Intertemporal Welfare Economics in Imperfect Economies

by

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Abstract

In this article, we make three contributions: i) wealth is the correct *linear* indicator of welfare (defined as the present value of future utilities) if population is constant, ii) there is no need to make assumption of an optimal economy and by using the abstract notion of a resource allocation mechanism, it is possible to develop intertemporal welfare economics in imperfect economies, iii) with changing population and constant returns to scale, wealth per capita is the appropriate linear welfare indicator. JEL D6, D9, H0, O47, Q01

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Over the past several years, we have benefited greatly from discussions with Kenneth Arrow on the subject matter of this article.

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1 Welfare Indices

1.1 Why

There are four reasons why we need quality of life indices. First, we need them if are to compare the lives of different groups of people. We may, for example wish to compare the quality of life of men with that of women, of female-headed households with that of male-headed households, of members of a particular denomination with that of the rest, in order to discover whether one group is doing badly relative to another. Secondly, we may wish to compare the state of affairs in different places (villages, districts, provinces, countries). Here too, the motivation could be to determine if public policy should be directed at some particular place or region. Thirdly, we may wish to know whether a community's long-term prospects are improving. For example, it could be that members of a community are currently living in an opulent style, but at the risk of a decline in future well-being.[1] We need welfare indices for use in detecting such possibilities. The concept of <u>sustainable development</u> was developed with this need in mind.[2]

The fourth reason we seek indices of social welfare is that policies have to be evaluated. We need criteria for determining whether a proposed change in policy is desirable. The criteria themselves need not be welfare indices; but they have to be founded on such indices; otherwise, choices would be based on objectives that are not congruent with social welfare. Of particular interest are investment projects. Evaluation criteria, such as a project's social rate of return, or the present discounted value of the flow of net social profits generated by it, are not themselves indices of social welfare, but they are based on such indices (see Section 9).

1.2 How

In recent years, debates on how to measure the quality of life have been influenced by two dichotomies: (1) the constituents versus the determinants of well-being, and (2) current versus intertemporal well-being. The constituents of well-being include health, happiness, freedom to be and do, and, more broadly, basic liberties. The determinants of well-being, on the other hand, are commodity inputs in the production of well-being; such as food, clothing, potable water, shelter, access to knowledge and information, and resources devoted to national security. Indices of the constituents of well-being measure output (health, the exercise of one's abilities, civil and political liberties), whereas those of the determinants of well-being

[1] In what follows we use the terms "quality of life", "well-being", and "welfare" interchangeably.

[2] IUCN (1980) and World Commission (1987).

are aggregates of the required inputs (expenditure on food, clothing, education, potable water, shelter, resources deployed for the protection and promotion of civil and political liberties). Publications from international organizations have made much of the dichotomy embodied in (1), and have reminded readers that there is a difference between "ends" and "means" (UNDP, 1994). However, working with models of timeless economies, the pioneers of the welfare economics of real national income demonstrated long ago that it does not matter whether use is made of the constituents or the determinants of well-being: welfare indices can be so constructed that those based on one are equivalent to those based on the other.[3] They also noted, more generally, that, indices that are based on the determinants of well-being are useful in practical work precisely because they can be so designed as to be linear in the quantities of goods and services. They showed that accounting prices can serve as weights in the construction of linear forms.

In contrast to (1), dichotomy (2) has not been much noted in the empirical literature published by international organizations.[4] It is however the case that the most well-known indices of social welfare - Gross National Product (GNP) per head and the United Nations Development Programme's Human Development Index (HDI) - are for all practical purposes measures of current well-being. The idea of sustainable development cannot be captured in such indices. We need an index of intertemporal well-being if we are to determine whether the quality of life is sustainable under alternative economic policies.

2 Summary

In this paper we show that there are systematic adaptations of a linear index of social welfare, namely wealth, that can serve all four purposes that we have for seeking quality of life indices. A community's wealth measures the social worth of its capital assets, where an asset's social worth is measured in terms of its accounting price (or shadow price). The notion of wealth we advance here is a comprehensive one, including, as it does, not only manufactured capital and knowledge, but also human and natural capital. We demonstrate below that, adjusting for (a) demographic differences, (b) differences in accounting prices arising from differences in institutional structures, and (c) the knowledge that is acquired freely from elsewhere, a society's wealth is an index of social well-being there. Of course, wealth itself is not meaningful, but differences in wealth are meaningful, involving as they do comparisons of states of affair.

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[3] The classics are Lindahl (1934), Hicks (1940), Samuelson (1961), and Mirrlees (1969).

[4] There is a sizeable theoretical literature on the subject though. Heal (1998) offers a clear account. Among international organisations, the World Bank is an exception in having explored the concept of sustainable development (Pezzey, 1992; World Bank, 1997; Hamilton and Clemens, 1999).

We show in particular that "sustainable development" should be interpreted as the maintenance of a society's wealth. This means that, in order to determine whether development is sustainable, we should estimate genuine investment, which is the accounting value of the net changes in a society's capital assets. Genuine investment measures changes in wealth at constant accounting prices. We demonstrate that, subject to certain qualifications that are identified below, social welfare increases over time if and only if genuine investment is positive. We show also that the correct way to evaluate investment projects in the public sector is to compare the reductions in consumption arising from the investment outlays with the increases in wealth the investments help to create. To put it in different words, projects should be accepted if they add to wealth, but not otherwise. That well-known criterion for project evaluation, namely, the present discounted value of the flow of a project's social profits, is shown to be implied by our finding.[5]

This chain of results unifies methods for valuing states of affairs and evaluating policies: they both involve wealth comparisons. Given that Adam Smith's inquiry into the wealth of nations is over two hundred years old, the conclusions could appear banal, but for the fact that in recent years the progress of nations has almost invariably been measured with every yardstick but wealth. Most common among them have been gross national product (GNP) and such ad-hoc measures of well-being as the United Nations Development Programmes' Human Development Index (HDI), neither of which is related to wealth. Thus, GNP could rise for an extended period even while wealth declines. GNP is a measure of economic activity, not social well-being. Our results imply also that unless an economy is in a stationary state, it is possible for net national product (NNP) to rise for an extended period, while wealth declines. As an index of social well-being, NNP too has to be rejected. Elsewhere we have shown (Dasgupta and Mäler, 2001) that the Human Development Index also is not suitable as an index of intertemporal well-being. HDI is in the main an index of current welfare; it does not accommodate future well-being. So, it should not cause surprise that HDI can increase over a period of time even while wealth declines. As an index of social welfare, HDI too has to be rejected.[6]

We use the Ramsey-Koopmans formulation of intertemporal welfare to establish the results summarized above. However, our use of their formulation differs from the use to which it has customarily

[5] A project's social profits are estimated on the basis of accounting prices, which, in effect provide the link between project evaluation criteria and indices of social welfare. See below in the text.

[6] Elsewhere we have shown that among the world's poorest regions, those that have recorded an improvement in their HDI during the past decade have in fact become even poorer (less wealthy) in per capita terms; moreover, those that have recorded a deterioration in their HDI have become wealthier in per capita terms. See Dasgupta (2001a) and Dasgupta and Mäler (2001).

been put in one significant respect:

Intertemporal welfare economics has mostly been developed for a society where the State is not only trustworthy, but it also optimizes on behalf of its citizens. Policy prescriptions emerging from the theory - including recommendations concerning "green" national accounts (see Lutz, 1993) - are for Utopia, or at worst, for what Meade (1989) called Agathotopia (the "good-enough society"). The theory does not apply directly to a world that is less good than Agathotopia.

In this paper we study non-optimizing economies - we will call them imperfect economies - of which Agathotopia is an agreeable special case. In an earlier paper (Dasgupta and Mäler, 2000) we showed that several basic principles of intertemporal welfare economics (e.g., the use of accounting prices to value goods and services) can be extended to imperfect economies. However, our earlier paper was concerned mostly with the concept of sustainable development. We did not prove there that the principles of intertemporal welfare economics can be extended to policy evaluation in imperfect economies. Nor did we show how population change could be accommodated when states of affairs are to be compared. In this paper we extend our previous results to provide a fuller account of the principles of intertemporal welfare economics in imperfect economies. Being imperfect, we suppose that the economies in question are capable only of policy reforms. It transpires that welfare comparisons can be made with the help of accounting prices in imperfect economies even if production possibilities are non-convex. So, in what follows there will be no presumption that transformation possibilities among goods and services are convex. We emphasize this because natural capital is included in our account, and some of the most significant ecological processes are known to be non-convex (see, e.g., Murray, 1993).

