

## **Net National Product and Social Well-Being**

by

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## **Abstract**

This paper is about net national product (NNP). We are concerned with what NNP means, what it should include, what it offers us and, therefore, why we may be interested in it. We show that NNP, properly defined, can be used to evaluate economic policies, but we also show that it should not be used in any of its more customary roles, such as in making intertemporal and cross-country comparisons of social well-being. We develop such indices as would be appropriate for making those comparisons. In particular, we show that welfare comparisons should involve comparisons of wealth. Writings on the welfare economics of NNP have mostly addressed economies pursuing optimal policies, and are thus of limited use. Our analysis generalizes this substantially by studying economies whose governments are capable of engaging only in policy reforms. We show how linear indices can be used for the evaluation of policy reform even in the presence of non-convexities in the economic environment. The analysis pertinent for optimizing governments are special limiting cases of the one we develop.

The literature on green NNP has widely interpreted NNP as "constant-equivalent consumption". We show that this interpretation is wrong. It is the Hamiltonian that equals constant-equivalent utility. Since both theory and empirics imply that the Hamiltonian is a non-linear function of consumption and leisure, the Hamiltonian should not be confused with NNP.

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## **1. Motivation: Why NNP?**

This article is on the concept of net national product (NNP). We are interested in the meaning of NNP, the items it should include, what NNP offers us and, therefore, why we may be interested in it. The concept is old. Even its modern version was developed over sixty years ago (Lindahl, 1934). Nevertheless, it has proved to be sufficiently intriguing to have appeared periodically on the research agenda of theoretical economists. In recent years there has been renewed interest in it because it has become clear that environmental pollution and resource depletion ought to find expression in NNP if NNP is to reflect what it is believed to reflect. The term "green NNP" is an expression of this belief. In the space of only a few years the term has gained such currency that it is today a commonplace to say that in estimating NNP deduction ought to be made from gross national product (GNP) of not only the depreciation of physical and human capital, but also the depreciation of natural capital and the social losses that are incurred owing to increases in the stock of environmental pollution.

A prior question arises: Why should we be interested in such a measure as NNP?

There are three potential reasons. The first has to do with the fact that there is need for an aggregate index of economic activity, of a kind that would help one to summarise a macroeconomy. GNP has been found to be useful in this role. The second reason arises because we need a quantitative measure of social well-being, not only for making welfare comparisons across space and time, but also for evaluating alternative economic policies. Criterion functions for social cost-benefit analysis of investment projects, such as the present discounted value of the flow of accounting profits (e.g. Little and Mirrlees, 1974), are examples of such indices. NNP is another example, for it is used routinely for making inter-country and intertemporal welfare comparisons.

The third reason for being interested in NNP is academic. It stems from a desire to estimate the levels of aggregate consumption an economy is capable of sustaining along alternative economic programmes. Early definitions of national income (Lindahl, 1934; Hicks, 1940; Samuelson, 1961; Weitzman, 1976) were designed to address this problem, and the bulk of recent theoretical explorations in green NNP have returned to it. But economic

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In what follows we use the terms "well-being" and "welfare" interchangeably.

<sup>1</sup> See, for example, Solow (1986, 1992), Hartwick (1990, 1994), Asheim (1994, 1997), Aronsson, Johansson, and Löfgren (1997), Aronsson and Löfgren (1998), and Weitzman (1998).

activity, social well-being, and sustainable consumption are not the same object. So their numerical measures are not necessarily the same. For example, in a market economy the wage bill for labour ought obviously to be included if the required index is to measure aggregate economic activity, as in GNP. But it is by no means obvious that this particular item ought to be included if the index is to measure social welfare (Nordhaus and Tobin, 1972; see Section 5). The moral is banal: the way an index ought to be defined, let alone estimated, is not independent of the purpose to which it is put.

It can be argued, of course, that if we seek a welfare index, we should measure welfare directly and not look for a surrogate and give it a different name, NNP or whatever. There is something in this. On the other hand, as there are several reasons for seeking a welfare measure, for many purposes the most convenient index could be something other than the thing itself. For example, we could be interested in some object X, but X may prove especially hard to measure (e.g. because it involves estimating non-linear functions of observable quantities). Suppose now that for some purposes X is known to correlate perfectly with Y and that Y is easier to measure than X (e.g. because Y is a linear function of observable quantities). Then we would wish to rely on Y for those purposes. As is well known, NNP is linear in quantities, with the weights being at least in part revealed by observable market prices. Therein lies its attraction.

In this paper we investigate if NNP is a suitable index for evaluating reforms in economic policy. We also construct indices which would enable one to check if the policies being pursued in an economy result in sustainable development. As these indices include autonomous technical change, we also look at the latter's role in generating sustainable development. Among other things, we will prove that NNP can indeed be used as an index for evaluating policy reform, but not for making intertemporal and cross-country comparisons of social well-being.

## **2. Plan of the Paper**

In Section 3 we develop the economic model in which our questions are to be explored. The model involves production with labour, reproducible capital and natural environmental resources. In order to maintain generality, we allow for the possibility that the resource base can be augmented to some extent by expenditure.

