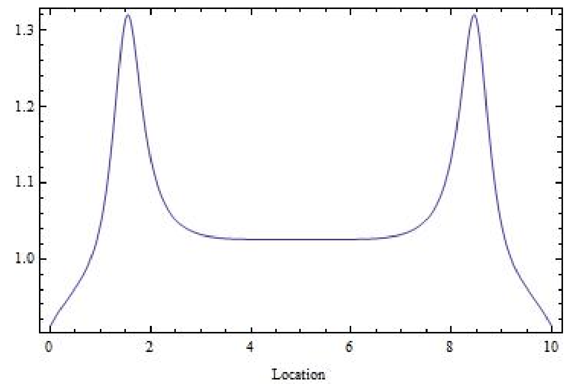
At this point it is of great interest to study the market allocations using the *same parameter values*. In Figure 2, we can see the same plots, i.e., labor, emissions, output, and land rents distribution. Without the assumption of pollution diffusion, which implies the enforcement of environmental policy, firms would concentrate around a central location in order to benefit from positive knowledge spillovers (see Kyriakopoulou and Xepapadeas, 2013). However, the trade-off between these spillovers and the ones associated with the environmental externalities make firms move further from the central area, which results in higher distributions of labor, emissions, and output close to the boundaries. The opposite is true for households, who prefer to locate in the rest of the region in order to avoid the polluted industrial sites. The comparison between residential and industrial land rents, under the condition that all agents should work and be housed in the region under study, leads to a mixed area at the city center, surrounded by two residential areas, which are followed by two industrial areas close to the boundaries. There are two peaks