A society's capital assets should be distinguished from its institutions. Admittedly, the latter are also sometimes called "capital" (as in "institutional capital", "social capital", "cultural capital"); however, institutions are distinct from capital assets, in that they guide the allocation of resources (among which are capital assets!). This is why it isn't useful to regard institutions as simply another form of capital. Institutions are more usefully viewed as resource allocation mechanisms, a concept central to welfare economics in imperfect economies. Together, a society's capital assets and institutions comprise its productive base. A society's productive base is the source of its well-being. We adopt this idea as our starting point.

3 Road Map

The plan of the paper is as follows:

Section 4 offers a stripped-down model of aggregate production and consumption possibilities with which we develop our analysis. Population is assumed to be constant; the future is taken to be deterministic;

the economy is closed; and there are only two types of capital assets, namely, manufactured and natural capital. We will refer to this model as the base case.

Resource allocation mechanisms are defined in Section 5. We offer a general definition. It includes in its domain economies that are riddled with inefficiencies and injustices. Attention is focussed on resource allocation mechanisms that are autonomous in time. In Sections 6 and 7 we develop the concepts of current and sustainable well-being, respectively. Accounting prices in imperfect economies are defined in Section 8. They are shown subsequently to play a central role in the welfare economics of imperfect economies.

Section 9 is concerned with project evaluation criteria. We show that the present discounted value of the flow of social profits of a project is the right evaluation criterion.[7] In Section 10 we show that wealth is a linear index of sustainable well-being. Section 11 contains a unification of the results in the previous two sections: it is shown that the present discounted value of the flow of social profits of a project is the change in an economy's wealth brought about by it.

In Lutz (1993) a number of social scientists suggested that net national product (NNP) is a measure of sustainable well-being. In Section 12 we show the limited sense in which NNP can be used in welfare economics. Section 13 contains a simple model economy for illustrating the theory. The model is so simple that it is possible to characterize the resource allocation mechanism explicitly and to illustrate our results.

Section 14 develops a criterion - based on wealth - for making cross-country comparisons of intertemporal well-being. Our base case is then used in Section 15 to make welfare comparisons in the presence of global public goods (e.g., the atmosphere).

In our base case, resource allocation mechanisms are assumed to be (time) autonomous. If they are not autonomous, adjustments have to be made to wealth if the latter is to serve as an index of social welfare. Autonomous technological change and population change are two reasons why resource allocation mechanisms may not be time-autonomous. Sections 16 and 17 extend the base case by introducing changes in technology and population, respectively. We show how the adjustments ought to be made.

Finally, in Section 18 we discuss further extensions of the basic model.

4 Modelling the Base Case

We assume, for expositional ease, that population is constant and the economy is closed and

[7] That it is the correct criterion for project evaluation in optimizing economies is well known (see, e.g., Little and Mirrlees, 1968, 1974; Arrow and Kurz, 1970). Dasgupta, Marglin, and Sen (1972) argued in favour of this same criterion for evaluating projects in imperfect economies, but did not offer formal proofs.

deterministic. This is the base case. Time is taken to be continuous. It is denoted as t (the present is $t = 0$). The horizon is assumed to be infinite.

There is an all-purpose, non-deteriorating durable good, whose stock at t is K_t (≥ 0). The good can be consumed or reinvested for its own accumulation. Production of this good requires labour, manufactured capital, and natural resources. For simplicity, we focus on one type of natural capital: extractive resources (oil and natural gas, timber, water, agricultural land).

The all-purpose good can be produced with its own stock (K), labour-hours (L) and the flow of natural resources (R) as inputs. Write aggregate output as Y and assume that $Y = F(K, L, R)$. GNP at t is then $Y_t = F(K_t, L_t, R_t)$. F is taken to be an increasing and continuously differentiable function of each of its variables. We do not assume F to be concave. Nor do we assume anything specific about substitution possibilities between K, L and R.

Let C_t (≥ 0) denote aggregate consumption at t. Net accumulation of manufactured capital is given by the condition

$$dK_t/dt = F(K_t, L_t, R_t) - C_t. \quad (1)$$

The aggregate stock of natural capital is denoted by S_t (≥ 0). We suppose that resources can be extracted costlessly. Let their natural rate of growth be $M(S_t)$, which we take to be a continuously differentiable function.[8]

The dynamics of the resource base can therefore be expressed as

$$dS_t/dt = M(S_t) - R_t. \quad (2)$$

As with F, we do not suppose that M is a concave function.

The initial conditions of the economy are given by the pair of capital assets (K_0, S_0).

Certain types of natural capital are directly valuable as stocks in production and consumption (e.g., resources having intrinsic value). For expositional ease, we assume that such "stock effects" are absent (but see Section 18). Current welfare is taken to depend on consumption and leisure (the negative of labour hours). We write this as $U(C, L)$, where U is strictly concave, increasing in C, decreasing in L (at least at large enough values of L), and continuously differentiable in both C and L.

$\{C_t, L_t, R_t, K_t, S_t\}_0^\infty$ is an economic programme - from the present to the indefinite future - if it satisfies equations (1) and (2). Social welfare at t (≥ 0) is defined to be

[8] If the resource in question were minerals or fossil fuels, S_t would denote known reserves at t and we would have $M(S) = 0$ for all S.

$V_t = {}_t\int^{\infty} U(C_\tau, L_\tau)e^{-\delta(\tau-t)}d\tau$, where $\delta > 0$. (3)

5 Resource Allocation Mechanisms

Consider an economy facing the technological and ecological constraints in equations (1) and (2). In addition, it faces institutional constraints (sometimes called transaction and information constraints). By the economy's "institutions" we mean market structures, the structure of property rights, tax rates, non-market institutions for credit, insurance and common property resources, the character of various levels of government, and so forth. For the moment, we assume that the institutional structure is given. It will be regarded as a "parameter" of the economy. The initial capital stocks (K_0, S_0) are a given too. If in addition, the behavioural characteristics of the various agencies in the economy (i.e., those of households, firms, the government, and so on) were known, it would be possible to make an economic forecast, by which we mean a forecast of the economic programme $\{C_t, L_t, R_t, K_t, S_t\}_0^\infty$ that would be expected to unfold. Call this relationship a resource allocation mechanism. A resource allocation mechanism is therefore a (many-one) mapping from initial capital stocks (K_0, S_0) into the set of economic programmes $\{C_t, L_t, R_t, K_t, S_t\}_0^\infty$ satisfying equations (1) and (2).

Let us now formalise this. Write

$\Omega_t \equiv (K_t, S_t)$, and (4)

$(\xi_\tau)_t^\infty \equiv \{C_\tau, L_\tau, R_\tau, K_\tau, S_\tau\}_t^\infty$, for $t \geq 0$. (5)

Next, let $\{t, \Omega_t\}$ denote the set of possible t and Ω_t pairs, and $\{(\xi_\tau)_t^\infty\}$ the set of economic programmes from t to the indefinite future. A resource allocation mechanism, α, can then be expressed as a (many-one) mapping

$\alpha: \{t, \Omega_t\} \rightarrow \{(\xi_\tau)_t^\infty\}$. (6)

α is time autonomous if for all $\tau \geq t$, ξ_τ is a function solely of Ω_t and $(\tau-t)$; that is, if the "state of the economy" at τ depends solely on Ω_t and $(\tau-t)$. α would be time autonomous if neither knowledge nor the terms of trade for a trading economy were to change autonomously over time. In certain cases exogenous changes in population size would also mean that α is not time autonomous (Section 17). For the moment we are considering a closed economy with constant population. Therefore, if knowledge does not display an exogenous shift, α is time autonomous. We will see presently that, computationally, it is simplest to study cases where α is time autonomous. Therefore, we assume that α is time autonomous. The assumption is dropped in Sections 16 and 17.

It bears re-emphasis that we do not assume α to sustain an optimum economic programme, nor even that it sustains an efficient programme. The following analysis is valid even if α is riddled with economic distortions and inequities.

