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Regime Shifts and Uncertainty in Pollution Control

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Abstract

We develop a simple model of managing a system subject to pollution damage under risk of an abrupt and random jump in the damage coefficient. The model allows the full dynamic characterization of the optimal emission policies under uncertainty. The results, that imply prudent behavior due to uncertainty, are compared with the ambiguous outcomes reported in the literature for similar models. The differences are explained in terms of the properties of the damage function associated with each model. The framework is used to analyze the adaptation vs. mitigation dilemma and provides a simple criterion to determine whether adaptation activities should be undertaken promptly, delayed to some future date, or avoided altogether.

Keywords: environmental pollution, optimal management, catastrophic transitions, uncertainty, adaptation, mitigation.

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1 Introduction

It has become clear over the last decades that many systems subject to pollution problems may suffer from abrupt and unexpected changes in their characteristics. At some threshold or tipping point, the underlying system dynamics may shift, directing the system into another domain of attraction with a substantial change in the services it provides. An example of such a regime shift is abrupt climate change where accelerated feedback mechanisms (such as ice melting) may trigger a transition to another climate regime as soon as a threshold greenhouse-gas concentration is crossed. This transition, once occurred, is believed to inflict substantial costs upon mankind. Other examples are the eutrophication of lakes which support the livelihood of the population along their shores and the destruction of coral-reef systems.

Many dynamic pollution control models assume smooth convex damage functions (e.g. van der Ploeg and de Zeeuw, 1992; Dockner and Long, 1993) and ignore the effect of a possible regime shift on the optimal emission policy. Accounting for possible shifts, different types of uncertainty may play a role. For example, the shift can be triggered when some a-priori unknown threshold is crossed (as in Tsur and Zemel, 1996; Nævdal, 2006). Alternatively, it may be due to a random occurrence controlled by some given hazard rate (Clarke and Reed, 1994; Tsur and Zemel, 1998; Gjerde et al., 1999; Haurie and Moresino, 2006). The models differ also in the specification of the post-occurrence outcome. Some papers focus on the system dynamics itself and model a regime shift as a sudden change in a parameter governing it; see Brozović and Schlenker (2011) for ecosystem control and Polasky et al. (2011) for a fishery. Alternatively, one can specify the damage incurred by the shift.

Obviously, this variety of modeling approaches gives rise to a wide range of policy responses to uncertainty, ranging from enhanced prudence, (i.e. less pollution) through ambiguous behavior (with the nature of the response depending on the system parameters) all the way to solutions implying enhanced pollution due to uncertainty. In this paper we present a model which is simple enough to allow a full dynamic characterization of the optimal processes and yet is able to obtain the full range of dynamic responses as special cases. Thus, we can trace the tradeoffs underlying the optimal response and explain the large differences among the outcomes of the models cited above in a clear and transparent manner. In order to achieve this goal, we construct the simplest model that displays these tradeoffs. We assume that the damage follows a quadratic law as a function of the pollution stock, with a coefficient that jumps to a larger value once the regime shift occurs. This is a gross simplification for systems that undergo a change in the underlying dynamics. For complex problems such as climate change, however, this approach can provide some useful insights, because the system dynamics is too complicated to be modeled precisely, but we may still have an idea about the damage induced by the catastrophic shift.

We begin with the benchmark case of a regime shift whose time of occurrence can be predicted in advance for every emission policy, and derive the condition under which it is optimal to trigger the shift in spite of the prior knowledge. We find that regardless of whether the shift is triggered or not, it is always optimal to lower emissions from the outset since the value decreases at the time of shift. In optimal control terminology, this is caused by a change in the transversality condition at this time. Then, we incorporate uncertainty by modeling the regime shift as a random event whose occurrence probability is specified via a given hazard rate. This formulation allows to convert the problem into a deterministic optimal control problem that can be treated using standard techniques (Kamien and Schwartz, 1971). The case of a hazard rate that depends on the pollution stock reflects endogenous uncertainty where our actions affect the shift probability. It seems reasonable that higher phosphorus loads promote lake eutrophication while lower fish stocks increase the probability of coral-reef collapse. One would expect that increasing hazard implies precaution (lower emissions) from the outset. This is indeed what we find here, but it is not the general conclusion in the literature. For example, several studies (Clarke and Reed, 1994; Tsur and Zemel, 1998; Aronsson et al., 1998; Gjerde et al., 1999; Polasky et al., 2011) consider "doomsday" regime shifts associated with a total loss of value. In this case the hazard rate is effectively added to the discount rate, increasing impatience and implying enhanced emissions (for a constant hazard rate) and ambiguous behavior (for increasing hazard rates). Incorporating the "doomsday" events into our framework, we reproduce these results and explain the difference in terms of the properties of the damage function associated with each specification.

While the result of uncertainty-induced precaution is intuitively appealing, the magnitude of the effect might appear surprising at first sight. For example, we show that if the hazard rate depends strongly on the pollution stock, precaution implies emissions at a rate that is even lower than in the case the system is already at the high-damage regime! Emission reduction, in this case, is aimed not only at reducing the direct pollution costs but also at decreasing the probability that the shift will occur at some future time.

Considerations regarding the appropriate degree of precaution become particularly relevant in the context of the climate debate, where different positions are taken on the optimal emission policy. Thus, the Stern Review (Stern, 2006) advocates intensive and early mitigation whereas Nordhaus (2008) advocates a more gradual policy. The different recommendations are mainly driven by different assumptions on the relevant discount rate, but as Weitzman (2009) has argued, "fat-tailed" probability distributions, which assign large probabilities to catastrophic impacts, dominate all other elements of the cost-benefit analysis. This leads to the so-called "dismal theorem", a sort of generalized precautionary principle. This principle has been part of the Rio Declaration in 1992, stating that "where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation". Gollier and Treich (2003) provide an economic justification to the precautionary principle by developing the conditions under which the trade-offs between learning first and acting first lead to precaution. However, it is hard to apply these conditions to random regime shifts. Our simple model, although abstracting from numerous important details, provides simple intuitive arguments to support precaution.