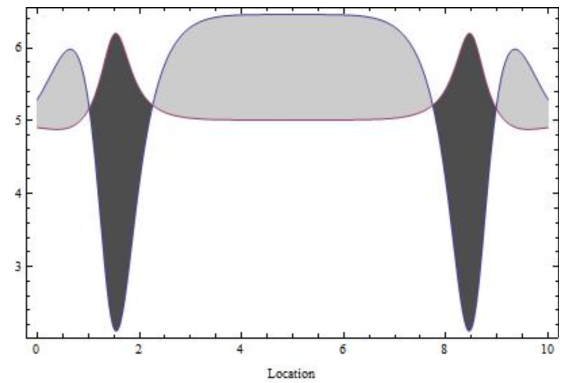
(a). Labor



(b). Emissions

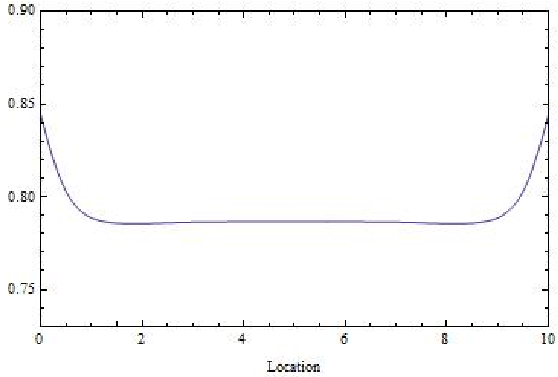


(c). Output

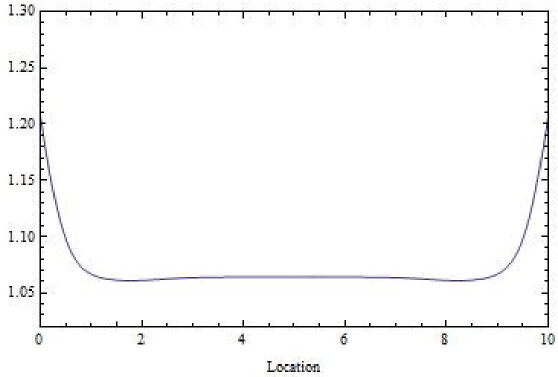


(d). Land Rents

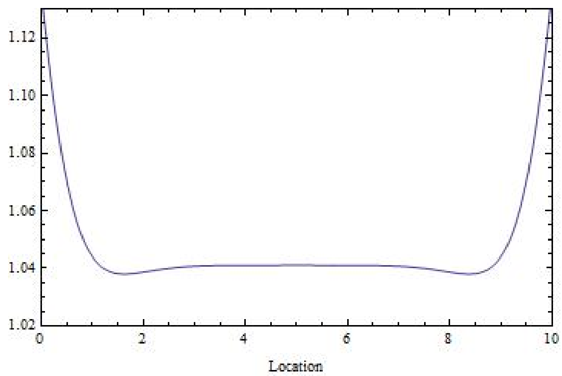
Figure 1: Optimal Densities



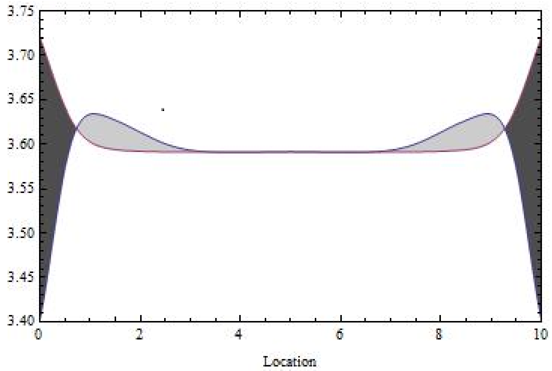
(a). Labor



(b). Emissions



(c). Output



(d). Land rents

Figure 2: Equilibrium Densities

in the residential areas, which can be explained as follows: In these areas workers are willing to pay higher land rents to avoid the high commuting costs that would result from locating further away, yet as we move close to the boundary, i.e., to industrial areas, the pollution discourages workers from paying high land rents. In the mixed areas we also need to specify the λ value so as to have the same number of residents and workers. In this numerical example, $\lambda = 0.35$, i.e., the 35% of the interval where agents and industries coexist is covered by the industrial sector and the remaining 65% by the residential sector.

The most apparent difference between the optimal and the equilibrium land use patterns is that, while mixed areas can emerge as an equilibrium outcome, a similar emergence of mixed areas at the optimum does not seem possible within our parameter range. This result is in line with previous literature studying optimal city patterns, such as Rossi-

Hansberg (2004), who proves that the optimal land use structure has no mixed areas. What we can also observe is the fact that industries operate in a much smaller interval covering 25% of the region in this numerical example, while in the market outcome firms operate in 40% of the given area. The full endogenization of the external effects at the optimum impedes firms from locating in central areas, which would be the "expected" result and seems to be the case in the market allocation. Contrary to this, the optimal solution seems to be a concentration of firms in small, spatial intervals, creating pure industrial clusters and hence restricting the diffusion of pollution across the region, which will reduce the damage to the residential areas. Some comparative analysis will help us understand which allocation is the most efficient in terms of the amount of generated emissions per unit of output calculated in the whole region. In the numerical experiment presented above, the optimal emissions per output equal 0.99 while the equilibrium rate is 1.36. Implementing the optimal policy instruments and deriving the optimum as an equilibrium outcome will significantly improve the generated emissions per unit of output by decreasing this rate by 27%.

6 Conclusion

This paper studies the optimal and market allocations in a spatial economy with pollution coming from stationary sources. It contributes to the literature by combining the assumption of pollution diffusion with two other forces that have been proven to significantly affect the spatial patterns: commuting costs and externalities in productions. The second difference compared with previous literature lies in the use of a novel approach of solving spatial models, which allows the full endogenization of the assumed external effects, i.e., the pollution and production externalities.

In order to model the above agglomeration and dispersion forces, we use a linear region where households and firms are free to choose where to locate. Firms produce by using land, labor, and emissions, enjoy positive knowledge spillovers, and pay an extra cost in the form of environmental taxation. Households work in the industrial sector, commute to work, consume the produced good and housing services, and derive negative utility from pollution. The optimal and the equilibrium spatial patterns are derived when considering the trade-off between the externalities in production, workers' commuting cost, and the consequences of aggregate pollution in terms of environmental policy and pollution damages.

A very general conclusion that comes from the incorporation of environmental issues in a general equilibrium model of land use is that the monocentric city result does not exist anymore. We show that firms have an incentive to create clusters in more than one location so as not to increase the cost of environmental policy even further by making a site very polluted. Also workers' incentive to locate close to firms to avoid high commuting costs has now changed, since pollution works to encourage them to locate in pure residential areas.

However, the most important result is that under the existence of pollution and production externalities, the optimal and equilibrium land uses differ a lot. This model allows us to identify the different allocations and suggest spatial policies that will close the gap between efficient and equilibrium outcomes. More specifically, we show that the joint implementation of a site-specific labor subsidy and a site-specific environmental tax can reproduce the optimum as an equilibrium outcome.

The numerical approach employed in this paper can be used to investigate further the role of pollution in spatial models of land use and provide insights on optimal spatial policies. The idea of two kinds of industries — polluting and a non-polluting ones — could be studied using the numerical tools presented here. Another possible extension of this model is to assume that pollution comes from non-stationary sources, like the transport sector, which is actually the case in modern cities. We leave these issues for future research.