To make the dependence of the economic forecast on α explicit, let $\{C_t(\alpha), L_t(\alpha), R_t(\alpha), K_t(\alpha), S_t(\alpha)\}_0^\infty$ denote the forecast at t = 0. Consider date t ($\geq$ 0). We may now write equation (3) as,

$$V_t(\alpha, \Omega_t) \equiv {}_t\int^\infty U(C_\tau(\alpha), L_\tau(\alpha))e^{-\delta(\tau-t)}d\tau. \tag{7}$$

V_t is also called the value function.[9]

Before putting the concept of resource allocation mechanism to work, it is as well to discuss examples by way of illustration. Imagine first that all capital assets are private property and that there is a complete set of competitive forward markets capable of sustaining a unique equilibrium. In this case α would be defined in terms of this equilibrium. (If equilibrium is not unique, a selection rule among the multiple equilibria would have to be specified.) A great deal of modern macroeconomics is founded on this mechanism, as are many writings on the intertemporal welfare economics of the environment.[10]

Of particular interest are situations where some of the assets are not private property. Consider, for example, the class of cases where K is private property and S is common property. It may be that S is a local common property resource, not open to outsiders. If the assets are managed efficiently, we are back to the case of a competitive equilibrium allocation, albeit one not entirely supported by market prices, but in part by, say, social norms.

On the other hand, it may be that local institutions are not functioning well (e.g., because social norms are breaking down, in that private benefits from using S exceed social benefits). Suppose in addition that decisions bearing on the net accumulation of K are guided by the profit motive. Then these behavioural rules together help determine α. In a similar manner, we could characterize α for the case where S is open-access.[11]

Institutional assumptions underlie the notion of resource allocation mechanism. Aspects of the concept of "social capital" (Putnam, 1993) appear in our framework as part of the defining characteristics of α, as do ideas relating to "social capability" (Adelman and Morris, 1965; Abramovitz, 1986), and "social infrastructure" (Hall and Jones, 1999). The prevalence (or absence) of trust and honest behaviour in the

[9] In all this, we take it that V_t is well defined. The assumption that $\delta > 0$ is crucial for this. Koopmans (1965; 1976; 1972) are the key papers on this requirement.

[10] Heal (1998) contains an account of the latter literature.

[11] In Section 13 we illustrate α by means of a formal example.

economy are embodied in α. Other aspects of the concept of social capital (personal networks) enter as factors in the production function F.

The crucial assumption we now make is that V_t is differentiable in each of the two components of Ω. It isn't easy to judge whether V_t is differentiable. The mathematical properties of V_t depend upon the mathematical properties of α. But it isn't easy to judge when α is "smooth". Problems are compounded because production and substitution possibilities in the economy are embedded in α, as is the economy's underlying institutional structure. Moreover, there are no obvious limits to the kinds of institutions one can envision. In many parts of the world the State has been known to act in bizarre and horrible ways. So one looks at what might be termed "canonical" institutions. Analytically, the most well understood are those which support optimum economic programmes. What do we know about them?

If U, F, and M are concave, V_t is concave along optimum economic programmes and is therefore differentiable almost everywhere in each component of Ω. This property holds even in those circumstances where the optimum programme is chaotic. Thus, chaotic αs don't rule out differentiable V_ts (almost everywhere).[12] However, if production functions are not concave, optimum resource allocation mechanisms can be discontinuous. Significantly, Skiba (1978) has proved for the case where production functions are "convex-concave" that at those values of Ω where V_t is non-differentiable with respect to Ω (such points are, however, non-generic), V_t is continuous. But if V_t possesses right- and left-partial derivatives (and it does in the examples we have studied), social cost-benefit analysis of policy reforms can be conducted at the optimum with the aid of accounting prices, using the present discounted value of the flow of social profits as the criterion of choice (Section 9). The same could be expected to be true for the case of market economies subject to fixed distortions, such as those considered by Little and Mirrlees (1968, 1974) in their account of social cost-benefit analysis.

Experience with non-linear dynamical systems suggests that if α is non-optimal, V_t is discontinuous at certain values of Ω. Accounting prices would not be definable at such points (see equations (10a,b)).[13] In those αs that have been studied in the literature, discontinuities would appear to be non-generic. So, unless

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[12] See Majumdar and Mitra (2000) for a fine account. We are grateful to Mukul Majumdar for discussions on this point.

[13] However, if the location of these points on the space of capital stocks is uncertain and the uncertainty a smooth probability distribution, the expected value of V_t would be continuous. We conjecture that in this case accounting prices exist.

the economy is by fluke at a point of discontinuity, V_t would be differentiable within a sufficiently small neighbourhood of the initial capital stocks. It would seem then that the demand that V_t be differentiable does not rule out much of practical significance. The theory we offer here is valid for a substantially more general set of environments than is usual in writings on intertemporal welfare economics.

Therefore, we assume that V_t is differentiable. In Section 13 we develop a model, based on Ramsey (1928) and Solow (1956), where the (imperfect) resource allocation mechanism has an explicit functional form. We show that V_t in that model is differentiable everywhere.

6 Current Welfare

Knowledge of the resource allocation mechanism α enables us to make an economic forecast. Current well-being along the forecast is $U(C_t, L_t)$. So

$$dU_t/dt = U_C dC_t/dt + U_L dL_t/dt. \tag{8}$$

Let us call U_C and $-U_L$ the accounting prices of consumption and labour effort, respectively, in well-being numeraire. Equation (8) yields a method for judging if current well-being is improving over time:

Proposition 1: If the accounting value of changes in the flow of consumption services is positive, current well-being can be said to be improving.

International time series of the quality of life, such as those published annually in the World Bank's World Development Report and UNDP's Human Development Report, are based implicitly on Proposition 1. The reports include such indices as private consumption and life expectancy at birth, which serve as surrogates for key components of what we are calling consumption and leisure here. The reports veer away from Proposition 1 when they include gross national product (GNP), in that GNP is neither a measure of current welfare nor a measure of intertemporal welfare. It is the "gross" bit in GNP that makes it an unsuitable index of social well-being.

7 Sustainable Welfare

IUCN (1980) and World Commission (1987) introduced the concept of sustainable development. Consider the following definitions[14]:

(A) The economic programme $\{C_t, L_t, R_t, K_t, S_t\}_0^\infty$ corresponds to a "sustainable development path" if $dU(C_t,L_t)/dt \geq 0$ for all $t \geq 0$.

(B) The economic programme $\{C_t, L_t, R_t, K_t, S_t\}_0^\infty$ corresponds to a "sustainable development path" if $dV_t/dt \geq 0$ for all $t \geq 0$.

[14] Pezzey (1992) contains a thorough discussion of what the concept may mean.

In contrast to A, which looks at current welfare (U), the focus of B is intertemporal welfare (V). Criterion A requires that U should never decline, whereas, criterion B requires that V should never decline. While A implies B, B does not imply A. So, B is more general.[15]

Criteria A and B offer contrasting ideas of what a consumption stream must look like if it corresponds to a path of sustainable development. Both involve conditions that must be satisfied at every date, a phenomenally strict requirement. If the notion of sustainability is to have practical force, it needs to be less ambitious. It should be enough for decision makers at any date if they knew that the choices they make would not compromise the prospects open to decision makers at future dates. Never mind what happens at dates far in the future, decision makers at any given date would justifiably feel satisfied if they were to do their own job satisfactorily. After all, or so they could argue, they will have no say over what will be chosen by decision makers in the distant future.

Consider then the following definitions:

(A′) The economic programme $\{C_t, L_t, R_t, K_t, S_t\}_0^\infty$ corresponds to a "sustainable development path" at t if $dU(C_t, L_t)/dt \geq 0$.

(B′) The economic programme $\{C_t, L_t, R_t, K_t, S_t\}_0^\infty$ corresponds to a "sustainable development path" at t if $dV_t/dt \geq 0$.