Writings on green NNP have addressed only such economies as those where

governments pursue optimum economic policies. The findings therefore have little practical applicability. In Section 4 we offer a substantial generalization by studying economies where governments are engaged in policy reform. Formally we consider perturbations to an arbitrary economic programme and ask if they improve social well-being. We call the perturbations "elementary policy reforms". Accounting prices of goods and services are constructed and are shown to reflect social scarcities. We show how linear indices can be used for the evaluation of policy reform even in the presence of non-convexities in the economic environment. The economic analysis pertinent for optimizing governments are special limiting cases of the one we develop. To be sure, the welfare economics of policy reforms has been discussed by a number of economists over the years. But to the best of our knowledge none have developed the mathematical apparatus necessary for studying an economy operating over time and involving non-convexities. The analysis in Section 4 may therefore be of independent interest to readers.

In Section 5 we show that NNP, measured in accounting prices, can be used for evaluating (elementary) reforms in economic policies, and that it can be so used even in the presence of significant non-convexities. To keep the analysis general, we avoid interpreting policy reforms in concrete terms. But readers can think of tax and subsidy changes, public investment, changes in property rights, and so forth, as examples. We show (Proposition 1) that a policy reform increases social well-being (in comparative dynamics terms) if and only if it registers an increase in NNP (again, in comparative dynamics terms). This is the sense in which NNP can serve as a welfare index.

In fact it has not been usual for those writing on green NNP to enquire if the index can be used for evaluating changes in economic policy. The overwhelming majority have found the concept's appeal elsewhere. Following Weitzman (1976) they have interpreted NNP as "constant-equivalent consumption". In Section 6 we show that this interpretation is wrong. What is true is that for any feasible economic programme the associated Hamiltonian

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Heal (1998) is a particularly good exposition of this mode of analysis.

See, in particular, Meade (1955), Dasgupta, Marglin and Sen (1972), Mäler (1974), Starrett (1988), Ahmad and Stern (1990), and Dreze and Stern (1990).

See, for example, the references in footnote 2.

at each date equals the constant-equivalent utility flow from that date onward (Proposition 2). But both theory and empirics tell us that the Hamiltonian is a non-linear function of consumption and leisure. This makes the Hamiltonian of no direct operational interest. In contrast, NNP is a linear function of quantities, the weights being accounting prices. This is why NNP appeals to economic statisticians. It is also why to identify the Hamiltonian with NNP is wrong. Nevertheless, those writing on green NNP (e.g. Weitzman, 1998) have urged economists to make this identification, presumably because it gives practical relevance to the equivalence result in Proposition 2. To us this isn't a sufficiently good justification. So we turn to other possible reasons why one could be interested in NNP.

In Section 7 we explore alternative notions of sustainable development. One particular notion, that the standard of living should never decline, readily yields as a criterion of sustainability that, at constant prices, the value of net consumption should never fall (Proposition 3). But as this particular notion of sustainable development is concerned only with the current standard of living, it is not easy to defend. So we adopt a notion which takes the future into account. In brief, we consider the formulation which requires that social well-being should never decline. The bulk of Section 7 develops methods for comparing social well-being across time and space. We show that, contrary to both popular belief and common practice, NNP comparisons are inappropriate for the task. So, in Propositions 4-7 we develop such indices as are appropriate. We identify a set of circumstances in which comparisons of social well-being involve comparisons of wealth.

In order to keep the exposition simple, until Section 8 we ignore exogenous technical change, of the kind that is frequently postulated in macroeconomic growth models (Romer, 1996). In Section 8, therefore, we extend the analysis to cover the case of economies capable of experiencing such change. We find that wealth is not an appropriate index of social well-being if the economy experiences exogenous technical change. We argue, however, that the assumption that technical change is exogenous is dubious. In Section 9 we collate our main conclusions and sketch a number of further extensions.

### 3. The Model

Consider a model economy where the production of goods and services requires labour, manufactured capital, and natural resources. The economy is deterministic. Time is continuous and is denoted by  $t (\geq 0)$ . Assume that there is an all-purpose, non-deteriorating durable good, whose stock at  $t$  is  $K_t (\geq 0)$ . The good can be either consumed, or spent in

increasing the stock of natural resources, or reinvested for its own accumulation. Assume that both population size and the stock of human capital are constant, which means that we may ignore them (but see Sections 8-9). The all-purpose good can be produced with its own stock ( $K$ ), labour ( $L$ ) and the flow of natural resources ( $R$ ) as inputs. We write the production function as  $F(K, L, R)$ . Production of the all-purpose durable good at date  $t$  is then  $F(K_t, L_t, R_t)$ . We take it that  $F$  is an increasing and continuously differentiable function of each of its variables. But we do not assume  $F$  to be concave. It transpires that we do not need to, given that our interest is in the welfare economics of policy reform.

Let  $C_t$  ( $\geq 0$ ) denote aggregate consumption at  $t$ , and  $E_t$  ( $\geq 0$ ) the expenditure on increasing the natural-resource base. Net accumulation of physical capital satisfies the condition:

$$dK_t/dt = F(K_t, L_t, R_t) - C_t - E_t. \quad (1)$$

It helps to interpret natural resources in broad terms. It enables us to consider a number of issues. We should certainly include in the natural-resource base the multitude of capital assets that provide the many and varied ecosystem services upon which life is based. But we should add to this minerals and fossil fuels. Note too that environmental pollution can be viewed as the reverse side of environmental resources. In some cases the emission of pollutants amounts directly to a degradation of ecosystems (e.g. loss of biomass); in others it amounts to a reduction in environmental quality (e.g. deterioration of air and water quality), which also amounts to degradation of ecosystems. This means that for analytical purposes there is no reason to distinguish resource economics from environmental economics, nor resource management problems from pollution management problems (Dasgupta, 1982). To put it crudely, "resources" are a "good", while "pollution" (the degrader of resources) is a "bad". So we work with an aggregate stock of natural resources, whose size at  $t$  is denoted by  $S_t$  ( $\geq 0$ ). For simplicity of exposition we assume that resource extraction is costless.