Acknowledgments

This research has been co-financed by the European Union (European Social Fund – ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) — Research

Funding Program: “Thalis – Athens University of Economics and Business — Optimal Management of Dynamical Systems of the Economy and the Environment: The Use of Complex Adaptive Systems.” Also, Efthymia Kyriakopoulou acknowledges support from the FORMAS research program COMMONS, as well as from the Swedish Government as part of the Sustainable Transport Initiative in cooperation between the University of Gothenburg and Chalmers University of Technology.

References

Arnott, R., Hochman, O., Rausser, G.C., 2008. Pollution and land use: optimum and decentralization. J. Urban Econ. 64, 390-407.

Bottazzi, L., Peri, G., 2003. Innovation and spillovers in regions: evidence from European patent data. Europ. Econ. Rev. 47, 687-710.

Bruvoll, A., Glosrød, S., Vennemo, H., 1999. Environmental drag: evidence from Norway. Ecol. Econ. 30, 235-249.

Carlino, G.A., Chatterjee, S., Hunt, R.M., 2007. Urban density and the rate of invention. J. Urban Econ. 61, 389-419.

Fujita, M., Ogawa, H., 1982. Multiple equilibria and structural transition of non-monocentric urban configurations, Reg. Sci. Urban Econ. 12, 161-196.

Fujita, M., Thisse, J.-F., 2002. Economics of Agglomeration: Cities, Industrial Location and Regional Growth, first ed. Cambridge University Press, Cambridge.

Henderson, J.V., 1977. Externalities in a spatial context: The case of air pollution. J. Public Econ. 7, 89-110.

Keller, W., 2002. Geographic localization of international technology diffusion. Amer. Econ. Rev. 92, 120-142.

Kolstad, C.D., 1986. Empirical properties of economic incentives and command-and-control regulations for air pollution control. Land Econ. 62, 250-268.

Kyriakopoulou, E., Xepapadeas, A., 2013. Environmental policy, first nature advantage and the emergence of economic clusters. Reg. Sci. Urban Econ. 43, 101-116.

Lucas Jr., R.E, 2001. Externalities and cities. Rev. Econ. Dynam. 4, 245-274.

Lucas Jr., R.E., Rossi-Hansberg, E., 2002. On the internal structure of cities. Econometrica 70, 1445-1476.

Maleknejad, K., Aghazadeh, N., Rabbani, M., 2006. Numerical solution of second kind Fredholm integral equations system by using a Taylor-series expansion method. Appl. Math. Comput. 175, 1229-1234.

Moiseiwitsch, B.L., 2005. Integral Equations, first ed. Dover Publications, New York.

Polyanin, A.D., Manzhirov, A.V., 1998. Handbook of Integral Equations, first ed. Chapman & Hall/CRC Press, Boca Raton.

Rossi-Hansberg, E., 2004. Optimal urban land use and zoning. Rev. Econ. Dynam. 7, 69-106.

Tietenberg, T.H., 1974. Derived demand rules for pollution control in a general equilibrium space economy. J. Environ. Econ. Manage. 1, 3-16.

Williams, R.III, 2002. Environmental tax interactions when pollution affects health or productivity. J. Environ. Econ. Manage. 44, 261-270.

Appendix A

We use the modified Taylor-series expansion method in order to solve a system of second kind Fredholm integral equation with symmetric kernels, and derive the optimal land use patterns.

The FONC for the optimum are given by (4) and (5).

The FONC with respect to $L(r)$ is:

$$pbe^{\gamma z(r)}X(r)^{-b\kappa}L(r)^{b-1}E(r)^c + \int_0^S pe^{\gamma z(s)}X(s)^{-b\kappa}L(s)^b E(s)^c \gamma \frac{\partial z(s)}{\partial L(r)} ds = w(r),$$

where $z(r) = \delta \int_0^S e^{-\delta(r-s)^2}\lambda(s)\ln(L(s))ds$

For different values of r, s the integral can be written as:[9]

[9] At this step, we assume that $\lambda(s) = 1$ for all $s \in S$.

$$\delta \left\{ \ln L(0) + e^{-\delta(0-r)^2} \ln L(r) + e^{-\delta(0-S)^2} \ln L(S) \mid_{r=0} + \ldots.. + e^{-\delta(r-0)^2} \ln L(0) + \ln L(r) + e^{-\delta(r-S)^2} \ln L(S) \mid_{r=r} + \ldots.. + \right.$$

$$\left. + e^{-\delta(S-0)^2} \ln L(0) + e^{-\delta(S-r)^2} \ln L(r) + \ln L(S) \mid_{r=S} \right\}$$