It is clear that B′ nails the idea of sustainable development, while A′ fails to do so. Being concerned only with comparisons of current well-being (U), criterion A′ could be satisfied at t even as the economy's productive base is allowed to shrink, jeopardizing life in the future. It could even be that current well-being increases at t only because prevailing policies discourage the accumulation of manufactured capital and are also rapacious in the use of the natural environment. Criterion A′ would be unable to detect this. Such measures of the quality of life as the Human Development Index of the United Nations Development Programme suffer from this weakness. In contrast, criterion B′ is able to detect if existing policies are myopic, because it involves a comparison of intergenerational well-beings (V). For these reasons, we adopt B′ as the definition of sustainable development.

8 Accounting Prices

As with optimizing economies, accounting prices are useful in imperfect economies. Let welfare be the numeraire. Define,

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[15] If economic policies were arbitrarily given, this is a trivial matter to confirm. Interestingly, Asheim (1994) has identified cases where even an optimum consumption stream may satisfy Y, while violating X.

$p_t(\alpha, \Omega_t) \equiv \partial V_t(\alpha, \Omega_t)/\partial K_t$; and $q_t(\alpha, \Omega_t) \equiv \partial V_t(\alpha, \Omega_t)/\partial S_t$. (9)

From equations (7) and (9) we have,

$$p_t(\alpha, \Omega_t) = {}_t\int^{\infty}[\partial U(C_\tau(\alpha), L_\tau(\alpha))/\partial K_t]e^{-\delta(\tau-t)}d\tau, \quad (10a)$$

and $$q_t(\alpha, \Omega_t) = {}_t\int^{\infty}[\partial U(C_\tau(\alpha), L_\tau(\alpha))/\partial S_t]e^{-\delta(\tau-t)}d\tau. \quad (10b)$$

$p_t(\alpha, \Omega_t)$ and $q_t(\alpha, \Omega_t)$ are the accounting prices of the capital assets. They are spot prices and measure the assets' social scarcities. Accounting prices are defined in terms of hypothetical perturbations to the economic forecast. Plainly, accounting prices at t are functions of Ω_t. Moreover, the structure of property rights also influences the accounting prices of assets. For example, accounting prices of "goods" can be negative if there is a dysfunctional set of property rights, such as those that lead to the tragedy of the commons. Finally, if α is time autonomous, accounting prices are not explicit functions of t. Having stressed the functional dependence of accounting prices on α and Ω, we drop α and Ω from the formulae so as to save on notation.

In this paper, we are concerned solely with developing the analytical foundations of intertemporal welfare economics in imperfect economies. While in principle accounting prices can be estimated from equations (10a-b), practical methods of estimation would involve short-cuts, such as a reliance on "observables" (e.g. market prices). However, it is as well to stress that in imperfect economies a clear distinction should be made between social welfare (as reflected in U) and market observables. The behaviour of households and firms are built into the resource allocation mechnaism, α. Using market observables to infer social welfare can be very misleading in imperfect economies.[16]

Recall that $V_t = V_t(\alpha, \Omega_t)$. Suppose for the moment that V_t is not time-autonomous. Then, using (7) and (9), we may conclude that

$$dV_t/dt = p_t dK_t/dt + q_t dS_t/dt + \partial V_t/\partial t. \quad (11)$$

If, as we are assuming here, V_t is time-autonomous, equation (11) reduces to the more amiable form:

$$dV_t/dt = p_t dK_t/dt + q_t dS_t/dt. \quad (12)$$

Equations (11) and (12) are fundamental. We will make much use of them below.

[16] Using the economic forecast based on α, it is possible to deduce the time trajectories of p_t and q_t. Along optimal programmes p_t and q_t would satisfy Pontryagin's equations. In Dasgupta and Mäler (2000) it was mistakenly claimed that they do so even in imperfect economies. In fact, they can be shown to satisfy Pontryagin's equations with adjustment terms, the adjustments reflecting the extent to which the economy is imperfect. Interested readers can easily deduce the dynamical equations from the definitions of p_t and q_t.

9 Project Evaluation

Imagine that even though the government does not optimize, it can bring about small changes to the economy by altering the existing resource allocation mechanism in minor ways. The perturbation in question could be small adjustments to the prevailing structure of taxes for a short while, it could be minor alterations to the existing set of property rights for a brief period, it could be a public investment, or whatever. Call any such perturbation a <u>policy reform</u>.[17]

Consider an investment project. It can be viewed as a perturbation to the resource allocation mechanism α for a brief period, after which the mechanism reverts back to its earlier form. We consider projects that are small relative to the size of the economy. How should they be evaluated?

The project involves small quantities of manufactured capital, labour, and natural resources to produce a small additional quantity of the all-purpose commodity Y. Denote the project's output and inputs at t by the vector $(\Delta Y_t, \Delta K_t, \Delta L_t, \Delta R_t)$.[18]

The project's acceptance would perturb aggregate consumption and labour supply under α. Let the perturbation at t be $(\Delta \mathcal{C}_t, \Delta \mathcal{L}_t)$. It would affect U_t by the amount $(U_C \Delta \mathcal{C}_t + U_L \Delta \mathcal{L}_t)$. It would be tiresome if the project evaluator was required to estimate $(\Delta \mathcal{C}_t, \Delta \mathcal{L}_t)$ for every project that came up for consideration. Accounting prices of capital assets are useful because they enable the evaluator to estimate $(\Delta \mathcal{C}_t, \Delta \mathcal{L}_t)$ indirectly.[19]

Recall that U is the unit of account. So, δ is the accounting rate of interest on well-being. Since the accounting price of manufactured capital is p_t (equation (10a)), the corresponding accounting rental rate is δp_t. It is most unlikely that consumption and investment have the same accounting price in an imperfect economy. So we decompose ΔY_t into two components: changes in consumption and in investments in

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[17] Over the years, economic evaluation of policy reform in imperfect economies has been discussed by a number of economists (Meade, 1955; Dasgupta, Marglin, and Sen, 1972; Mäler, 1974; Starrett, 1988; Ahmad and Stern, 1990; Dreze and Stern, 1990; and Edwards and Keen, 1996, to name only a few). But they did not develop a formal account of <u>intertemporal</u> welfare economics in a reformist economy. This article is an attempt to fill that gap.

[18] If the project has been designed efficiently, we would have $\Delta Y_t = F_K \Delta K_t + F_L \Delta L_t + F_R \Delta R_t$. The analysis that follows in the text doesn't require the project to have been designed efficiently.

[19] The arguments in the text develop the theory of social cost-benefit analysis in Dasgupta, Marglin, and Sen (1972).

manufactured capital. Denote them as ΔC_t and $\Delta(dK_t/dt)$, respectively.[20]

Let w_t denote the accounting wage rate. How would we measure it? If we knew α, we would be able to estimate $\Delta\breve{L}_t/\Delta L_t$. Now, $w_t = -(\Delta\breve{L}_t/\Delta L_t)U_L$. So, $w_t = -U_L$ if $\Delta\breve{L}_t = \Delta L_t$, and $w_t = 0$ if $\Delta\breve{L}_t = 0$. In "labour-surplus economies" one would typically find $0 < -(w_t/U_L) < 1$.

It follows that:

$$U_C\Delta\breve{C}_t + U_L\Delta\breve{L}_t = U_C\Delta C_t + p_t\Delta(dK_t/dt) - w_t\Delta L_t - \delta p_t\Delta K_t - q_t\Delta R_t. \qquad (13)$$

But the right-hand-side (RHS) of equation (13) is the project's social profit at t. Let ΔV_0 be the change in social well-being if the project, starting at t = 0, were accepted. We then have

$$\Delta V_0 = {}_0\!\int^{\infty}[U_C\Delta C_\tau+p_\tau\Delta(dK_\tau/d\tau)-w_\tau\Delta L_\tau-\delta p_\tau\Delta K_\tau-q_\tau\Delta R_\tau]e^{-\delta\tau}d\tau. \qquad (14)$$

Equation (14) yields the criterion we seek:

Proposition 2: A project should be accepted if and only if the present discounted value of its social profits is positive.