Let the natural rate of regeneration of the resource base be  $M(S)$ , where  $M(S)$  is a continuously differentiable function. We suppose that the base can also be augmented by

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We ignore intermediate goods for expositional simplicity. They can easily be incorporated (Dasgupta, Krström and Mäler, 1998).

<sup>2</sup> If the resource in question were minerals or fossil fuels,  $S_t$  would denote known reserves at  $t$  and we would have  $M(S) = 0$  for all  $S$ .

expenditure  $E_t$  (exploration costs in the case of minerals and fossil fuels, clean-up costs in the case of polluted water, and so forth). Define

$$Z_t = \int_{-\infty}^t E_t dt. \quad (2)$$

In certain applications of the model,  $Z_t$  would be a measure of the stock of knowledge at  $t$ . This interpretation enables us to connect our model with one where there is endogenous technical progress, a matter to be discussed in Section 9. Let us now re-express equation (2) in the more useable form,

$$dZ_t/dt = E_t. \quad (3)$$

There are a number of ways in which one can model the process by which the resource base is deliberately augmented. Let  $N(E_t, Z_t, S_t)$  denote the rate at which this augmentation occurs, where  $N$  is taken to be a continuously differentiable function. It is natural to assume that  $N$  is non-decreasing in both  $E$  and  $Z$ . We therefore assume it to be so. Two special forms of  $N$  are considered in Section 6.

The dynamics of the resource base can be expressed as:

$$dS_t/dt = M(S_t) - R_t + N(E_t, Z_t, S_t). \quad (4)$$

We formulate the idea of social welfare in a conventional manner and ignore those many matters which arise when households are heterogeneous. We do this so as to keep the notation tidy. Following the classic articles of Koopmans (1960, 1972), we assume that social well-being at  $t$  ( $\geq 0$ ) is of the "utilitarian" form,  $\int_0^\infty U(C_t, L_t) e^{-dt} dt$ , where  $U$  is strictly concave, increasing in  $C$ , decreasing in  $L$  (at least at large enough values of  $L$ ), and continuously differentiable in both  $C$  and  $L$ .  $d$  ( $> 0$ ), a constant, is the "utility" discount rate. Our analysis does not require that  $U$  be concave. We assume it nonetheless to be concave for ethical reasons.

#### 4. The Analytics of Policy Reform

Let  $(C_t, L_t, R_t, E_t, K_t, Z_t, S_t)_0^\infty$  denote an economic programme, from the present ( $t = 0$ ) to the indefinite future. A theory of economic policy capable of speaking only to optimizing governments would be of very limited interest. For it to be of practical use, a theory should be able to cover economies where governments not only do not optimize, but perhaps cannot even ensure that economic programmes resulting from its policies are

<sup>3</sup>  $Z_0$  is part of the data of the economy. Like  $K_0$  and  $S_0$ , it is an "initial condition".

The implications of household heterogeneity and other extensions are sketched in Section 10.

intertemporally efficient. Consider then such an economy. To have a problem to discuss, imagine that even though the government does not optimize, it can bring about small changes to the economy by altering its existing, sub-optimal policies in minor ways. The perturbation in question may, for example, consist of small adjustments to the prevailing structure of taxes, or it could be minor alterations to the existing set of property rights, or it could be a public investment, or whatever. We call any such perturbation a policy reform.

We proceed to develop the mathematics of policy reforms.

#### **4.1 Formalising Policy Reforms**

For concreteness, consider an economy facing the technological constraints in equations (1), (3) and (4). In addition, it faces institutional constraints (sometime called transaction and information constraints) which we will formalize presently. The initial capital stocks ( $K_0, Z_0, S_0$ ) are given and known. By the institutional structure of the economy we will mean market structures, the structure of property-rights, tax rates, and so forth. We take it that the institutional structure is given and known. If in addition we knew the behavioural characteristics of the various agencies in the economy (i.e. those of households, firms, the government, and so on) it would be possible to make a forecast of the economy, by which we mean a forecast of the economic programme  $(C_t, L_t, R_t, E_t, K_t, Z_t, S_t)_0^\infty$  that would be expected to unfold. We call this relationship a resource allocation mechanism. So, a resource allocation mechanism is a mapping from initial capital stocks  $(K_0, Z_0, S_0)$  into the set of economic programmes  $(C_t, L_t, R_t, E_t, K_t, Z_t, S_t)_0^\infty$  satisfying equations (1), (3)-(4).