The model can also be used to shed light on the adaptation vs. mitigation dilemma. It is now recognized that mitigating global pollution problems (such as climate change) requires international cooperation which is hard to achieve and sustain. Suspecting that mitigation policies may not suffice to prevent the shift, an economy can invest resources in means and methods to moderate the damage associated with the shift (e.g. Kane and Shogren, 2000; Smit et al, 2000; Shalizi and Lecocq, 2010). For example, a flat coastal country such as the Netherlands might judge that global efforts to reduce CO_2 emissions are not intense enough to avoid a sea-level rise and the catastrophic floods it entails. A possible response can take the form of investments in the protective dike system as well as in improved pumping capacities and inland transfer of essential infrastructure in order to decrease the damage from future floods, should they occur in spite of the efforts to mitigate the risk. When the regime shift time can be predicted in advance, it is relatively simple to evaluate the benefits of adaptation measures and compare them with their cost. However, when this time is subject to uncertainty, interesting tradeoffs arise. First, we have the usual considerations of investment under uncertainty, namely the tradeoff between the advantage of early investment and the value of the option to wait. Second, investment resources must be allocated between adaptation and mitigation. Indeed, these two kinds of tradeoffs interact, and the adaptation option affects the optimal mitigation policy (Kane and Shogren, 2000).

The regime shift model presented above allows to study these considerations in a simple and tractable manner. Consistent with our modeling approach, we consider the simplest possible form of adaptation that displays these tradeoffs within a dynamic model. In our pollution control framework, adaptation is seen as an investment in lowering the coefficient of the high-damage regime. This investment can be usefully carried out only prior to the shift. We find a rich variety of dynamic behavior, starting from the case where prompt adaptation is optimal, via cases in which adaptation is worthwhile but should be delayed, to the extreme case where adaptation is too costly to be implemented. We establish the complex interaction between the two policy measures: the adaptation option increases emissions (and reduces mitigation) even before this option is actually realized. At the same time, the mitigation policy enters the condition for the cost-effectiveness of adaptation. Thus, the optimal response must consider these two measures simultaneously.

The following section presents the pollution control problem under a regime shift threat. Section 3 characterizes the full dynamics of the post-event solution which enters the formulation of the uncertainty problem. Section 4 considers the "certainty" case, where the threshold is a-priori known. In section 5 we solve the problem under uncertainty and section 6 explains the difference between the outcomes of the regime shift and the "doomsday" events. Section 7 introduces the adaptation option and investigates the interaction between mitigation and adaptation. Section 8 concludes the paper.

2 The regime-shift problem

Economic activities involving emissions at the rate E give rise to the instantaneous concave benefit $\beta E - E^2/2$ (where $\beta > 0$ is the maximal marginal benefit) and at the same time increase the pollution stock P according to

$$\dot{P} = E - \alpha P \tag{2.1}$$

where $\alpha > 0$ denotes the natural decay rate. The pollution stock implies a stream of damages at the rate $\gamma P^2/2$ where γ is the damage coefficient which can take one of two values $\gamma_2 > \gamma_1 > 0$. During the initial, "clean" period, the regime of low damage, with $\gamma = \gamma_1$, holds. However, at some point the system might shift abruptly and irreversibly to the "dirty" regime, governed by the high damage flow with $\gamma = \gamma_2$. This shift may occur at some known or estimated level of the pollution stock but it may also be subject to uncertainty. In that case we model the transition time T to be random, controlled by the hazard rate $h(\cdot)$ so that the survival probability at time t is $S(t) = \exp\left(-\int_0^t h(\tau)d\tau\right)$. We refer to the shift in regimes as the (random) "event".

Given the occurrence time T, the damage stream is governed by γ_1 for $t \leq T$ and by γ_2 for $t > T$. The corresponding welfare is given by

$$\int_0^T [\beta E - E^2/2 - \gamma_1 P^2/2] \exp(-\rho t)dt + \exp(-\rho T)v_2(P_T)$$

where $v_2(\cdot)$ is the post-event value function corresponding to the "dirty" regime, P_T is the pollution stock at the regime shift time, and $\rho > 0$ is the discount rate. Taking the expectation with respect to the occurrence time T, we obtain the optimal pre-event emission policy in case of uncertainty as the outcome of

$$v_{uc} = \max_{\{E\geq 0\}} \left\{ \int_0^\infty [\beta E - E^2/2 - \gamma_1 P^2/2 + h(t) v_2(P(t))] S(t) \exp(-\rho t) dt \right\} \tag{2.2}$$

subject to (2.1) and $P(0) = 0$. To specify v_{uc} completely, we must derive the post-event value $v_2(\cdot)$ which is obtained as the outcome of

$$v_2(P_0) = \max_{\{E\geq 0\}} \left\{ \int_0^\infty [\beta E - E^2/2 - \gamma_2 P^2/2] \exp(-\rho t) dt \right\} \tag{2.3}$$

subject to (2.1) and $P(0) = P_0$.[1] We turn now to characterize the solution of this problem.

3 The post-event problem

3.1 Steady state

The optimal emission rate corresponding to (2.3) is bounded, hence the optimal pollution process is bounded as well. Since the problem is defined in terms of one state variable, the evolution of the state process is monotonic in time and the process must converge to a steady state. By considering small deviations in time and level of the steady-state policy, Tsur and Zemel (2001) show that the optimal steady state can be characterized as a root of what they call "the evolution function" (this can be compared to the first-order condition in the dynamic programming approach). This method leads to the following.

The steady-state policy corresponding to any state P is $E(P) = \alpha P$ and the corresponding steady-state value is

$$W(P) = [\alpha\beta P - (\alpha^2 + \gamma_2) P^2/2]/\rho. \tag{3.1}$$

The evolution function becomes

$$L(P) = \rho W'(P) + \rho(\beta - \alpha P) = (\rho + \alpha)\beta - (\alpha^2 + \gamma_2 + \alpha\rho) P. \tag{3.2}$$

This function is linear in P and hence has a unique positive root

$$\hat{P} = \frac{(\rho + \alpha)\beta}{\gamma_2 + \alpha(\rho + \alpha)} \tag{3.3}$$

which must be the unique steady state for this problem.[2] The steady state emission rate

$$\hat{E} = \frac{\alpha(\rho + \alpha)}{\gamma_2 + \alpha(\rho + \alpha)} \beta \tag{3.4}$$

[1] For notational convenience we have reset the time origin in the formulation of (2.3) so that the event occurrence time corresponds to $t = 0$.