How is project evaluation related to optimum planning? Imagine that at each date projects are evaluated as a tatônnement. The accounting prices used to evaluate projects along the tatônnement are those that would prevail if all acceptable projects in the queue to date had been accepted and all unacceptable ones rejected. This sequence of hypothetical choices is often called the "gradient process" (also called the "hill-climbing method"). Arrow and Hurwicz (1958) proved in the context of a finite-dimensional economy that, provided the set of economic possibilities has a sufficiently strong convex structure, the gradient process converges to the optimum. Given that we are considering infinite-dimensional economic programmes, a corresponding result for our model economy would be harder to prove. It is reasonable to conjecture that despite this, a sequence of project selections in the form of a suitably defined gradient process would converge to an optimum economic programme if the economy had a strong convex structure.[21] If the economy does not have a convex structure, the gradient method can at best be expected to lead to a local optimum.

10 Wealth and Sustainable Welfare

UNDP (1994: 14-15) castigates those who regard GNP to be an index of social well-being on the grounds that it is a measure of a country's opulence. The criticism is faulty in two ways. First, opulence is

[20] See Dasgupta, Marglin, and Sen (1972) for practical methods of estimation.

[21] In referring to an optimum economic programme, we include "second-best" optima.

a stock concept, and GNP is not a return on any index of opulence that we are aware of.[22] Secondly, and more importantly, the connection we drew earlier between the constituents and determinants of well-being tells us that it isn't a mistake to seek to measure a society's well-being in terms of an index of opulence. The point isn't that opulence misleads, but rather that we should search for the <u>right measure</u> of opulence.

A country's wealth is the social worth of its capital assets. It is a measure of the nation's opulence. We confirm below that, subject to a well-defined set of qualifications, it is simultaneously a measure of social welfare.

Define,

$$I_t^K \equiv p_t dK_t/dt \text{ and } I_t^S \equiv q_t dS_t/dt. \quad (15)$$

They are net investments in the three types of capital assets, respectively. Define <u>genuine investment</u> to be,

$$I_t = I_t^K + I_t^S.\text{[23]} \quad (16)$$

Now use equations (12), (15), and (16) to conclude that

$$dV_t/dt = I_t. \quad (17)$$

Equation (17) gives us an index of sustainable development:

<u>Proposition 3: $dV_t/dt \geq 0$ if and only if there is genuine investment at t</u>.[24]

This is a local result and has intuitive appeal. It says that social welfare is higher today than it was yesterday if the economy is <u>wealthier</u> today. An economy's wealth is the accounting value of all its capital assets. Wealth comparisons are made, locally, at constant prices.

Proposition 3 is an <u>equivalence</u> result. It doesn't on its own say if development is sustainable. Whether the economy is capable of growing wealthier depends on the extent to which different assets are

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[22] One can even argue that, because it doesn't take note of capital depreciation, GNP <u>cannot</u> be a measure of opulence.

[23] Note that the summation in equation (17) does <u>not</u> imply any assumptions regarding substitution possibilities among the three kinds of capital assets. Whatever substitution possibilities there may be would be reflected in the accounting prices.

[24] This result has been known for some while to be a property of optimum economic programmes. It originated in Solow (1974, 1992) and Hartwick (1977) in their work on intergenerational maxi-min consumption streams. Dasgupta and Heal (1979: ch. 10) integrated their results with those implied by the Ramsey-Koopmans formulation and offered a finding that was an early version of Proposition 3, but restricted to optimum economic programmes.

substitutable in production. Accounting prices reflect substitution possibilities, among other things.[25]

Imagine that substitution possibilities are limited, and the resource allocation mechanism in place is profligate in the use of natural resources. Under these circumstances the quality of life will not be sustainable. At some date in the future accounting prices will assume such values as to make it impossible for genuine investment to be positive. Social welfare declines when genuine investment is negative.

11 Valuing and Evaluating

How is genuine investment related to the changes in future consumption brought about by it? To answer this, imagine that the capital base at t isn't Ω_t, but $\Omega_t+\Delta\Omega_t$, where Δ is an operator signifying a small difference. In the obvious notation:

$$V_t(\alpha, \Omega_t+\Delta\Omega_t) - V_t(\alpha, \Omega_t) \approx {}_t\int^{\infty}(U_C\Delta C_\tau + U_L\Delta L_\tau)e^{-\delta(\tau-t)}d\tau. \qquad (18)$$

Now suppose that at t there is a small change in the resource allocation mechanism α, but only for a brief moment, Δt, after which the resource allocation mechanism reverts back to α. We write the increment in the capital base at $t+\Delta t$ consequent upon the brief increase in genuine investment as $\Delta\Omega_t$. So $\Delta\Omega_t$ is the consequence of an increase in genuine investment at t and $\Omega_{t+\Delta t}+\Delta\Omega_t$ is the resulting capital base at $t+\Delta t$. Let Δt tend to zero. From equation (18) we obtain

Proposition 4: Genuine investment measures the present discounted value of the changes to consumption services brought about by it.

Notice the connection between equation (18) and equations (10a-b). They say the same thing. Proposition 4 brings out the connection between wealth as a measure of social well-being (Proposition 3) and the present discounted value of consumption services as a criterion for social cost-benefit analysis (Proposition 2). The way to evaluate an investment project is to compare reductions in short-term well-being resulting from it to the increase in wealth the reductions help to create.

12 The Welfare Significance of NNP

Net national product (NNP) in our model economy is

$$NNP_t = U_CC_t + U_LL_t + p_tdK_t/dt + q_tdS_t/dt. \qquad (19)$$

More generally, NNP is the sum of genuine investment and the accounting values of all consumption services (including direct consumption services from the natural environment and "negative" consumptions arising from pollution; Nordhaus and Tobin, 1972).

Dasgupta and Mäler (1991; 2000) and Mäler (1991) proved that the contribution investment projects

[25] Dasgupta and Heal (1979: chs. 7 and 10).

make to NNP (as defined in equation (19)), is the appropriate project evaluation criterion, provided that projects are small and are of the briefest duration. The intuition underlying the result is that accounting prices do not change if the period is short. But as investment projects are not of the briefest of durations, the result is of no practical interest. It can be shown more generally that, if relative prices remain constant, NNP is indeed an index of intertemporal welfare. Dasgupta and Mäler (2000) showed that NNP is the return on wealth if and only if U is linear and accounting prices are constant over time. Since neither condition is believable, this result too is of no practical interest.

However, NNP does have normative significance. Use equation (19) to restate Proposition 3 as,

Proposition 5: $dV_t/dt \geq 0$ if and only if $U_C C_t + U_L L_t \leq NNP_t$.

Taken together, Propositions 3 and 5 say that if social welfare is to increase, the value of consumption services must not exceed NNP.[26]

13 Illustration

To illustrate the theory, it is simplest to build on the models of capital accumulation in Ramsey (1928) and Solow (1956). The model is really stylized. For example, it assumes that substitution possibilities between manufactured and natural capital are so large that the latter can be ignored when building macroeconomic models. However, the construct is the only one for which we have been able to obtain an explicit description. So we study it here. It is meant to interpret the theory in simple terms, nothing more.

Imagine that there is an all-purpose durable good, whose stock at t is K_t (≥ 0). The good can be consumed or reinvested for its own accumulation. There are no other assets. Population size is constant and labour is supplied inelastically. Write output (GNP) as Y. Technology is linear. So $Y = \mu K$, where $\mu > 0$. μ is the output-wealth ratio. GNP at t is $Y_t = \mu K_t$.

A constant proportion of GNP is saved at each moment. There is no presumption though that the saving rate is optimum. It is a behavioural characteristic of consumers, reflecting their response to an imperfect credit market. Other than this imperfection, the economy is assumed to function well. At each moment, expectations are fulfilled and all markets other than the credit market clear. This defines the resource allocation mechanism, α. Clearly, α is autonomous in time. We now characterise α explicitly.

Let the saving ratio be s ($0 < s < 1$). Write aggregate consumption as C_t. Therefore,

$$C_t = (1-s)Y_t = (1-s)\mu K_t. \tag{20}$$

Capital depreciates at a constant rate γ (> 0). Genuine investment is,

[26] Lindahl (1934), Hicks (1940), and Samuelson (1961).

$$dK_t/dt = (s\mu-\gamma)K_t. \quad (21)$$

K_0 is the initial capital stock. The economy grows if $s\mu > \gamma$, and shrinks if $s\mu < \gamma$. To obtain a feel for orders of magnitude, suppose $\gamma = 0.05$ and $\mu = 0.25$. The economy grows if $s > 0.2$, and shrinks if $s < 0.2$.