We now formalise this. Write

$$O_t \equiv (K_t, Z_t, S_t), \text{ and} \quad (5)$$

$$(\cdot)_t^\infty \equiv (C_t, L_t, R_t, E_t, K_t, Z_t, S_t)_t^\infty, \text{ for } t \geq 0. \quad (6)$$

Next let  $\{O_t\}$  denote the set of possible  $O_t$ s and  $\{t, (\cdot)_t^\infty\}$  the set of pairs of dates,  $t$ , and economic programmes from  $t$  to the indefinite future. A resource allocation mechanism,  $a$ , can then be expressed as a mapping

$$a: \{t, O_t\} \rightarrow \{(\cdot)_t^\infty\}. \quad (7)$$

$a$  would depend on calender time if knowledge, or population, or terms of trade were to change autonomously over time. If they were not to display any exogenous shift,  $a$  would

There are exceptions to this statement in extreme cases, namely, closed economies where production is subject to constant-returns-to-scale, population changes exponentially, technical change is Harrod-neutral,

be independent of  $t$ . For reasons discussed in Section 8, we pay particular attention to the case where  $a$  is autonomous. So let us assume that  $a$  does not depend on calendar time (i.e. it is time-consistent).

It bears re-emphasis that we do not assume  $a$  to sustain an optimum economic programme, nor even do we assume that it sustains an efficient programme. The following analysis is valid even if  $a$  is riddled with economic distortions and inequities.

To make the dependence of the economic forecast on  $a$  explicit, let  $(C_t(a), L_t(a), R_t(a), E_t(a), K_t(a), Z_t(a), S_t(a))_0^\infty$  denote the forecast at  $t = 0$ . Consider date  $t (\geq 0)$ . Use (5)-(7) to define

$$V_t(a, O_t) \equiv \int_t^\infty e^{-d(t-t)} U(C_t(a), L_t(a)) dt. \quad (8)$$

The right-hand-side (RHS) of equation (8) is social welfare at  $t$ . In the theory of optimum programming  $V_t$  is called the value function at  $t$  (Bellman, 1957).

Before putting the concept of resource allocation mechanism to work, it is as well that we discuss examples by way of illustration. Imagine first that all capital assets are private property and that there is a complete set of competitive forward markets capable of sustaining a unique equilibrium. In this case  $a$  would be defined in terms of this equilibrium. (If equilibrium were not unique, a selection rule among the multiple equilibria would need to be specified.) Most studies on green accounting (e.g. Heal, 1998) are implicitly based on this mechanism.

Of particular interest are situations where some of the assets are not private property. Consider, for example, the class of cases where  $K$  and  $Z$  are private property, but  $S$  is not. It may be that  $S$  is a local common-property resource, not open to outsiders. If  $S$  is managed efficiently, we are back to the case of a competitive equilibrium allocation, albeit one not entirely supported by market prices, but in part by, say, social norms.

On the other hand, it may be that local institutions are not functioning well (e.g. because social norms are breaking down), in that the marginal private benefits from the use of  $S$  exceed the corresponding marginal social benefits. Suppose in addition that decisions

there are no environmental resources, and social well-being is based on classical utilitarianism (Mirrlees, 1967). In such an economy  $a$  would be a mapping from the set of capital assets per efficiency unit of labour into the set of economic programmes, where the programmes are expressed in efficiency units of labour.

In all this, we take it that  $V_t$  is well defined. The assumption that  $d > 0$  is crucial for this.

bearing on the net accumulation of  $K$  and  $Z$  are guided by the profit motive. Then these behavioural rules together help determine  $a$ . In a similar manner, we could characterize  $a$  for the case where  $S$  is open-access.

These observations imply that institutional assumptions underlie our notion of resource allocation mechanism. Aspects of the concept of "social capital" (Putnam, 1993) would appear in our framework as part of the defining characteristics of  $a$ ; other aspects would be reflected as factors in the production functions  $F$  and  $N$ .

The crucial assumption we now make is that  $V_t$  is differentiable in each of the three components of  $O$ . We apologise for imposing a technical condition on something which is endogenous, but space forbids us exploring here the various conditions on an economy's fundamentals (behavioural characteristics of the various agencies and properties of the various production functions and ecological processes; initial set of property rights; and so forth) which would guarantee a differentiable value function.

It is not easy to judge if differentiability of  $V_t$  is a strong assumption. What is certainly true is that if  $a$  is a differentiable mapping, then  $V_t$  is differentiable. We should therefore ask if  $a$  is differentiable. This is not easy to answer. An economy's underlying institutional structure is incorporated in  $a$ , and there are no obvious limits to the kinds of institutions one can envision. So one looks at what might be termed "canonical" institutions. Analytically, the most well understood are those which support optimum economic programmes. What do we know of the mathematical properties of the corresponding  $a$ ?

We know that if the production functions are concave and differentiable everywhere, then for optimum economic programmes  $V_t$  is differentiable in each of the components of  $O$ . Interest therefore lies in cases where the production functions are not concave. Now, we know that even if such production functions are differentiable, not only could optimum economic programmes be discontinuous in each of the components of  $O$ , so could  $V_t$  be discontinuous (Skiba, 1978). But at points where  $V_t$  is discontinuous, social cost-benefit analysis of policy reforms cannot be conducted solely with the aid of accounting prices: the relevant "consumer surpluses" need to be estimated.

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The analytics underlying the idea of social capital are explored in Dasgupta (1999).

The analysis that follows can be extended to cover cases where  $V_t$  possesses right- and left-derivatives everywhere, but is not differentiable everywhere.

Having noted this, it should be stressed that such discontinuities as we are alluding to are non-generic. So, unless the optimising economy were by fluke at one of the points of discontinuity (they are called "bifurcation points"),  $V_t$  would be differentiable within a sufficiently small neighbourhood of the initial capital stocks. The same could be expected to be true for other "canonical" institutions, such as market economies subject to fixed distortions. It would seem, therefore, that the demand that  $V_t$  be differentiable would not appear to rule out much of practical significance. The theory we offer here about the role of NNP in social cost-benefit analysis of policy reforms is valid for a considerably more general set of environments than is usual in writings on NNP.