[2] The corner state $P = 0$ can be ruled out as an optimal steady state because $L(0) = (\rho + \alpha)\beta > 0$.

is smaller than the rate $E = \beta$ that maximizes the instantaneous benefit $\beta E - E^2/2$ due to the presence of the damage term γ_2 in the denominator of (3.4).

3.2 Dynamic behavior

We characterize now the dynamic process that leads to the steady state $\hat{P}$ of (3.3). The current-value Hamiltonian corresponding to (2.3) is

$$\mathcal{H} = \beta E - E^2/2 - \gamma_2 P^2/2 + \lambda(E - \alpha P). \tag{3.5}$$

Maximizing $\mathcal{H}$ with respect to E gives

$$E = \beta + \lambda, \tag{3.6}$$

while

$$\dot{\lambda} = (\rho + \alpha)\lambda + \gamma_2 P. \tag{3.7}$$

Taking the time derivative of (2.1) and using (3.6) and (3.7) to eliminate $\dot{\lambda}$ and λ we find

$$\ddot{P} - \rho\dot{P} - [\gamma_2 + \alpha(\rho + \alpha)]P + (\rho + \alpha)\beta = 0. \tag{3.8}$$

In a steady state, the time derivatives of P vanish, yielding a solution to the inhomogeneous equation

$$\hat{P} = \frac{(\rho + \alpha)\beta}{\gamma_2 + \alpha(\rho + \alpha)}$$

in agreement with (3.3).

To obtain the general solution of (3.8) we add the solution of the homogeneous equation to the inhomogeneous solution $\hat{P}$

$$P(t) = \hat{P} + p^+ \exp(r^+ t) + p^- \exp(r^- t), \tag{3.9}$$

where $r^{+,-}$ are the roots of the characteristic equation

$$r^2 - \rho r - [\gamma_2 + \alpha(\rho + \alpha)] = 0. \tag{3.10}$$

Thus, we can rewrite (3.3) in terms of any of these roots as

$$\hat{P} = \frac{(\rho + \alpha)\beta}{r(r - \rho)}. \tag{3.11}$$

The integration constant p^- can be determined using the initial condition $P(0) = P_0$ or $P_0 = \hat{P} + p^+ + p^-$. Thus,

$$P(t) = \hat{P} + (P_0 - \hat{P})\exp(r^- t) + p^+[\exp(r^+ t) - \exp(r^- t)], \tag{3.12}$$

and

$$\begin{aligned} E(t) &= \alpha P(t) + \dot{P}(t) = \\ &= \alpha P(t) + (P_0 - \hat{P}) r^- \exp(r^- t) + p^+ [r^+ \exp(r^+ t) - r^- \exp(r^- t)]. \end{aligned} \tag{3.13}$$

The determination of the second integration constant p^+ depends on the appropriate final time or transversality condition. In the following section we will consider the case of a threshold that is reached in some finite time but first our interest is focused on the post-event problem which extends over an infinite time horizon. In this case, the exponential term corresponding to the positive root r^+ must be discarded, because it yields an unbounded emission rate in the long term. Thus, $p^+ = 0$ and we can omit for brevity the superscript "$-$" and write

$$P(t) = \hat{P} + (P_0 - \hat{P}) \exp(rt), \tag{3.14}$$

where

$$r = [\rho - \sqrt{(\rho + 2\alpha)^2 + 4\gamma_2}\ ]/2. \tag{3.15}$$

We can now write the shadow price $\lambda(t)$ as a linear function of the corresponding pollution stock $P(t)$

$$\lambda(t) = E(t) - \beta = \alpha P(t) + \dot{P}(t) - \beta = P(t)(r + \alpha) + \hat{\Lambda}, \tag{3.16}$$

where the constant $\hat{\Lambda}$ is defined, using (3.11), as

$$\hat{\Lambda} \equiv -r\hat{P} - \beta = \beta \frac{r + \alpha}{\rho - r}. \tag{3.17}$$

Since $r + \alpha < 0$, it follows that $\hat{\Lambda} < 0$.

The linear relation (3.16) also holds at the time when the regime shift occurs. Since the derivative of the value function is equal to the shadow price, we have established that the post-event value function $v_2(\cdot)$ is quadratic in the pollution stock with negative coefficients ($(r + \alpha)/2$ and $\hat{\Lambda}$) for the quadratic and linear terms, respectively. The constant term (which measures the value at the clean state $P = 0$) is determined by the condition $v_2(\hat{P}) = W(\hat{P})$ where $W(\cdot)$ is the steady state value given by (3.1). This yields $(r\hat{P})^2/(2\rho)$ for the constant term, so that the post-event value function can be written as[3]

$$v_2(P) = \frac{r + \alpha}{2} P^2 + \frac{\beta(r + \alpha)}{\rho - r} P + \frac{(r\hat{P})^2}{2\rho}. \tag{3.18}$$

[3]Strictly speaking, the result does not hold for very large P_0 because the constraint $E \geq 0$ is violated by our solution in this case. It is verified that the solution (3.14) is feasible so long as $P_0 < \hat{P}|r|/|r + \alpha|$ and the bound exceeds $\hat{P}$. In the following we assume that the relevant pollution process never exceeds the bound so the quadratic solution for $v_2(\cdot)$ can be used in the analysis of the pre-event problem.

Observe that the same characterization is obtained for the risk-free problem, where the regime shift can never occur. The latter problem is identical to the post-event problem, with γ_1 replacing γ_2 as the relevant damage coefficient for the "clean" regime. To distinguish between the solutions of these two problems, we denote variables corresponding to the "non-event" problem (with $\gamma = \gamma_1$) by the subscript "ne".