Integrating equation (21), we obtain,

$$K_\tau = K_t\exp[(s\mu-\gamma)(\tau-t)], \qquad \text{for all } \tau \text{ and } t,\ \tau \geq t \geq 0. \quad (22)$$

from which it follows that,

$$C_\tau = (1-s)\mu K_\tau = (1-s)\mu K_t\exp[(s\mu-\gamma)(\tau-t)], \text{ for all } \tau \text{ and } t,\ \tau \geq t \geq 0. \quad (23)$$

If the capital stock was chosen as numeraire, wealth would be K_t, and NNP would be $(\mu-\gamma)K_t$. Each of wealth, GNP, NNP, consumption and genuine investment expands at the exponential rate $(s\mu - \gamma)$ if $s\mu > \gamma$; they all contract at the exponential rate $(\gamma - s\mu)$ if $s\mu < \gamma$. We have introduced capital depreciation into the example so as to provide a whiff (albeit an artificial whiff) of a key idea, that even if consumption is less than GNP, wealth declines when genuine investment is negative. Wealth declines when consumption exceeds NNP.

As labour is supplied inelastically, current well-being can be expressed as $U(C_t)$. Consider then the iso-elastic form:

$$U(C_t) = B - C_t^{-(\eta-1)}, \qquad \text{where } \eta > 1 \text{ and } B > 0. \quad (24)$$

δ is the social rate of discount if current well-being is numeraire. Let ρ_t be the social rate of discount if consumption is the unit of account. It follows that

$$\rho_t = \delta + \eta[dC_t/dt]/C_t = \delta + \eta(s\mu - \gamma). \quad (25)$$

The sign of ρ_t depends upon the resource allocation mechanism α. ρ_t can be negative. To see why, suppose the unit of time is a year, and $\gamma = 0.05$, $s = 0.10$, $\eta = 2$, and $\mu = 0.25$. Then equation (33) says that $\rho_t = -0.04$ per year.

Intertemporal welfare at t is,

$$V_t = {}_t\int^\infty U(C_\tau)e^{-(\tau-t)}d\tau. \quad (26)$$

Using equations (23) and (24) in equation (26), we have:

$$V_t = -[(1-s)\mu K_t]^{-(\eta-1)}{}_t\int^\infty \exp[(\eta-1)(s\mu-\gamma)+\delta]d\tau,$$

or, $$V_t = -[(1-s)\mu K_t]^{-(\eta-1)}/[(\eta-1)(s\mu-\gamma)+\delta]. \quad (27)$$

V is differentiable in K everywhere. Moreover, $\partial V_t/\partial t = 0$. Equations (22) and (27) confirm Proposition 3.[27]

We turn now to accounting prices. Notationally, it is simplest to regard well-being as numeraire. Let

[27] As the economy has a single asset, Proposition 3 is trivially true.

the accounting price of the capital asset be p_t. From equations (10) and (27), we have,

$$p_t = (\eta-1)[(1-s)\mu]^{-(\eta-1)}K_t^{-\eta}/[(\eta-1)(s\mu-\gamma)+\delta]. \quad (28)$$

Using equations (23), (24) and (26), we find that $p_t \neq U'(C_t)$, except when $s = (\mu+(\eta-1)\gamma-\delta)/\mu\eta$. It is simple to confirm directly from equation (27) that the saving ratio is optimum when,

$$s = s^* \equiv (\mu+(\eta-1)\gamma-\delta)/\mu\eta. \quad (29)$$

Note that $p_t < U'(C_t)$ if $s > s^*$, which means there is excessive saving. Conversely, $p_t > U'(C_t)$ if $s < s^*$, which means there is excessive consumption.

The example can also be used to show that Frank Ramsey's precept of setting δ equal to zero can imply unreasonably high savings. To see this, suppose $\gamma = 0$ (capital doesn't depreciate), $\delta = 0$, and $\eta = 1.5$. In this case, the $s^* = 2/3$ (the optimum rate of saving is about 66 percent of GNP), a high figure.

Although intergenerational equity is nearly always discussed in terms of the rate at which future well-being is discounted (see, e.g. Portney and Bryant, 1998), equity is best discussed in terms of the curvature of U. Let us see how.

Let the unit of time be a year. Suppose $\gamma = 0$, $\delta = 0.02$, and $\mu = 0.32$. Consider two alternative values of η: 25 and 50. It is simple to confirm that $s^* = 0.038$ if $\eta = 25$ and $s^* = 0.019$ if $\eta = 50$. Intergenerational equity in both the determinants and constituents of well-being (the latter is a concave function of the former) can be increased indefinitely by making η larger and larger: C_t becomes "flatter" as η is increased. In the limit, as η goes to infinity, s^* tends to zero (equation (29)), which reflects the Rawlsian maxi-min consumption as applied to the intergenerational context.[28]

14 Cross-Country Comparisons

Cross-country comparisons of GNP per head are today a commonplace exercise. The previous sections have confirmed that the practice is mistaken. So, then, how should cross-country comparisons be made?

Imagine that we are to compare current well-being. There is a strong case for comparing the "representative" person's current quality of life in each place. We may ask: which economy would you choose to inhabit if in each you faced an equal probability of occupying every citizen's position?[29] It is simplest to

[28] Solow (1974) and Hartwick (1977) are the key articles on this limiting case.

[29] The key paper on this is Harsanyi (1955). Since the person conducting the thought-experiment could choose to be very risk averse, to focus on the representative person isn't to be insensitive to distributional concerns.

think of a continuum of closed economies, parametrized by x (a scalar).[30] Economies differ in their endowments, population size, and resource allocation mechanisms. However, because we are comparing current well-being, none of these differences is relevant for the exercise.

The representative person's current well-being in x is U_x. It is an explicit function of x if "cultures" differ and culture is a direct determinant of well-being. Replacing time by space in equation (8), we have,

$$dU_x/dx = U_C dC_x/dx + U_L dL_x/dx + \partial U_x/\partial x. \qquad (30)$$

Earlier we argued that the worth of the constituents of well-being (e.g., health) is independent of culture. Cultural coordinates are a component of what we are calling resource allocation mechanisms, but they aren't well-being's determinants in any other respect. So we set $\partial U_x/\partial x = 0$ in equation (30), which then implies

Proposition 6: Current well being in x is higher than in any of its immediate neighbours if and only if the accounting value of the commodity determinants of current well-being in x is greater.

Making cross-country comparisons of intergenerational well-being is far harder. It involves deep conceptual problems, arising out of the fact that countries differ in population size.[31] We side-step the problems and assume that the size of the population remains constant in each country and is the same everywhere. Countries differ only in their capital endowments and resource allocation mechanisms. x is now to be interpreted as a composite index of the allocation mechanism.

Let V_x be the value function in x. We may then replace time by space in equation (11) to obtain,

$$dV_x/dx = p_x dK_x/dx + q_x dS_x/dx + \partial V_x/\partial x. \qquad (31)$$

If the resource allocation mechanisms are the same as well, $\partial V_x/\partial x = 0$, and equation (31) implies,

Proposition 7: Social well-being in a country is higher than in any of its immediate neighbours if and only if it is wealthier than its neighbours.[32]

Proposition 7 parallels Proposition 3. A corresponding "discrete" result holds for non-neighbouring countries. It involves integrating the RHS of equation (31). It can be shown that the desired integrals are path independent.

15 Global Public Goods

[30] We assume a continuum of economies in order to make use of the calculus. It simplifies the computations. The analysis that follows can be easily adapted for the case where there is a discrete number of economies.

[31] See Section 17.

[32] The structure of accounting prices differs across countries because endowments differ.

Countries interact with one another not only through trade in international markets, but also via transnational externalities. Of particular interest are externalities arising from the use of global public goods in a world where nations don't cooperate.[33] We ignore trade and concentrate on global commons. At date t, let G_t be the stock of a global common (the atmosphere). We imagine that G is measured in terms of a "quality" index (carbon dioxide concentration). Being a global public good, G is determined by the resource allocation mechanism in each nation. Forecasts can be made of the economic programme in each country on the basis of knowledge of the resource allocation mechanisms in all countries. This is in effect what is attempted in economic models of global warming.