#### **4.2 Local Accounting Prices and their Dynamics**

Define,

$$p_t(a) \equiv \partial V_t(a, O_t)/\partial K_t; \quad q_t(a) \equiv \partial V_t(a, O_t)/\partial Z_t; \quad \text{and} \quad r_t(a) \equiv \partial V_t(a, O_t)/\partial S_t. \quad (9)$$

We refer to them as local accounting prices. They measure social scarcities of the economy's capital assets along the economic forecast.

How might local accounting prices be estimated? If households are not rationed in any market and externalities are negligible, market prices would be the reasonable estimates. However, when households are rationed or externalities are rampant, estimating local accounting prices involves more complicated work. For example, in the presence of environmental externalities market prices need to be augmented by the external effects (see, for example, Freeman, 1992, for an excellent account of current evaluation techniques). If households are rationed, one has to estimate "willingness-to-pay". And so on. We will presently show that NNP, computed on the basis of local accounting prices, can be used to evaluate policy reform.

What are the dynamics of local accounting prices? To study this, note that the current-value Hamiltonian associated with  $a$  can be expressed as

$$H_t = U(C_t, L_t) + p_t(F(K_t, L_t, R_t) - C_t - E_t) + q_t E_t + r_t(M(S_t) - R_t + N(E_t, Z_t, S_t)). \quad (10)$$

Recall equation (8), which we re-write here as:

$$V_t(a, O_t) \equiv \int_t^\infty e^{-d(t-t)} U(C_t, L_t) dt. \quad (11)$$

$V_t$  is social well-being at  $t$ . Differentiating  $V_t$  with respect to  $t$  we obtain

$$dV_t/dt = dV_t - U(C_t, L_t). \quad (12)$$

But  $V_t = V_t(a, O_t)$ . Using (9), we conclude also that

$$dV_t/dt = p_t dK_t/dt + q_t dZ_t/dt + r_t dS_t/dt + \partial V_t/\partial t. \quad (13)$$

Now combine equations (10), (12)-(13) to obtain

$$H_t = dV_t - \partial V_t/\partial t. \quad (14)$$

We can use equations (9) and (14) to conclude that

$$dp_t/dt = -\partial H_t/\partial K_t + dp_t; dq_t/dt = -\partial H_t/\partial Z_t + dq_t; \text{ and } dr_t/dt = -\partial H_t/\partial S_t + dr_t. \quad (15)$$

The equations embodied in (15) define the dynamics of local accounting prices. It will be noticed that they are formally the same as the Pontryagin conditions for the evolution of accounting prices in an optimizing economy. Note also that all future effects on the economy of changes in the structure of assets are reflected in local accounting prices. That is why they are useful objects.

As  $a$  has been assumed not to depend on calendar time,  $V_t$  does not depend on it either. So equation (14) reduces to

$$H_t = dV_t. \quad (16)$$

Equation (16) is fundamental in intertemporal welfare economics. It says that the Hamiltonian equals the return on social well-being even in a non-optimizing economy.

## 5. Using NNP to Evaluate Policy Reforms

Recall that  $a$  is being assumed not to depend on calendar time. Let us now think of an "elementary policy reform" as a perturbation to  $a$  over the short interval  $[0, t]$ . The perturbation is expressed as  $\delta a$ . During  $[0, t]$  the resource allocation mechanism is denoted as  $(a + \delta a)$ . From  $t$  onward the economy is assumed to be governed by  $a$  again. Note now that if the reform were undertaken, the economic variables during  $[0, t]$  would be slightly perturbed ( $(C_t + \delta C_t)$  rather than  $C_t$ , and so forth). Note too that at  $t$  stocks of capital assets would be slightly different from what they would have been had the reform not been undertaken.

Let the stocks at  $t$  be  $(O_t + \delta O_t)$  as a consequence of the elementary reform. The change in  $V_0$  arising from the reform can then be expressed as

$$\begin{aligned} \delta V_0 &= V_0(a + \delta a, O_0) - V_0(a, O_0) \\ &= \int_0^t e^{-dt} [U(C(a + \delta a), L(a + \delta a)) - U(C(a), L(a))] dt + e^{-dt} [V_t(a, O_t + \delta O_t) - V_t(a, O_t)]. \end{aligned} \quad (17)$$

It is here that we are invoking the assumption that  $a$  is a differentiable mapping. Seierstad and Sydsæter (1987) offers a rigorous account of the reasoning involved here.

On using equation (9) and the accumulation equations (1), (3) and (4), equation (17) can be expressed as:

$$\Delta V_0 = t e^{-dt} (U_C \Delta C + U_L \Delta L) + e^{-dt} (V_K \partial K_t + V_Z \partial Z_t + V_S \partial S_t) + e(t), \quad (18)$$

where  $e(t)$  is an error term with the property that  $[e(t)/t] \rightarrow 0$  as  $t \rightarrow 0$ .

Equation (18) is simple to interpret. A policy reform undertaken during  $[0, t]$  has two effects on  $V_0$ . First, the reform affects consumption and leisure during the period of the reform. Second, it affects the asset structure of the economy at  $t$ , when the reform ends. The right-hand-side (RHS) of equation (18) measures the combined effect of the two sets of changes on  $V_0$ .