So, $\frac{\theta z(s)}{\theta L(r)} = \delta \frac{1}{L(r)} \, [e^{-\delta(0-r)^2} + ... + 1 + ... + e^{-\delta(S-r)^2}] = \delta \frac{1}{L(r)} \int_0^S e^{-\delta(r-s)^2} ds.$

For the numerical analysis, we approximate the value of the integral that expresses the aggregate impact on all sites from a change in site r, by valuing the aggregate impact with the marginal valuation at site r. Then the FONC wrt $L(r)$ becomes:

$$bpe^{\gamma z(r)} X(r)^{-b\kappa} L(r)^{b-1} E(r)^c + pe^{\gamma z(r)} X(r)^{-b\kappa} L(r)^b E(r)^c \gamma\delta \frac{1}{L(r)} \int_0^S e^{-\delta(r-s)^2} ds = w$$

so

$$pe^{\gamma z(r)} X(r)^{-b\kappa} L(r)^{b-1} E(r)^c (b + \gamma\delta \int_0^S e^{-\delta(r-s)^2} ds) = w.$$

Taking logs,

$$\ln p + \gamma\delta \int_0^S e^{-\delta(r-s)^2} \ln(L(s)) ds - b\kappa \int_0^S e^{-\zeta(r-s)^2} \ln(E(s)) \; ds + (b-1) \ln L(r) + c \ln E(r)$$

$$+ \ln(b + \gamma\delta \int_0^S e^{-\delta(r-s)^2} ds) = \ln w.$$

Next, we differentiate with respect to $E(r)$:

$$pce^{\gamma z(r)} X(r)^{-b\kappa} L(r)^b E(r)^{c-1} - \int_0^S \left[pb\kappa e^{\gamma z(s)} X(s)^{-b\kappa-1} L(s)^b E(s)^c - \phi X(s)^{\phi-1} \right] \frac{\partial X(s)}{\partial E(r)} ds = 0.$$

Aggregate pollution, $X(r)$, is described by: $\ln X(r) = \int_0^S e^{-\zeta(r-s)^2} \ln(E(s))\ ds$ or $e^{\ln X(r)} = e^{\int_0^S e^{-\zeta(r-s)^2} \ln(E(s))\ ds}$ or

$X(r) = e^{\int_0^S \left[e^{-\zeta(r-s)^2} \ln E(s)\right] ds}$.

For different values of r, s the exponential term can be written as:

$$e^{[\ln E(0)+e^{-\zeta(0-r)^2} \ln E(r)+e^{-\zeta(S)^2} \ln E(S)]} \mid_{r=0} +\ldots\ldots+ e^{[e^{-\zeta(r)^2} \ln E(0)+\ln E(r)+e^{-\zeta(r-S)^2} \ln E(S)]} \mid_{r=r}$$

$+\ldots\ldots+$

$$+e^{[e^{-\zeta(S)^2} \ln E(0)+e^{-\zeta(S-r)^2} \ln E(r)+\ln E(S)]} \mid_{r=S} .$$

So, differentiating this expression wrt $E(r)$, we have:

$$\frac{\theta X(s)}{\theta E(r)} = \left(\frac{e^{-\zeta(0-r)^2}}{E(r)} + \ldots. + \frac{1}{E(r)} + \ldots. + \frac{e^{-\zeta(S-r)^2}}{E(r)}\right) e^{\int_0^S \left[e^{-\zeta(r-s)^2} \ln E(s)\right] ds} =$$

$$\frac{1}{E(r)}\ e^{\int_0^S \left[e^{-\zeta(r-s)^2} \ln E(s)\right] ds} \left(e^{-\zeta(0-r)^2} + \ldots. + 1 + \ldots. + e^{-\zeta(S-r)^2}\right) =$$

$$\frac{1}{E(r)}\ e^{\int_0^S \left[e^{-\zeta(r-s)^2} \ln E(s)\right] ds} \int_0^S e^{-\zeta(s-r)^2} ds.$$