4 Certainty: a known transition state

Consider now the pre-event problem. We begin with the case in which it is known with certainty that the regime shift occurs as soon as the pollution state exceeds a given threshold level $\bar{P}$ and express the value function as

$$v_c = \max_{\{E\geq 0, T\}} \left\{ \int_0^T [\beta E - E^2/2 - \gamma_1 P^2/2] \exp(-\rho t) dt + \exp(-\rho T) v_2(P_T) \right\} \tag{4.1}$$

subject to (2.1), $P(0) = 0$, $P(t) \leq \bar{P}$ for $t < T$ and $P(T) = \bar{P}$, where the transition time T is a free control variable that can take the value $T = \infty$ if the regime shift never occurs. Obviously, the case $\bar{P} > \hat{P}_{ne}$ is not interesting, because the pollution process will follow $P_{ne}(t)$ all the way approaching $\hat{P}_{ne}$ and the threshold pollution state will never be reached. However, in case $\bar{P} < \hat{P}_{ne}$ we can ask whether a regime shift will be avoided or will occur at some finite time.

Suppose that the pollution state equals $\bar{P}$ at some time and the policy maker must decide whether to increase pollution further, enjoying the post-event value $v_2(\bar{P})$, or to stay at the present level indefinitely, with the corresponding steady-state value $W_{ne}(\bar{P})$. Introducing the damage function

$$\psi(P) = v_2(P) - W_{ne}(P), \tag{4.2}$$

we see that avoiding the event is optimal only if $\psi(\bar{P}) \leq 0$. Using (3.18) for v_2 and (3.1) with $\gamma = \gamma_1$ for W_{ne} we can write ψ explicitly as

$$\psi(P) = [(r^2 + \gamma_1 - \gamma_2)P^2 - 2r^2\hat{P}P + r^2\hat{P}^2]/(2\rho). \tag{4.3}$$

Now, $\psi(0) = (r\hat{P})^2/(2\rho) > 0$. However, at the steady state $\hat{P}$ of the post-event problem $\psi(\hat{P}) = (\gamma_1 - \gamma_2)\hat{P}^2/(2\rho) < 0$. It follows that $\psi(\cdot)$ must vanish at some state $0 < \tilde{P} < \hat{P}$, given by the solution of

$$(\tilde{P} - \hat{P})^2 = \frac{\gamma_2 - \gamma_1}{r^2}\tilde{P}^2.$$

The solution $\tilde{P}$ depends on the parameter $f \equiv \sqrt{\gamma_2 - \gamma_1}/|r|$ and is given by

$$\tilde{P} = \hat{P}/(1 + f). \tag{4.4}$$

(The second solution $\hat{P}/(1-f)$ is either negative or in excess of $\hat{P}$ and can be discarded.) The regime shift depends on the location of the threshold pollution state $\bar{P}$ vis-à-vis the indifference state $\tilde{P}$: no regime shift if $\bar{P} \geq \tilde{P}$ and shifting the regime at some finite time otherwise. This result is in contrast with the conclusions of Tsur and Zemel (2004) in which crossing the threshold state under certainty is never optimal. The difference can be traced to the features of the post-event value under the regime shift considered here, which allows for positive values of the damage function at low pollution levels.

Observe that regardless of whether the regime shift occurs or not, the mere presence of the threshold state $\bar{P} < \hat{P}_{ne}$ affects the optimal emission policy. To see this, note that although the process begins under the low damage coefficient γ_1, it cannot follow the process $P_{ne}(t)$ all the time because the latter proceeds to $\hat{P}_{ne}$, crossing the threshold state, which would trigger the shift. A steady state below $\bar{P}$ cannot be optimal under γ_1, hence the process must reach $\bar{P}$ in a finite time T. We must relax the condition in Section 3.2 that the coefficient p^+ of (3.12) vanishes and replace it with the transversality condition associated with the free value of T

$$\rho V(T) = \mathcal{H}(\bar{P}, E(T), \lambda(T)),$$

where $V(T) = \max\{v_2(\bar{P}), W_{ne}(\bar{P})\}$, i.e. $V(T) = v_2(\bar{P})$ if $\bar{P} < \tilde{P}$ implies a regime shift at time T and $V(T) = W_{ne}(\bar{P})$ if the system is kept at the steady state $\bar{P}$ in order to avoid the shift. Using the dynamic characterization above, the transversality condition specifies to

$$2\rho V(T) = (E(T) - \alpha\bar{P})^2 - \alpha^2\bar{P}^2 + 2\alpha\beta\bar{P} - \gamma_1\bar{P}^2, \tag{4.5}$$

which determines the emission rate $E(T)$ in terms of the given values of $V(T)$ and $P(T) = \bar{P}$. Note that if $V(T)$ were equal to the non-event value $v_{ne}(\bar{P})$, the condition could be satisfied with $p^+ = 0$. However, $V(T)$ is in fact smaller, so the term $(E(T) - \alpha\bar{P})^2$ must be reduced. Writing (3.12) as[4]

$$(P_0 - \hat{P}_{ne})\exp(r_1^- T) = \bar{P} - \hat{P}_{ne} - p^+[\exp(r_1^+ T) - \exp(r_1^- T)],$$

we find, using (3.13)

$$E(T) - \alpha\bar{P} = -r_1^-(\hat{P}_{ne} - \bar{P}) + p^+(r_1^+ - r_1^-)\exp(r_1^+ T). \tag{4.6}$$

The first term on the right hand side is positive, so we must have $p^+ < 0$ in order to decrease the value of $(E(T) - \alpha\bar{P})^2$ and satisfy condition (4.5). Inspecting (3.13) we establish that the emission rate $E(t)$ must be smaller than its non-event counterpart at all $0 \leq t \leq T$. The possibility of crossing the threshold implies a more prudent emission policy at all times, both if the crossing is avoided as well as if the damage implied by shifting regimes is insufficient to prevent the crossing.

[4]The roots $r_1^{+,-}$ correspond here to the damage coefficient γ_1.