Consider now the perturbation to the asset structure at  $t$  as a consequence of the elementary reform. Observe that

$$\Delta K_t = \int_0^t (\partial K_t / dt) dt = t (\partial K_t / dt)_{t=0} + \epsilon(t),$$

where  $\epsilon(t)$  is an error term with the property that  $[\epsilon(t)/t] \rightarrow 0$  as  $t \rightarrow 0$ . Perturbations to  $Z_t$  and  $S_t$  can be estimated in a similar manner. Therefore, equation (18) can be re-written as

$$\Delta V_0/t = e^{-dt} (U_C \Delta C + U_L \Delta L + p_0 \Delta K_t + q_0 \Delta Z_t + r_0 \Delta S_t) + \epsilon(t), \quad (19)$$

where  $\epsilon(t)$  is an error term with the property that  $\epsilon(t) \rightarrow 0$  as  $t \rightarrow 0$ . The left-hand-side (LHS) of (19) is the change in social well-being per unit of time during  $[0, t]$ . As we are interested in small perturbations, we let  $t \rightarrow 0$ . The LHS of equation (19) then becomes the change in social welfare occasioned by the elementary reform, and the RHS tends in the limit to:

$$U_C \Delta C_0 + U_L \Delta L_0 + p_0 \Delta K_t + q_0 \Delta Z_t + r_0 \Delta S_t. \quad (20)$$

Choose consumption as numeraire and write

$$n_0 = -U_L/U_C; m_0 = p_0/U_C; u_0 = q_0/U_C; \text{ and } v_0 = r_0/U_C.$$

On dividing expression (20) by  $U_C$ , we obtain

$$\Delta C_0 - n_0 \Delta L_0 + m_0 \Delta K_t + u_0 \Delta Z_t + v_0 \Delta S_t. \quad (21)$$

Now use equations (1), (3) and (4) to convert expression (21) into:

$$\Delta C_0 - n_0 \Delta L_0 + m_0 (F(K_t, L_t, R_t) - C_t E_t)_{t=0} + u_0 (E_t)_{t=0} + v_0 (M(S_t) - R_t + N(E_t, Z_t, S_t))_{t=0}. \quad (22)$$

If expression (21), or equivalently (22), is positive, the elementary reform increases social

$U_C$  and  $U_L$  are evaluated at  $t=0$ .  $V_K$  is the partial derivative of  $V$  with respect to  $K$  at  $t=0$ , and so forth. We have now dropped writing the dependence of the economic forecast on  $a$ . This saves on notation.

Since the economic programme sustained by  $a$  is not a first-best,  $m_0$  is typically not equal to 1.

welfare, so it is desirable; if it is negative, the reform decreases social welfare, so it is undesirable. Define

$$\hat{\Delta}_t \equiv U_C C_t + U_L L_t + p_t dK_t/dt + q_t dZ_t/dt + r_t dS_t/dt, \quad (23a)$$

and thereby

$$\Delta_t \equiv C_t - n_t L_t + m_t dK_t/dt + u_t dZ_t/dt + v_t dS_t/dt. \quad (23b)$$

If the right-hand-sides of equations (23a,b) have a familiar ring to them, it is because they represent NNP at  $t$  (in utility and consumption numeraires, respectively), measured in local accounting prices. Observe now that expression (21) is the change in NNP at  $t = 0$  occasioned by the elementary policy reform at  $t = 0$ . So we have

Proposition 1: An elementary policy reform increases social well-being if and only if it registers an increase in net national product measured in local accounting prices.

Note that NNP as defined here is not NNP as it is usually defined. Conventional NNP is the sum of aggregate consumption and net investment in physical capital, with both measured at market prices. Expressions (23a,b) tell us that all components of NNP should be valued at the local accounting prices given in equation (9), and that the accounting value of net investment in the stocks of all durable capital goods (manufactured, natural, human, and knowledge capital) should be included in NNP. The NNP we are studying here is "green NNP".

Note that autonomous changes in  $a$  would not affect our result. Being exogenous, such changes would be unaffected by elementary policy reforms, so they are irrelevant for social cost-benefit analysis of policy reform.

The policy reforms we have envisaged here are confined to a short interval. That is why we have labelled them "elementary". But what if a reform were small but irreversible (e.g. a small permanent change in fuel tax)? The answer is that if the accounting cost of the irreversibility constraint were to be estimated and used in the computation of NNP, Proposition 1 would continue to hold (Dasgupta, Kriström and Mäler, 1999). If those accounting prices were not put to use (Johansson and Löfgren, 1996), future changes in consumer surpluses would need to be estimated for the purposes of social cost-benefit analysis. This is because a permanent reform, no matter how small, would have cumulative

Dasgupta and Mäler (1991), Mäler (1991), and Dasgupta, Kriström and Mäler (1999) contain a more detailed account of the various components of NNP.

effects on the size of capital stocks.

How are elementary policy reforms related to optimum planning? Consider an indefinite sequence of elementary reforms at every  $t$ , each of which increases NNP at  $t$ , where NNP is computed at the prevailing local accounting prices. We take it that the entire sequence is conducted in a counter-factual manner; that is, as a  $\hat{t}$  at  $t$ . Such an adjustment process is called a "gradient process" (it is also called the "hill-climbing method"). So far we have not needed to assume convexity of the production possibility set. Now we do. In a well-known paper, Arrow and Hurwicz (1958) proved in the context of a finite-dimensional economy that, provided the set of production possibilities has a sufficiently convex structure, the gradient process converges to the optimum. A corresponding result for our model economy would be harder to prove, given that we are considering infinite-dimensional consumption streams. Our conjecture is that, despite this, a sequence of elementary policy reforms in the form of a suitably defined gradient process would converge to the optimum economic programme if the economy had a strong convex structure.