For the numerical analysis, we approximate the value of the integral that expresses the aggregate impact on all sites from a change in site r by valuing the aggregate impact with the marginal valuation at site r. Then the FONC wrt $E(r)$ becomes:

$$cp\, e^{\gamma z(r)} X(r)^{-b\kappa} L(r)^b E(r)^{c-1} - bkpe^{\gamma z(r)} X(r)^{-b\kappa-1} L(r)^b E(r)^c \frac{1}{E(r)}\ e^{\int_0^S \left[e^{-\zeta(r-s)^2} \ln E(s)\right] ds} \int_0^S e^{-\zeta(s-r)^2} ds$$

$$-\phi X(r)^{\phi-1} \frac{1}{E(r)}\ e^{\int_0^S \left[e^{-\zeta(r-s)^2} \ln E(s)\right] ds} \int_0^S e^{-\zeta(s-r)^2} ds = 0 \Rightarrow$$

$$p\ e^{\gamma z(r)} X(r)^{-b\kappa} L(r)^b E(r)^{c-1} \left(c - b\kappa \int_0^S e^{-\zeta(s-r)^2} ds\right) = \phi X(r)^{\phi} \frac{1}{E(r)} \int_0^S e^{-\zeta(s-r)^2} ds.$$

Taking logs,

$$\ln p + \gamma\delta \int_0^S\ e^{-\delta(r-s)^2} \ln(L(s)) ds - b\kappa \int_0^S e^{-\zeta(r-s)^2} \ln(E(s))\ ds + b \ln L(r) + (c-1) \ln E(r) =$$

$$\ln\phi+\phi\int_0^S e^{-\zeta(r-s)^2}\ln(E(s))\,ds-\ln E(r)+\ln\left[\int_0^S e^{-\zeta(s-r)^2}ds\right]-\ln\left(c-b\kappa\int_0^S e^{-\zeta(s-r)^2}ds\right)\Rightarrow$$

$$\ln p+\gamma\delta\int_0^S e^{-\delta(r-s)^2}\ln(L(s))ds+b\ln L(r)+c\ln E(r)=$$
$$\ln\phi+(\phi+b\kappa)\int_0^S\left[e^{-\zeta(r-s)^2}\ln E(s)\right]ds+\ln\left(\int_0^S e^{-\zeta(s-r)^2}ds\right)-\ln\left(c-b\kappa\int_0^S e^{-\zeta(s-r)^2}ds\right).$$

So, the first-order conditions are:

$$\ln p+\gamma\delta\int_0^S e^{-\delta(r-s)^2}\ln(L(s))ds-b\kappa\int_0^S e^{-\zeta(r-s)^2}\ln(E(s))\,ds+(b-1)\ln L(r)+c\ln E(r)$$

$$+\ln(b+\gamma\delta\int_0^S e^{-\delta(r-s)^2}ds)=\ln w$$

and

$$\begin{aligned}&\ln p+\gamma\delta\int_0^S e^{-\delta(r-s)^2}\ln(L(s))ds+b\ln L(r)+c\ln E(r)\\ =\;&\ln\phi+(\phi+b\kappa)\int_0^S\left[e^{-\zeta(r-s)^2}\ln E(s)\right]ds+\ln\left(\int_0^S e^{-\zeta(s-r)^2}ds\right)-\ln\left(c-b\kappa\int_0^S e^{-\zeta(s-r)^2}ds\right).\end{aligned}$$

Setting $\ln L=y$ and $\ln E=\varepsilon$, we obtain the following system:

$$\gamma\delta\int_0^S e^{-\delta(r-s)^2}y(s)ds-b\kappa\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)\,ds+(b-1)y(r)+c\varepsilon(r)=\ln w-\ln p-\ln(b+\gamma\delta\int_0^S e^{-\delta(r-s)^2}ds)$$

$$\begin{aligned}&\gamma\delta\int_0^S e^{-\delta(r-s)^2}y(s)ds+by(r)+c\varepsilon(r)-(\phi+b\kappa)\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)\,ds\\ =\;&\ln\phi-\ln p+\ln\left(\int_0^S e^{-\zeta(s-r)^2}ds\right)-\ln\left(c-b\kappa\int_0^S e^{-\zeta(s-r)^2}ds\right).\end{aligned}$$