5 Uncertainty

Having derived the functional form of the post-event value function $v_2(\cdot)$, we can also characterize the solution of the pre-event problem v_{uc} of (2.2) under uncertainty. In fact, modeling the threat of a regime shift with the hazard rate h allows us to formulate the problem as a standard optimal-control problem that can also be tackled with the evolution function method. We compare the steady state of (2.2) with the steady state corresponding to the risk-free (non-event) problem where the regime shift can never occur. Under the steady-state policy, P and $h(P)$ are fixed. The survival probability at time t becomes $S(t) = \exp(-h(P)t)$ and the steady-state value for the uncertainty problem is given by

$$W_{uc}(P) = [\alpha\beta P - (\alpha^2 + \gamma_1)P^2/2 + h(P)v_2(P)]/[\rho + h(P)]. \tag{5.1}$$

5.1 Constant hazard

We begin with the simple case in which the hazard rate h is constant, hence the distribution of the occurrence time is exponential, independent of the emission policy. The survival probability at time t becomes $S(t) = \exp(-ht)$. We denote variables and parameters associated with this case of constant hazard by the subscript "ch". Using $v_2'(P) = \lambda(P)$, the evolution function for this problem becomes

$$\begin{aligned} L_{ch}(P) &= (\rho + h)W_{ch}'(P) + (\rho + h)(\beta - \alpha P) = \\ &= (\rho + \alpha)\beta - (\alpha^2 + \gamma_1 + \alpha\rho)P + h[\beta - \alpha P + v_2'(P)] = \\ &= L_{ne}(P) + h[\beta - \alpha P + \lambda(P)], \end{aligned} \tag{5.2}$$

where W_{ch} is the value W_{uc} of (5.1) with $h(P) \equiv h$, L_{ne} is the evolution function for the non-event problem (with $\gamma = \gamma_1$), and $\lambda(P)$ is the shadow price associated with the post-event problem (with $\gamma = \gamma_2$). Using (3.11), (3.16) and (3.17), we can write

$$\beta - \alpha P + \lambda(P) = \beta + rP + \hat{\Lambda} = r(P - \hat{P}),$$

so that we can simplify the second term of the right-hand side of (5.2). It follows that $L_{ch}(\cdot)$ is also linear in P:

$$L_{ch}(P) = L_{ne}(P) + hr(P - \hat{P}), \tag{5.3}$$

where $\hat{P}$ is the root of the post-event evolution function. Evaluated at the root $\hat{P}_{ne}$ of $L_{ne}(\cdot)$, the first term on the right-hand side of (5.3) vanishes, yielding

$$L_{ch}(\hat{P}_{ne}) = hr(\hat{P}_{ne} - \hat{P}) < 0$$

because $\hat{P}_{ne} > \hat{P}$ (recall that $\gamma_1 < \gamma_2$) and $r < 0$. It follows that the root $\hat{P}_{ch}$ falls short of $\hat{P}_{ne}$, which means that uncertainty implies a more conservative policy (less pollution) than in case the regime shift can never take place. On the other hand, $\hat{P}_{ne} > \hat{P}$ also implies that $L_{ne}(\hat{P}) > 0$ hence $L_{ch}(\hat{P}) = L_{ne}(\hat{P}) > 0$ and $\hat{P}_{ch} > \hat{P}$, so that

$$\hat{P} < \hat{P}_{ch} < \hat{P}_{ne}. \tag{5.4}$$

The extra prudence implied by uncertainty is insufficient to bring the steady state pollution stock down to the level $\hat{P}$ implied by the high damage coefficient γ_2. It can be verified that

$$\lim_{h\to\infty} \hat{P}_{ch} = \hat{P},$$

because a very high hazard rate implies an immediate shift to the "dirty" regime.

The effect of uncertainty is manifest during the full evolution of the dynamic process on its path to the steady state. Following the same steps as in Section 3.2 we find that the equation governing the optimal process $P_{ch}(\cdot)$ is analogous to (3.8):

$$\ddot{P}_{ch} - (\rho + h)\dot{P}_{ch} - [r_1(r_1 - \rho) - hr_2]P_{ch} - r_2(\rho + h - r_2)\hat{P} = 0, \tag{5.5}$$

(here r_1 and r_2 are the negative roots of (3.10) corresponding to γ_1 and γ_2, respectively.) The general solution (initiated at $P_{ch}(0) = 0$) is

$$P_{ch}(t) = \hat{P}_{ch}[1 - \exp(r_{ch}^{-}t)] + p_{ch}^{+}[\exp(r_{ch}^{+}t) - \exp(r_{ch}^{-}t)], \tag{5.6}$$

where

$$r_{ch}^{+,-} = [\rho + h \pm \sqrt{(\rho + h - 2r_1)^2 + 4h(r_1 - r_2)}\ ]/2 \tag{5.7}$$

and

$$\hat{P}_{ch} = \frac{\rho + h - r_2}{(r_1/r_2)(\rho - r_1) + h}\hat{P}, \tag{5.8}$$

which is the inhomogeneous solution and the root of L_{ch}. To approach this state in the long run, the integration constant p_{ch}^{+} must vanish, yielding $P_{ch}(t) = \hat{P}_{ch}[1 - \exp(r_{ch}^{-}t)]$. Using (5.7), it can be verified that $0 > r_1 > r_{ch}^{-}$, because $h > 0$, and that $r_{ch}^{-} > r_2$, because $\gamma_2 > \gamma_1$. It follows that (5.4) can be extended to arbitrary times so that

$$P(t) < P_{ch}(t) < P_{ne}(t) \tag{5.9}$$

for all $t > 0$. ($P(\cdot)$ represents here the optimal process under γ_2.)