## **6. The Hamiltonian as Constant-Equivalent Utility**

In the previous section we showed that NNP can be used as an index for conducting social cost-benefit analysis of policy reforms. But the theoretical literature on green NNP has been directed toward a quite different end (see especially Weitzman, 1998). It has argued that NNP measures "constant-equivalent consumption". We now look into this interpretation. In order to do that we have to assume that  $V_t$  is differentiable everywhere. So we do so.

Continue to assume that  $\partial V_t / \partial t = 0$ . Since

$d[\int_t^\infty e^{-d(t-t)} dt] = 1$ , equation (15) can be written as

$$H_t = H_t \{ d [ \int_t^\infty e^{-d(t-t)} dt ] \} = d [ \int_t^\infty e^{-d(t-t)} H_t dt ] = dV_t,$$

from which we have

$$H_t \left[ \int_t^{\infty} e^{-d(t-t')} dt' \right] = \int_t^{\infty} e^{-d(t-t')} H_t dt = V_t \equiv \int_t^{\infty} e^{-d(t-t')} U(C_t, L_t) dt. \quad (24)$$

Equation (24) can be summarized as:

Proposition 2: Along any economic programme the Hamiltonian at each date equals the constant-equivalent flow of utility starting from that date.

This result was proved for optimum economic programmes by Weitzman (1976), who restricted his analysis to linear utility functions (specifically that  $U(C,L) = C$ ). Since in this

case the Hamiltonian is NNP, Weitzman interpreted NNP as the constant-equivalent consumption. The interpretation is today in wide usage.

But a linear utility function is ethically flawed: it is insensitive to distributional issues. Furthermore, large bodies of evidence concerning household saving behaviour are at odds with linear utility functions. One could nevertheless be tempted to imagine that a simple recalibration of the utility function would enable one to interpret NNP as constant-equivalent consumption. For recall that the ethical ordering of economic programmes represented by  $V$  is invariant under positive affine transformations of  $U$ . Thus, if  $U$  is a utility function, one could as well use  $(aU + b)$ , where  $a$  and  $b$  are constants and  $a > 0$ . This means that there are two degrees of freedom when  $U$  is calibrated. For simplicity, imagine that utility depends solely on consumption; that is,  $U = U(C)$ , with  $U'(C) > 0$  and  $U''(C) < 0$ . Let  $a$  be the resource allocation mechanism and  $C_0$  the initial rate of aggregate consumption resulting from it. Choose  $a$  and  $b$  so that  $aU'(C_0) = 1$  and  $(aU(C_0) + b) = C_0$ . The idea, therefore, would be to so calibrate  $U$  that initial utility equals initial consumption (expressed in utility numeraire) and initial marginal utility equals unity. This makes the Hamiltonian at  $t = 0$  equal to NNP at  $t = 0$ . It follows from equation (24) that at  $t = 0$  NNP can indeed be interpreted as the constant-equivalent utility stream associated with  $a$ .

But there is a problem with this device. For note that a high or low value of  $U$  in itself carries no significance ( $a$  and  $b$  are freely chooseable, remember). So to be told that today's NNP, expressed in utility numeraire, is high (or low) because the constant-equivalent utility is high (or low), in itself has no meaning. As we discuss in the next section, what would have meaning would be comparisons of  $U$ ; across time, or space, or groups of people, or whatever. It would certainly be informative if we could be told, say, that because NNP is expected to be greater tomorrow than it is today, tomorrow's constant-equivalent utility can be expected to be greater than what it is today. If we were to be told that, we would be able to infer that social well-being tomorrow should be expected to be higher than what it is today. Unfortunately, we cannot in general be told that. The reason is that once  $U$  has been calibrated at  $t = 0$ , it must not be recalibrated ever again. For to do so would be to alter the underlying ethical ordering of economic programmes, which would render intertemporal comparisons of social well-being meaningless. But unless  $U$  were to be constant over time, it would have to be recalibrated continuously if Weitzman's interpretation of NNP were to be preserved at each date.

In a recent piece where he has re-iterated the interpretation, Weitzman offers a different justification. He says:

"First of all, it is assumed that, in effect, there is just one composite consumption good. It might be calculated as an index number with given price weights, or as a multiple of some fixed basket of goods, or, more generally, as a cardinal utility-like aggregator function. The important thing is that the consumption level in period  $t$  can always be registered unambiguously by the single number  $C(t)$ ." (Weitzman, 1998: 1,583. Italics ours).

There is a problem with this. Measuring non-linear utility functions for an entire economy would involve estimating a battery of "consumer surpluses". The practical appeal of such objects as NNP is precisely that they are linear functions of quantities. We are being asked, however, to abandon such practical advantages in order to accomodate a particular interpretation of the Hamiltonian. And we are asked to do it by simply re-naming the "utility of consumption" as "consumption". We know of no practical reason why we should do so. Moreover there are no theoretical advantages either. In the following section we will see why.