Imagine now that we are to make cross-country comparisons of current well-being. We focus on a date and drop the time subscript. Although G is available equally to all, its worth differs from country to country. So U is an explicit function of x. An argument identical to the one leading to equation (30) now yields,

$$dU_x/dx = U_C dC_x + U_L dL_x + \partial U_x/\partial x.$$

Proposition 6 continues to hold, but now it is corrected for differences among countries in the worth of the global public good.[34]

16 Technological Change, Institutions, and Growth Accounting

How should technical change be incorporated in the index of wealth?

Imagine that research and development (R&D) expenditure can be directed at natural resource augmentation and at shifts in the aggregate production function. Let E_{1t} and E_{2t}, respectively, denote expenditures on the two types of technological progress. Define Z_{1t} and Z_{2t} by the equations

$$dZ_{1t}/dt = E_{1t}, \quad (32)$$

and

$$dZ_{2t}/dt = E_{2t}. \quad (33)$$

Z_1 and Z_2 can be thought of as stocks of knowledge.

Let the resource augmentation function be $J(E_1, Z_1, S)$. It is reasonable to assume that the partial derivatives of J with respect to E_1 and Z_1 are non-negative. Imagine next that output of the produced consumption good is

$$Y_t = e^{\lambda t}Q(Z_{2t})F(K_t, L_t, R_t), \quad (34)$$

[33] We are grateful to Gordon Munro for suggesting that we discuss this case.

[34] For further discussion on this, see Dasgupta, Kriström, and Mäler (1995).

where $\lambda \geq 0$ and $Q'(Z_2) \geq 0$. Technical progress in the production of the final good appears here as the term $e^{\lambda t}Q(Z_{2t})$. It combines exogenous factors (λ) with endogenous ones (Z_2). We now re-write equations (1) and (2) as

$$dK_t/dt = F(K_t, L_t, R_t) - C_t - E_1t - E_{2t} \tag{35}$$

and
$$dS_t/dt = M(S_t) - R_t + J(E_1, Z_1, S). \tag{36}$$

The economy's capital assets are K, S, Z_1, and Z_2. Wealth is the sum of their accounting values. Stock adjustments are given by equations (32), (33), (35) and (36). $\partial V_t/\partial t > 0$ if $\lambda > 0$; but $\partial V_t/\partial t = 0$ if $\lambda = 0$. How plausible is to assume that λ is not zero? To address this question it is useful to identify the factors that contribute to changes in GNP over time. GNP in our model economy is given by (34). Differentiating both sides of the equation with respect to time, re-arranging terms, and dropping the time subscript from variables for the sake of notational simplicity, the growth accounting identity is

$$(dY/dt)/Y \equiv \lambda + (Q'(Z_2)dZ_2/dt)/Q(Z_2) + (F_K dK/dt)/F + (F_L dL/dt)/F + (F_R dR/dt)]/F. \tag{37}$$

The first term on the RHS of equation (37) measures the residual, while the remaining terms together represent the contributions of changes in the factors of production to the percentage rate of change in GNP. By definition, λ is an exogenous factor, it is unexplained within the model.

Solow (1957) used a reduced-form of the production function in (34) to estimate the contribution of changes in the factors of production to growth of non-farm GNP per "man-hour" in the US economy over the period 1909-1949, and discovered that it was a mere 12 percent of the average annual rate of growth.[35] In other words, 88 percent of the growth was attributable to the residual. (Solow's estimate of λ was 1.5 percent per year.) A significant empirical literature since then has shown that when K is better measured (e.g., by accounting for changes in the utilization of capacity and changes in what is embodied in capital) and when account is taken of human-capital formation, the residual has been small in the US economy.[36]

This is congenial to intuition. It isn't prudent to imagine an everlasting residual, which would be rather like receiving an indefinite flow of manna from heaven. It is hard to believe that serendipity, unbacked by R&D effort and investment, can be a continual source of productivity growth. The assumption that total factor productivity can grow indefinitely would no doubt permit us to imagine that growth in aggregate consumption is sustainable. That indeed would be its attraction: it would enable us to assume away problems

[35] Solow assumed in particular that $Q'(Z_2) = 0$.

[36] Jorgenson (1995) contains a masterly account of this complex literature.

of environmental and resource constraints. But there are no theoretical or empirical grounds for presuming that it is a reasonable assumption. Recall too that most environmental resources go unrecorded in growth accounting. If the use of natural capital has in fact been growing, estimates of the residual would have an upward bias. At this point in our understanding of the process by which discoveries are made, it makes greater sense to set $\lambda = 0$ in (37). This would imply that $\partial V_t/\partial t = 0$.

The residual can have short bursts in imperfect economies. Imagine that a government reduces economic inefficiencies by improving the enforcement of property rights, or by reducing centralized regulations (import quotas, price controls, and so forth). We would expect the factors of production to find better uses. As factors realign in a more productive fashion, total factor productivity would increase.

In the opposite vein, the residual could become negative for a period. Increased government corruption could be a cause, or civil strife, which destroys capital assets and damages a country's institutions. When institutions deteriorate, assets are used even more inefficiently than previously. The residual declines. This would appear to have happened in sub-Saharan Africa over the past forty years (Collins and Bosworth, 1996).

Collins and Bosworth (1996) have estimated that the residual in the Indian subcontinent during the past thirty years has been about 0.8 per year. But as this is undoubtedly an overestimate (natural capital was not included), we are unclear whether there has been any growth in total factor productivity in that part of the world. One can but conclude that two of the poorest regions of the world (South Asia and sub-Saharan Africa) haven't improved their institutional capabilities over four decades, nor have they been able to improve productivity by making free use of knowledge acquired in advanced industrial nations.

This isn't to suggest that there is no such thing as technical change, but rather that, of the first two terms on the RHS of equation (37), it is the latter which is significant. It denotes the contribution of of research and development to the growth in public knowledge, and thereby to growth in output. If the residual is nil, $\partial V_t/\partial t = 0$.

17 Population Change and Sustainable Development

How should demographic change be incorporated in the index of sustainable development? It has been customary in growth accounting to regard changes in population to be exogenously given. However, future population should be expected to depend on the resource allocation mechanism α. Here we are taking α to be given. So it is reasonable to assume that the population profile is exogenously given.

We seek to determine how population change influences the final term on the RHS of equation (11). To illustrate how to do this, we take it, to begin with, that intertemporal welfare is the average well-being of all who ever come into being (Harsanyi, 1955).

Let N_t be population size at t and n_t the rate of change of N_t. Thus, $N_t = N_0\exp({}_0\int^t n_\tau d\tau)$.[37] For notational simplicity, we ignore intragenerational inequality and changes in the age composition of the population. Let y_t denote per capita consumption at t. Thus $y_t = C_t/N_t$. Assume as well that labour is supplied inelastically in each period. Current well-being of the representative person can then be expressed as $U(y_t)$. Intertemporal welfare is

$$V_t = {}_t\int^\infty N_\tau U(y_\tau)e^{-\delta(\tau-t)}d\tau/[{}_t\int^\infty N_\tau e^{-\delta(\tau-t)}d\tau]. \quad (38)$$

If V_t is to be well-defined, it must be that $\delta > [{}_0\int^t n_\tau d\tau]/t$ for large enough t. Let k_t and s_t denote the capital stocks per head. Thus, $k_t = K_t/N_t$ and $s_t = S_t/N_t$. This means that V_t is a function of k_t, s_t and N_t. Define $\nu_t = \partial V_t/\partial N_t$, which is the contribution of an additional person at t to social well-being. Let p_t and q_t denote the accounting prices of k_t and s_t, respectively, in well-being numeraire. Equation (11) then reads as,

$$dV_t/dt = p_t dk_t/dt + q_t ds_t/dt + \nu_t dN_t/dt. \quad (39)$$

The RHS of equation (39) is the index of sustainable development. Proposition 3 remains valid so long as wealth comparisons mean comparisons of wealth per capita, adjusted for demographic changes. The adjustment is given by the last term in equation (39).

There are resource allocation mechanisms where V_t is not an explicit function of N_t. It can be shown, for example, that if population grows at a constant exponential rate, the production functions F and M in equations (1) and (2) are subject to constant returns to scale, and the saving rate is constant, then $\nu_t = 0$. In such cases the last term on the RHS of equation (39) is zero and we have an exact result: social well-being increases if and only if wealth per head accumulates.