5.2 Variable hazard

Next we generalize the results of the previous subsection by allowing for a dependence of the hazard rate on the pollution stock $h = h(P)$, with $h'(P) > 0$. When choosing an emission policy, the policy maker must also account for the effect of pollution on the occurrence hazard. This introduces an additional $h'(\cdot)$ term to the corresponding evolution function:

$$\begin{aligned} L_{uc}(P) &= [\rho + h(P)]W'_{uc}(P) + [\rho + h(P)](\beta - \alpha P) = \\ &= L_{ne}(P) + h(P)r[P - \hat{P}] + h'(P)[v_2(P) - W_{uc}(P)] \\ &= \tilde{L}_{ch}(P) + h'(P)[v_2(P) - W_{uc}(P)]. \end{aligned} \tag{5.10}$$

(The "~ " symbol over L_{ch} indicates a slight abuse of notation, since $\tilde{L}_{ch}$ is defined using a variable hazard rate.) Let $\hat{P}_{ch}$ denote the root of $\tilde{L}_{ch}(\cdot)$, i.e. the optimal steady state of the uncertainty problem under the *fixed* hazard $h \equiv h(\hat{P}_{ch})$. For this fixed-hazard problem the steady-state policy $E = \alpha P$ is optimal at $\hat{P}_{ch}$ and therefore the steady-state value $\tilde{W}_{ch}(\hat{P}_{ch})$ is certainly larger than the value $v_2(\hat{P}_{ch})$ which is obtained if the regime shift occurs immediately. Since $\tilde{W}_{ch}(\hat{P}_{ch}) = W_{uc}(\hat{P}_{ch})$, the last term of (5.10) is negative at this state, so that $L_{uc}(\hat{P}_{ch}) < 0$. It follows that

$$\hat{P}_{uc} < \hat{P}_{ch},$$

where the root $\hat{P}_{uc}$ of $L_{uc}(\cdot)$ is the optimal steady state of the uncertainty problem with variable hazard. We see, therefore, that the state-dependence of the hazard rate implies extra prudence relative to the emission behavior under fixed hazard. The desire to reduce the risk pushes the pollution process towards cleaner states.

Observe that $\psi(P) \equiv v_2(P) - W_{uc}(P)$ measures the damage incurred when the regime shift occurs at the state P, reducing the value from the uncertainty value $W_{uc}(P)$ to the post-event value $v_2(P)$ and $h(P)\psi(P)$ measures the expected damage. Thus, the last term of (5.10) is a measure of the change in expected damage due to the hazard variation. Indeed, when the marginal hazard is large, this term can give rise to a surprising result.

Note that $\psi(P)$ can also be written in the form $\psi(P) = \rho[v_2(P) - W_{ne}(P)]/[\rho + h(P)]$ (which agrees with (4.2) when $h \equiv 0$). Now $\hat{P}$ is the steady state of the post-event problem. It follows that $v_2(\hat{P}) = W(\hat{P})$ and thus $\psi(\hat{P}) = (\gamma_1 - \gamma_2)\hat{P}^2/2[\rho + h(\hat{P})]$. On the other hand, $L_{ne}(\hat{P}) = L(\hat{P}) + (\gamma_2 - \gamma_1)\hat{P} = (\gamma_2 - \gamma_1)\hat{P}$, reducing (5.10) to

$$L_{uc}(\hat{P}) = L_{ne}(\hat{P}) + h'(\hat{P})\psi(\hat{P}) = (\gamma_2 - \gamma_1)\hat{P}\left(1 - \frac{h'(\hat{P})\hat{P}}{2[\rho + h(\hat{P})]}\right).$$

If $h'(\hat{P}) > 2[\rho + h(\hat{P})]/\hat{P}$, it follows that $L_{uc}(\hat{P}) < 0$ and thus

$$\hat{P}_{uc} < \hat{P}.$$

If the marginal dependence of the hazard rate on the pollution stock is sufficiently high, the effect on precaution is such that pollution is even further reduced than in the case of an immediate shift to the dirty regime! (cf. 5.4)

6 Catastrophic damage

In our model a catastrophic event corresponds to an exceedingly large damage, which is obtained when the post-event damage coefficient γ_2 tends to infinity. In contrast, earlier models of such events (Clarke and Reed, 1994; Tsur and Zemel, 1998; Aronsson et al., 1998; Gjerde et al., 1999; Polasky et al., 2011) refer to a "doomsday event" in which the catastrophic occurrence abruptly ceases all economic activities, giving rise to the post-event value $v_2 \equiv 0$. The latter specification offers the obvious advantage of simplicity, because in this case the hazard effect is equivalent to increasing the discount rate from ρ to $\rho + h(P)$.[5]

The distinction among the two specifications might appear insignificant, since both seem to describe highly undesirable occurrences which should be avoided as far as it is possible. However, they entail very different pre-event behavior. In our model, the threat of a catastrophic occurrence implies ultra-cautious behavior that would bring the long-term pollution level down to the clean state $P = 0$. In contrast to this intuitive behavior, the "doomsday event" gives rise to ambiguous results, and in the simple case of a constant positive hazard it actually calls for *more* pollution relative to the case in which occurrence is not possible. In order to explain this difference, we turn to look at each case in more detail.

6.1 Catastrophic regime shift

In the limit $\gamma_2 \to \infty$, we find from (3.3) that $\hat{P} \to 0$. The extremely high pollution damage calls for pollution reduction and directs the post-event policy towards the clean state. In the same limit, (3.15) implies that the negative root $r \to -\infty$. Under a constant hazard, the second term of the right-hand side of (5.3) will drive $L_{ch}(P)$ to large negative values for all states P exceeding $\hat{P}$. It follows that the steady state $\hat{P}_{ch}$ also tends to zero in this limit. Finally, we recall that a pollution dependent hazard calls for enhanced prudence relative to the case of a constant hazard and hence the pre-event policy will tend to the clean state also in this case. We see the damage effect at work here: a very high damage implies extreme precaution and diminishing pollution.

[5] For the economic implications of endogenizing the effective discount rate via a pollution-dependent hazard in another environmental context, see Tsur and Zemel (2009).

6.2 "Doomsday" event

With $v_2 \equiv 0$, we see from (5.2) that under constant hazard the uncertainty evolution function takes the form

$$L_{ch}^{dd}(P) = L_{ne}(P) + h(\beta - \alpha P).$$

Evaluated at the steady state $\hat{P}_{ne}$ of the non-event problem, this gives

$$L_{ch}^{dd}(\hat{P}_{ne}) = h(\beta - \alpha \hat{P}_{ne}) = h\beta \left(1 - \frac{\alpha(\rho+\alpha)}{\gamma_1 + \alpha(\rho+\alpha)}\right) > 0,$$

so that the steady state of this uncertainty problem lies *above* $\hat{P}_{ne}$. We see that the constant hazard actually implies *more* pollution than the outcome of the problem without occurrence risk. This result is explained in terms of discounting at the increased rate, which is known to encourage myopic behavior and short term benefits (from enhanced pollution) at the expense of long term considerations. With a pollution-independent hazard rate, the occurrence probability cannot be diminished by reduced emissions and the high discount rate dominates the pollution policy (Gjerde et al., 1999).