We need to do the following transformation in order to obtain a system of second kind Fredholm integral equations with symmetric kernels:

$$\underbrace{\left.\begin{cases} \begin{pmatrix} \gamma\delta & -b\kappa \\ \gamma\delta & -\phi - b\kappa \end{pmatrix} \begin{pmatrix} \int\limits_0^S e^{-\delta(r-s)^2} y(s)ds \\ \int\limits_0^S e^{-\zeta(r-s)^2} \varepsilon(s)ds \end{pmatrix} + \\ \begin{pmatrix} \ln\left(b + \gamma\delta \int\limits_0^S e^{-\delta(r-s)^2} ds\right) + \ln p - \ln w \\ \ln p - \ln\phi - \ln\left(\int\limits_0^S e^{-\zeta(s-r)^2} ds\right) + \ln\left(c - b\kappa \int\limits_0^S e^{-\zeta(s-r)^2} ds\right) \end{pmatrix} \end{cases}\right\}}_{\mathrm{B}} =$$

$$\underbrace{\begin{pmatrix} 1-b & -c \\ -b & -c \end{pmatrix}}_{\mathrm{A}} \underbrace{\begin{pmatrix} y(r) \\ \varepsilon(r) \end{pmatrix}}_{\mathrm{Z}}$$

$$B = AZ$$

$$A^{-1}B = Z,$$

where $A^{-1} = \begin{pmatrix} 1 & -1 \\ -\frac{b}{c} & -\frac{1-b}{c} \end{pmatrix}$

$$\begin{pmatrix} 1 & -1 \\ -\frac{b}{c} & -\frac{1-b}{c} \end{pmatrix} \left\{ \begin{pmatrix} \gamma\delta & -b\kappa \\ \gamma\delta & -\phi - b\kappa \end{pmatrix} \begin{pmatrix} \int_0^S e^{-\delta(r-s)^2} y(s)ds \\ \int_0^S e^{-\zeta(r-s)^2} \varepsilon(s)ds \end{pmatrix} + \right.$$

$$\left. \begin{pmatrix} \ln(b + \gamma\delta \int\limits_0^S e^{-\delta(r-s)^2} ds) + \ln p - \ln w \\ \ln p - \ln\phi - \ln\left(\int\limits_0^S e^{-\zeta(s-r)^2} ds\right) + \ln\left(c - b\kappa \int\limits_0^S e^{-\zeta(s-r)^2} ds\right) \end{pmatrix} \right\} = \begin{pmatrix} y(r) \\ \varepsilon(r) \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} 0 & \phi \\ -\frac{\gamma\delta}{c} & \frac{(1-b)\phi + b\kappa}{c} \end{pmatrix} \begin{pmatrix} \int_0^S e^{-\delta(r-s)^2} y(s)ds \\ \int_0^S e^{-\zeta(r-s)^2} \varepsilon(s)ds \end{pmatrix} +$$

$$
\begin{pmatrix}
\ln\left(b+\gamma\delta\int\limits_0^S e^{-\delta(r-s)^2} ds\right) + \ln p - \ln w - \ln p + \ln\phi + \ln\left(\int\limits_0^S e^{-\zeta(s-r)^2} ds\right) - \\
-\ln\left(c - b\kappa\int\limits_0^S e^{-\zeta(s-r)^2} ds\right) \\
\\
-\frac{b}{c}\left[\ln\left(b+\gamma\delta\int\limits_0^S e^{-\delta(r-s)^2} ds\right) + \ln p - \ln w\right] - \\
-\frac{1-b}{c}\left[\ln p - \ln\phi - \ln\left(\int\limits_0^S e^{-\zeta(s-r)^2} ds\right) + \ln\left(c - b\kappa\int\limits_0^S e^{-\zeta(s-r)^2} ds\right)\right]
\end{pmatrix}
$$

$$
= \begin{pmatrix} y(r) \\ \varepsilon(r) \end{pmatrix}
$$

So, the system of second kind Fredholm integral equations is:

$$
\phi\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_1^*(r) = y(r) \tag{A1}
$$

$$
-\frac{\gamma\delta}{c}\int_0^S e^{-\delta(r-s)^2}y(s)ds + \frac{(1-b)\phi+b\kappa}{c}\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_3^*(r) = \varepsilon(r), \tag{A2}
$$

where

$$
\begin{aligned}
g_1^*(r) \quad = \quad & \ln\left(b+\gamma\delta\int\limits_0^S e^{-\delta(r-s)^2} ds\right) + \ln p - \ln w - \ln p + \ln\phi + \ln\left(\int\limits_0^S e^{-\zeta(s-r)^2} ds\right) \\
& - \ln\left(c - b\kappa\int\limits_0^S e^{-\zeta(s-r)^2} ds\right)
\end{aligned} \tag{A3}
$$

$$g_2^*(r) = -\frac{b}{c}\left[\ln\left(b+\gamma\delta\int_0^S e^{-\delta(r-s)^2}ds\right)+\ln p-\ln w\right]- \tag{A4}$$
$$-\frac{1-b}{c}\left[\ln p-\ln\phi-\ln\left(\int_0^S e^{-\zeta(s-r)^2}ds\right)+\ln\left(c-b\kappa\int_0^S e^{-\zeta(s-r)^2}ds\right)\right].$$