It is interesting to consider how the analysis would change if instead of expression (38), intertemporal welfare is regarded to be the total well-being of all who ever comes into being. This is to subscribe to the Classical Utilitarian form of intertemporal social welfare. Thus, assume that,

$$V_t = {}_t\int^\infty N_\tau U(y_\tau)e^{-\delta(\tau-t)}d\tau. \quad (40)$$

In other respects the economy is assumed to be the same as the one we have just considered. From expression (40), we have $\nu_t = \partial V_t/\partial N_t = V_t/N_t$. Equation (39) therefore reduces to,

$$dV_t/dt = p_t dk_t/dt + q_t ds_t/dt + n_t V_t. \quad (41)$$

Equation (41) says that the adjustment term to genuine investment per head (the adjustment term being the final term on the RHS of equation (41)) is proportional to social welfare itself. This means that the index of

[37] It would be reasonable to suppose that N_t is a logistic function.

sustainable development is sensitive to whether social welfare is positive or negative.

Define welfare subsistence to be the lowest level of current well-being such that it isn't a bad thing that someone's current well-being is at that level. Despite the obvious sensitivity such a concept involves, it is essential in normative discussions of population policies.[38] But equation (41) says that welfare subsistence must enter discussions of sustainable development as well if social welfare is taken to be the total well-being of all who ever come into being. This is why (38) is the more natural expression for social welfare. If future population is exogenously given, expression (40) doesn't capture what we should be after.

It should be noted that in an optimizing economy, it is a matter of indifference whether expression (38) or (40) is used to reflect social welfare: the optimum policy is invariant to this choice. In contrast, the choice matters for welfare comparisons in imperfect economies.

18 Further Extensions

A number of important features of actual economies were missing in the economic models developed earlier so far. We comment on a few of them and show how they can be included in the theory.

1. Intragenerational distribution. The distribution of well-being within a generation has been ignored so far. Theoretically it is not difficult to include this. If there are N people in each generation and person i consumes C_i, while supplying L_i units of labour, her well-being would be $U(C_i,L_i)$.[39] A simple way to express intragenerational welfare would be to "concavify" U. Let G be a strictly concave, increasing function of real numbers. We may then express intragenerational welfare as $\Sigma_i[G(U(C_i,L_i))]$.[40] Some people would be well-off, others badly-off. The formulation ensures that at the margin, the well-being of someone who is badly off is awarded greater weight than that of someone well-off.

The social worth of the determinants of well-being (C and L) depends on who gets what. To accommodate this idea, we have to enlarge the set of commodities so as to distinguish, at the margin, a good consumed or supplied by one person from that same good consumed or supplied by another. This is the idea

[38] See Meade (1955) and Dasgupta (1969; 2001: chs. 13-15). Strictly, welfare subsistence ought to be defined in regard to a person's lifetime well-being. We are referring to it in the context of current well-being for expositional simplicity.

[39] Person-specific factors (e.g., age, health status, gender) can be included in the well-being function. This is routinely done in applied economics.

[40] A more general formulation would have us define a symmetric, strictly concave function from M-vectors into the reals: $G(U(C_1,L_1), ..., U(C_M,L_M))$.

of "named goods".[41] It means, for example, that a piece of clothing worn by a poor person should be regarded as a different commodity from that same type of clothing worn by someone who is rich. Accounting prices of named goods would depend on the names attached to them. With this re-interpretation of goods and services, the results we have obtained continue to hold.

2. Environmental externalities. Global public goods were introduced in Section 15. More general environmental externalities can be incorporated by a device identical to the one devised to incorporate distributional considerations. To describe who is affected, in which manner, and by whose actions involves the use of named goods and services. Accounting prices should be "named", so as to distinguish private costs from social costs and private benefits from social benefits. They are a generalization of Lindahl prices in first-best economies.[42] Pigovian taxes and subsidies on externalities can be computed on the basis of named accounting prices.[43] The accounting prices of the capital assets are functions of such taxes and subsidies.

3. Stock effects. Some natural resources have value as a stock, qua stock, either because the stock provides a flow of services (ecosystems) or because it has intrinsic value (the great apes). The way to accomodate this would be to enlarge the domain on which current well-being is defined, by including S. Assume for simplicity of exposition that population is constant. Write $U(C, L, S)$, where $\partial U/\partial S > 0$. The resource's accounting price would reflect this "stock effect".[44]

Stock pollutants can be introduced in a similar manner. Suppose pollution (carbon emission) is a byproduct of production. Imagine that it is a constant proportion (g) of Y. Let P be the stock of pollutants. Assume that it depreciates at a rate π. Then the dynamics of the pollutant would be given by

$$dP_t/dt = gY_t - \pi P_t. \tag{42}$$

[41] Hahn (1971).

[42] Lindahl (1958) and Arrow (1971).

[43] Sandmo (2000) has a good discussion of this.

[44] Kurz (1968) and Uzawa (1974a,b) were the first to analyse the intertemporal consequences of stock effects in social welfare. Purists would say that including a resource' intrinsic value in U is a misuse of terminology since U is being interpreted as human well-being. It is a misuse of terminology, but no harm is done to our understanding of matters so long as we know it is a misuse.

We may write current well-being as U(C, L, P), where $\partial U/\partial P < 0$ and retrace the formal arguments in earlier sections. Accounting prices would include the stock effect of pollutants. Wealth would include the value of P.

4. Defensive expenditure. Let us generalize the ideas developed in the previous point. Denote by Q_t the stock of defensive capital and X_t investment in its accumulation. We can write equation (42) as,

$$dP_t/dt = G(Y_t, Q_t) - \pi P_t, \text{ where } G(Y_t, Q_t) \geq 0, \partial G/\partial Y > 0 \text{ and } \partial G/\partial Q < 0. \quad (43)$$

Moreover,

$$dQ_t = X_t - \xi Q_t, \quad \text{where } \xi > 0. \quad (44)$$

Equation (1) has now to be re-written as,

$$dK_t/dt = F(K_t, L_t, R_t) - C_t - X_t. \quad (45)$$

Technological possibilities for the economy are describable by equations (2) and (43)-(45). Let the economy otherwise be as in the base case. Denote by m_t the accounting price of defensive capital and h_t (< 0) the accounting price of the pollutant. Wealth can then be expressed as,

$$p_tK_t + q_tZ_t + r_tS_t + m_tQ_t + h_tP_t,$$

and genuine investment at t as,

$$I_t = p_tdK_t/dt + q_tdZ_t/dt + r_tdS_t/dt + m_tdQ_t/dt + h_tdP_t/dt. \quad (46)$$

Equation (46) says that defensive expenditure against pollution ought to be included in the estimation of genuine investment (m_tdQ_t/dt), but, then, so should changes in the quality of the environment be included (h_tdP_t/dt). To include the former, but not the latter, is a mistake.

5. Uncertainty. The economy has so far been assumed to be deterministic. Let intertemporal welfare at t=0 be the expected value of the present discounted flow of utility. Now define contingent goods, which are goods produced or consumed contingent on identifiable events (Debreu, 1959). From this one can define contingent accounting prices. It is not presumed that there is a complete set of markets for contingent goods. Our account of the welfare economics of imperfect economies, nevertheless, goes through.

6. Human capital. We have abstracted from human capital. The way to incoporate human capital is to define labour in units of effective labour. Someone, whose human capital is H is capable of offering HL units of effective labour if they work L hours. The creation of human capital requires investment (time and resources). If we were to include the formation of human capital in our model, we would specify an accumulation equation for its aggregate stock. The common practice of regarding expenditure in education as consumption is based on an error: the expenditure is in great part an investment activity. However, unlike physical capital, human capital is frequently non-transferable. They are named goods. Moreover, it

"depreciates" when people die.

7. International trade: But for Section 15, where a global public good was introduced, our analysis has been for closed economies. International trade is formally an additional form of production and exchange. International prices are a given if the country in question is small. If they are not constant, the resource allocation mechanism, α, is not time autonomous. Price changes should be included in the measure of the country's wealth.[45] Otherwise the analysis remains the same.

[45] Sefton and Weale (1996).

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