Turning to the case of a variable hazard rate, we find that the evolution function has an additional term:

$$L_{uc}^{dd}(P) = L_{ne}(P) + h(P)[\beta - \alpha P] - h'(P)W_{uc}(P).$$

The function $L_{ne}(\cdot)$ vanishes at the steady-state $\hat{P}_{ne}$ of the non-event policy and the question whether the pollution policy is more or less conservative relative to the risk-free situation depends on the relative magnitudes of $h'(\hat{P}_{ne})/h(\hat{P}_{ne})$ and $(\beta - \alpha\hat{P}_{ne})/W_{uc}(\hat{P}_{ne})$. When the hazard rate depends strongly on the pollution state, the h'/h term dominates and $L_{uc}^{dd}(\hat{P}_{ne}) < 0$, implying less pollution under uncertainty. Otherwise, the discounting effect dominates, and the uncertainty steady state of the pollution process lies above its non-event counterpart. This type of ambiguity has been discussed by Clarke and Reed (1994); Tsur and Zemel (1998) and Aronsson et al. (1998).

It is now easy to trace the difference between the ambiguous "doomsday" results and the cautious policy implied by the regime shift model to the different specifications of the damage function (see also Polasky et al., 2011, for a fishery model with a sudden loss of total value versus a negative jump in the carrying capacity). Under catastrophic regime shifts, the loss increases with the parameter γ_2, calling for pollution reduction. Under the "doomsday" scenario, the loss is equal to the steady-state value W_{ne} which actually decreases with the pollution stock P, encouraging more pollution. For high pollution states which render W_{ne} negative, occurrence is actually a desirable event because it increases the value. It follows that in spite of its great simplicity, the results of the "doomsday" model should be evaluated with care when one sets to design policies under catastrophic risks. The "doomsday" model leads to higher pollution, in the case of a constant hazard rate, and to ambiguous results in the case of a variable hazard rate, but the regime shift model always calls for more precaution.

7 Adaptation vs. mitigation

Our discussion so far has considered "mitigation" efforts (in the form of emission reduction) as the sole response to the pollution damage and the regime shift risk. However, other response measures, known as "adaptation", are also possible. These include investments in means and methods to moderate the damage associated with the shift, should it occur in spite of the efforts to mitigate the risk.

In this section we consider the simplest possible form of adaptation that displays the adaptation-mitigation tradeoffs within a dynamic framework. We assume that the hazard rate h is constant and that the policy maker holds the option to buy, at any time prior to the shift, a technology or equipment that will reduce the damage implied by the shift by decreasing the post-event coefficient γ_2. The purchase, at the given cost of R per unit change in γ_2, has no effect on the pre-event coefficient γ_1. It cannot, however, be delayed until the shift occurs, because then it would not be able to undo the shift damage and affect γ_2 (in other words, we consider *proactive adaptation*, see Smit et al, 2000; Shalizi and Lecocq, 2010). Under this specification, uncertainty regarding the time of shift plays a major role in the determination of the adaptation policy.

We consider a small reduction of the damage coefficient $\gamma_3 = \gamma_2 - d\gamma$ and evaluate first the corresponding change in the post-event value $v_2(\cdot)$. Using the dynamic Envelope Theorem (or alternatively, taking the derivative of (3.18) with respect to γ_2) we find

$$dv = \frac{1}{\rho - 2r_2}\left[\frac{1}{2}P_0^2 + \frac{-r_2\hat{P}}{\rho - r_2}P_0 + \frac{(r_2\hat{P})^2}{\rho(\rho - r_2)}\right]d\gamma, \tag{7.1}$$

where P_0 is the initial pollution state of the post-event problem and the constants $r_2 < 0$ and $\hat{P}$ correspond to γ_2. Note that the coefficients of the quadratic and linear terms in P_0 are both positive, implying that the higher is the pollution state corresponding to the shift, the larger is the gain in the post-event value from the adaptation investment $Rd\gamma$.

We return now to the pre-event problem under uncertainty (2.2) and denote by $v_{ch}(P)$ the value obtained under constant hazard for this problem with the original post-event damage coefficient γ_2 and by $v^*_{ch}(P)$ the corresponding value obtained with the reduced coefficient γ_3 (i.e. with $h(v_2 + dv)$ replacing hv_2 in the objective integral). The argument P represents an arbitrary initial state for these problems. Suppose that at $t = 0$ (when $P = 0$) the policy maker commits to invest in adaptation once the pollution state will reach some predetermined level $\bar{P}$ (unless the regime randomly shifts prior to reaching this level, rendering adaptation useless). The remaining value at the adaptation time τ will be $v^*_{ch}(\bar{P}) - Rd\gamma$. If, however, the regime shifts at some state $P < \bar{P}$ prior to adaptation, the remaining value at the regime shift time T will be $v_2(P)$. Taking the expectation over the distribution of the random regime shift time, we find that the optimal emission policy is the outcome of

$$\begin{aligned} v_a &= \max_{\{E\geq 0,\tau\}} \{\int_0^\tau [\beta E - E^2/2 - \gamma_1 P^2/2 + hv_2(P)] \exp[-(\rho + h)t]dt \qquad (7.2) \\ &+ \exp[-(\rho + h)\tau][v^*_{ch}(\bar{P}) - Rd\gamma]\} \end{aligned}$$

subject to (2.1), $P(0) = 0$, $P(t) < \bar{P}$ for $t < \tau$ and $P(\tau) = \bar{P}$, where the adaptation time τ is a free control variable that can take the value $\tau = \infty$ if the state $\bar{P}$ is never reached. Observe that $\tau = \infty$ implies $v_a = v_{ch}(0)$ while $\tau = 0$ turns v_a into $v^*_{ch}(0) - Rd\gamma$.