We use a modified Taylor-series expansion method for solving Fredholm integral equations systems of second kind (Maleknejad et al., 2006).[10] So, a Taylor-series expansion can be made for the solutions $y(s)$ and $\varepsilon(s)$:

$$y(s) = y(r) + y'(r)(s-r) + \frac{1}{2}y''(r)(s-r)^2$$

$$\varepsilon(s) = \varepsilon(r) + \varepsilon'(r)(s-r) + \frac{1}{2}\varepsilon''(r)(s-r)^2.$$

Substituting them into (1), (2), and (3):

$$\phi\int_0^S e^{-\zeta(r-s)^2}\{\varepsilon(r)+\varepsilon'(r)(s-r)+\frac{1}{2}\varepsilon''(r)(s-r)^2\}\ ds + g_1^*(r) = y(r)$$

$$-\frac{\gamma\delta}{c}\int_0^S e^{-\delta(r-s)^2}\ \{y(r)+y'(r)(s-r)+\frac{1}{2}y''(r)(s-r)^2\}\ ds+$$
$$\frac{(1-b)\phi+b\kappa}{c}\int_0^S e^{-\zeta(r-s)^2}\{\varepsilon(r)+\varepsilon'(r)(s-r)+\frac{1}{2}\varepsilon''(r)(s-r)^2\}\ ds + g_2^*(r) = \varepsilon(r).$$

Rewriting the equations, we have:

$$y(r) - \left[\phi\int_0^S e^{-\zeta(r-s)^2}ds\right]\varepsilon(r)- \tag{A5}$$

[10] K. Maleknejad, N. Aghazadeh, and M. Rabbani, Numerical solution of second kind Fredholm integral equations system by using a Taylor-series expansion method, Appl. Math. Comput. 175, 1229-1234 (2006).

$$\left[\phi\int_0^S e^{-\zeta(r-s)^2}(s-r)ds\right]\varepsilon'(r)-\left[\frac{1}{2}\,\phi\int_0^S e^{-\zeta(r-s)^2}(s-r)^2ds\right]\varepsilon''(r)=g_1^*(r)$$

$$\left[\frac{\gamma\delta}{c}\int_0^S e^{-\delta(r-s)^2}ds\right]y(r)+\left[\frac{\gamma\delta}{c}\int_0^S e^{-\delta(r-s)^2}(s-r)ds\right]y'(r)+$$

$$\left[\frac{1}{2}\,\frac{\gamma\delta}{c}\int_0^S e^{-\delta(r-s)^2}(s-r)^2ds\right]y''(r)\;+\left[1-\frac{(1-b)\phi+b\kappa}{c}\int_0^S e^{-\zeta(r-s)^2}ds\right]\varepsilon(r)- \tag{A6}$$

$$\left[\frac{(1-b)\phi+b\kappa}{c}\int_0^S e^{-\zeta(r-s)^2}(s-r)ds\right]\varepsilon'(r)-\left[\frac{1}{2}\,\frac{(1-b)\phi+b\kappa}{c}\int_0^S e^{-\zeta(r-s)^2}(s-r)^2ds\right]\varepsilon''(r)=g_2^*(r).$$

If the integrals in equations (3)-(4) can be solved analytically, then the bracketed quantities are functions of r alone. So (3)-(4) become a linear system of ordinary differential equations that can be solved if we use an appropriate number of boundary conditions.

To construct boundary conditions, we differentiate (1), (2):

$$y'(r)=\phi\int_0^S -2\zeta\;(r-s)\;e^{-\zeta(r-s)^2}\;\varepsilon(s)\;ds+g_1^{*'}(r) \tag{A7}$$

$$y''(r)=\phi\int_0^S\left[-2\zeta+4\zeta^2\;(r-s)^2\right]\;e^{-\zeta(r-s)^2}\;\varepsilon(s)\;ds+g_1^{*''}(r) \tag{A8}$$

$$\varepsilon'(r)=-\frac{\gamma\delta}{c}\int_0^S -2\delta\;(r-s)\;e^{-\delta(r-s)^2}\;y(s)\;ds+ \tag{A9}$$