Apart from the presence of the h-dependent terms, (7.2) is similar to the certainty problem (4.1), with τ replacing T as the free transition time, where "transition" refers here to the adaptation investment rather than to the regime shift. Indeed, the optimal process $P_a(\cdot)$ of (7.2) satisfies (5.5) and the general solution is written as

$$P_a(t) = \hat{P}_{ch}[1 - \exp(r^-_{ch}t)] + p^+_a[\exp(r^+_{ch}t) - \exp(r^-_{ch}t)]. \qquad (7.3)$$

If the adaptation value $v^*_{ch}(\bar{P}) - Rd\gamma$ were equal to $v_{ch}(\bar{P})$, the policy maker would be indifferent about adaptation and the solution $P_a(\cdot)$ would coincide with the policy corresponding to v_{ch} which extends over an infinite horizon, so that the integration constant p^+_a would have to vanish. In fact, for adaptation to be worthwhile at $\bar{P}$, the relation $v^*_{ch}(\bar{P}) - Rd\gamma > v_{ch}(\bar{P})$ must hold, because otherwise the policy of never investing in adaptation outperforms adaptation at the state $\bar{P}$. It follows that the transversality condition associated with the free choice of the adaptation time τ implies $p^+_a > 0$ (c.f. discussion in section 4). The adaptation option *increases* the emission rate and the corresponding pollution stock *even before this option is actually realized at* τ.

The optimal adaptation time τ is obtained in terms of the adaptation state $\bar{P}$ via the transversality condition. How is the state $\bar{P}$ determined? Suppose that the unit adaptation cost is so large that

$$R > [v^*_{ch}(P) - v_{ch}(P)]/d\gamma \qquad (7.4)$$

for all relevant pollution states (i.e. for all $P \leq \hat{P}_{ch}$). The considerations above show that investment in adaptation is never worthwhile in this case.

The value $\tau = \infty$ can be secured by setting $\bar{P} > \hat{P}_{ch}$ and the expensive investment will never take place. The optimal policy in this case is to follow the process associated with v_{ch} at all times.

Suppose now that the converse of (7.4) holds at $P = 0$. The Envelope Theorem result (7.1) indicates that the benefits from adaptation increase with pollution. It follows that (7.4) is violated for all P and adaptation at every pollution state is better than the policy of never investing. We now show that prompt adaptation, (i.e. setting $\bar{P} = 0$) is optimal in this case, yielding the value $v_a = v_{ch}(0)$. Consider $\bar{P} > 0$ and note that the objective of (7.2) would increase if $v^*_{ch}(\bar{P}) - Rd\gamma$ in the second term of the right-hand side were replaced by $v_{ch}(\bar{P})$. This replacement would equalize this term to the tail of the v_{ch} problem initiated (not necessarily optimally) at time τ at the state $\bar{P}$. The first term of the right-hand side of (7.2) gives the objective of the first part of $v_{ch}(0)$. It follows that $v_a < v_{ch}(0)$ for all $\bar{P} > 0$ and therefore prompt adaptation is optimal.

Finally, we consider the intermediate case in which (7.4) holds at $P = 0$ but is violated at $P = \hat{P}_{ch}$. Obviously, prompt adaptation is sub-optimal. By continuity, there exists a state $0 < \tilde{P}_a < \hat{P}_{ch}$ where $R = [v^3_{ch}(\tilde{P}_a) - v^2_{ch}(\tilde{P}_a)]/d\gamma$. The considerations above imply that it is optimal to set $\bar{P} = \tilde{P}_a$. Reaching the state $\tilde{P}_a$ requires some time, hence τ is finite in this case. We conclude, therefore, that intermediate adaptation costs imply *delayed adaptation*. Observe that the assumption of fixed unit adaptation costs is not essential and the results above also hold for pollution-dependent costs, as long as condition (7.4) changes sign only at a unique pollution state.

Condition (7.4), which is formulated here as a comparison between the costs and benefits of the adaptation investment, reflects the complex dynamic tradeoffs associated with the problem. First, the term $v^*_{ch}(P) - v_{ch}(P)$ results from the change introduced by adaptation to the hv_2 term in the objective of (2.2). This observation manifests the role of uncertainty (in the form of the hazard rate h) in optimizing the timing of adaptation investments. Moreover, $v^*_{ch}(P) - v_{ch}(P)$ depends on the optimal mitigation policies under the different values of γ. We have shown before how the adaptation option affects the mitigation policy. Here we show the interaction in the opposite direction, with mitigation policies entering the condition for the cost effectiveness of adaptation. In a dynamic setting, the two response measures are strongly connected, and the optimal policy requires that both are addressed simultaneously.

8 Conclusions

Regime shifts and uncertainty have become important aspects of pollution control. This paper presents a very simple model in which the pollution damage function may experience an instantaneous and significant increase due to a random shift into a high-damage regime at a time which is subject to uncertainty. The simple structure of the model allows to derive the full dynamic solution of the optimal emission process and to compare the resulting precautionary policy with the ambiguous behavior reported in the literature for similar models. The differences are explained in terms of the corresponding damage function in a clear and intuitive manner. In particular, we show that in case of endogenous uncertainty (when the occurrence hazard depends on the emission rate) the implied precaution may lead even to lower emissions compared to the rates that are optimal in the high-damage regime. This observation manifests the fact that policy considerations should include not only direct pollution cost reductions but also the need to decrease the shift probability.

Applying the model to study adaptation/mitigation tradeoffs, we obtain again a full characterization of the dynamic processes and derive a condition that determines the optimal time to initiate adaptation activities as well as whether or not the extreme solutions (of prompt adaptation investment or avoiding adaptation at all times) are optimal. The condition can be interpreted as a cost-benefit criterion, comparing the long-term damage reduction due to adaptation with its cost. Obviously, the simple formulation suggested here cannot pretend to accuracy and important elements are left out. Nevertheless, the results presented here clearly display the tradeoffs at work when one set to determine the optimal mix of mitigation and adaptation activities.

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