$$\frac{(1-b)\phi+b\kappa}{c}\int_0^S -2\zeta\;(r-s)\;e^{-\zeta(r-s)^2}\;\varepsilon(s)\;ds+g_3^{*'}(r)$$

$$\varepsilon''(r) = -\frac{\gamma\delta}{c}\int_0^S \left[-2\delta + 4\delta^2\ (r-s)^2\right]\ e^{-\delta(r-s)^2}\ y(s)\ ds + \tag{A10}$$

$$\frac{(1-b)\phi + b\kappa}{c}\int_0^S \left[-2\zeta + 4\zeta^2\ (r-s)^2\right]\ e^{-\zeta(r-s)^2}\ \varepsilon(s)\ ds + g_3^{*''}(r).$$

We substitute $y(r), \varepsilon(r)$ for $y(s), \varepsilon(s)$ in equations (5)-(8):

$$y'(r) = \left[\phi\int_0^S -2\zeta\ (r-s)\ e^{-\zeta(r-s)^2}\ ds\right]\varepsilon(r) + g_1^{*'}(r) \tag{A11}$$

$$y''(r) = \left[\phi\int_0^S \left[-2\zeta + 4\zeta^2\ (r-s)^2\right]\ e^{-\zeta(r-s)^2}\ ds\right]\varepsilon(r) + g_1^{*''}(r) \tag{A12}$$

$$\varepsilon'(r) = \left[-\frac{\gamma\delta}{c}\int_0^S -2\delta\ (r-s)\ e^{-\delta(r-s)^2}\ ds\right]y(r) + \tag{A13}$$

$$\left[\frac{(1-b)\phi + b\kappa}{c}\int_0^S -2\zeta\ (r-s)\ e^{-\zeta(r-s)^2}\ ds\right]\varepsilon(r) + g_3^{*'}(r)$$

$$\varepsilon''(r) = \left[-\frac{\gamma\delta}{c}\int_0^S \left[-2\delta + 4\delta^2\ (r-s)^2\right]\ e^{-\delta(r-s)^2}\ ds\right]y(r) + \tag{A14}$$

$$\left[\frac{(1-b)\phi + b\kappa}{c}\int_0^S \left[-2\zeta + 4\zeta^2\ (r-s)^2\right]\ e^{-\zeta(r-s)^2}\ ds\right]\varepsilon(r) + g_3^{*''}(r).$$

From equations (A11)-(A14), $y'(r)$, $y''(r)$, $\varepsilon'(r)$, $\varepsilon''(r)$ are functions of $y(r)$, $\varepsilon(r)$, $g_1^{*'}(r)$, $g_1^{*''}(r)$, $g_3^{*'}(r)$, $g_3^{*''}(r)$. Substituting them into (A5) and (A6), we have a linear system of two algebraic equations that can be solved using Mathematica.

Appendix B

The same method of modified Taylor-series expansion was used in order to solve for the market allocations. We take the logs of the system (14) and (15) and follow the same process as the one described in Appendix A.

Appendix C

Transformation of the system of equations (6)-(7) to a single Fredholm equation of 2nd kind (Polyanin and Manzhirov, 1998).

We define the functions $Y(r)$ and $G(r)$ on $[0, 2S]$, where $Y(r) = y_i(r - (i-1)S)$ and $G(r) = g_i(r - (i-1)S)$ for $(i-1)S \le r \le iS$.[11] Next, we define the kernel $\Gamma(r, \bar{r})$ on the square $[0, 2S] \times [0, 2S]$ as follows: $\Gamma(r, \bar{s}) = k_{ij}(r - (i-1)S,\ \bar{r} - (j-1)S)$ for $(i-1)S \le r \le iS$ and $(j-1)S \le \bar{r} \le jS$.

So, the system of equations(6)-(7) can be rewritten as the single Fredholm equation $Y(r) - \frac{1}{1-b-c} \int_0^{2S} \Gamma(r, s)\ Y(s)\ ds = G(r)$, where $0 \le r \le 2S$.

If the kernel $k_{ij}(r, s)$ is square integrable on the square $[0, S] \times [0, S]$ and $g_i(r)$ are square integrable functions on $[0, S]$, then the kernel $\Gamma(r, s)$ is square integrable on the new square: $[0, 2S] \times [0, 2S]$ and $G(r)$ is square integrable on $[0, 2S]$. Functions $g_i(r)$, as described in Appendix A by equations (A3)-(A4) are square integrable.

[11] We assume that $y_1 = y$ and $y_2 = \varepsilon$, so as to follow the notation of our model.