## 7. Social Well-Being and the Concept of Sustainability

World Commission (1987) defined "sustainable development" as an economic programme in which, loosely speaking, the well-being of future generations is not jeopardized. There are a number of possible interpretations of this. Consider the following:

- (a) An economic development is sustainable if  $dU_t/dt \geq 0$ , where  $U_0 \geq \lim U_t$  as  $t \rightarrow -0$ .
- (b) An economic development is sustainable if  $dU_t/dt \geq 0$ .
- (c) An economic development is sustainable if  $dV_t/dt \geq 0$ ,

where  $V_t(a, O_t) \equiv \int_t^\infty e^{-d(t-t)} U(C_t, L_t) dt$ .

If  $U$  is homogeneous of degree  $h$  ( $0 < h < 1$ ), one can construct a measure which looks like NNP and is proportional to the Hamiltonian. However, it involves using prices that do not reflect social scarcity values. To see how this can be done, notice that, for the case in hand, equation (23a) can be written as:

$$\hat{U}_{-t} \equiv h[U_C C_t + U_L L_t] + p_t dK_t/dt + q_t dZ_t/dt + r_t dS_t/dt.$$

Define  $p_t^* = p_t/h$ ;  $q_t^* = q_t/h$ ; and  $r_t^* = r_t/h$ . So

$$\hat{U}_{-t} \equiv h[U_C C_t + U_L L_t + p_t^* dK_t/dt + q_t^* dZ_t/dt + r_t^* dS_t/dt].$$

Notice, however, that in this expression the prices of investment goods relative to those of consumption goods do not reflect social scarcity values. This is why the result has no merit.

See Pezzy (1992) for a thorough treatment. It should be noted that to ask if economic development is sustainable is different from asking if a given level of consumption is sustainable. See below in the text.

It is clear that (a) lacks ethical foundation. For example, it may be desirable to reduce  $U$  in the short run in order to accumulate assets in order that the flow of  $U$  is still higher in the future. In this sense (b) offers greater flexibility in ethical reasoning: it permits initial sacrifices in the current standard of living,  $U$  (a burden assumed by the generation engaged in the reasoning), but requires that no future generation should have to experience a decline in their standard of living.

Consider the resource allocation mechanism a. The mechanism allows one to make an economic forecast. Suppose (b) were to be adopted as the definition of sustainable development. Now

$$dU_t/dt = U_C dC_t/dt + U_L dL_t/dt. \quad (25)$$

From equation (25) we may conclude with:

Proposition 3: If sustainable development is taken to mean that, starting from now, utility must never decline, then an economic programme corresponds to sustainable development if, and only if, the value of changes in the flow of consumption services is always non-negative.

### 7.1 Comparisons of Social Welfare Across Time

In contrast to (b), the focus of (c) as a notion of sustainable development is social well-being,  $V$ . The criterion permits the first generation to make initial sacrifices in  $V$  (relative to the past), but requires that social well-being should never decline in the future. Note that, while (b) implies (c), (c) does not imply (b). In short, (c) is more general. In what follows, we adopt (c) as our notion of sustainable development and develop criteria for judging if a given economic programme represents sustainable development.

Continue to assume that  $\partial V_t/\partial t = 0$ . Differentiating both sides of equation (15) with respect to time, we have

$$dH_t/dt = ddV_t/dt. \quad (26)$$

Use (23b) to define

$$I_t^K \equiv p_t dK_t/dt; I_t^Z \equiv q_t dZ_t/dt; \text{ and } I_t^S \equiv r_t dS_t/dt, \quad (27)$$

which are net investments in the three types of capital assets, respectively, expressed in utility numeraire. We may then define aggregate net investment as,

For an arbitrary  $a$  this is a trivial matter to confirm. Interestingly, Asheim (1994) has identified cases where even an optimum economic programme may satisfy (c), while violating (b).

$$I_t = I_t^K + I_t^Z + I_t^S. \quad (28)$$

It follows from equations (10), (13) and (26)-(28) that

$$U_C dC/dt + U_L dL/dt + dI_t/dt = dI_t. \quad (29)$$

Equation (29) enables us to obtain two alternative indicators of sustainable development. The first can be obtained from the RHS of equation (29). For it implies

Proposition 4: An economic programme increases social well-being over time if, and only if, along the programme net investment in the economy's capital assets is always non-negative.

The result has intuitive appeal. It says that social welfare is higher today than it was yesterday if the economy is wealthier today. Here, an economy's "wealth" is interpreted as the accounting value of all its capital assets, and wealth comparisons are made at constant prices. In a famous article Samuelson (1961) argued in connection with national income accounting that welfare comparisons should deal with "wealth-like" entities. Proposition 4 formalizes that insight.

Note, however, that what we have obtained is an equivalence result: Proposition 4 cannot on its own tell us if sustainable development is feasible. Whether the economy is capable of growing wealthier indefinitely depends, among other things, on the extent to which different assets are substitutable in production.

An equivalent way of characterising sustainable development is to use the LHS of equation (29). We state the result as:

Proposition 5: Social welfare increases (decreases) over a short interval of time if, and only if, during the interval the value of net changes in the flow of consumption services plus

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Note that the summation in equation (28) does not imply any assumptions regarding substitution possibilities among the three kinds of capital assets. Whatever substitution possibilities there may be would be reflected in the local accounting prices.

This result, shown to be a property of optimum economic programmes, originated in Solow (1974) and Hartwick (1977), who determined the investment rule that would sustain the maximum constant utility stream. Pearce, Hamilton and Atkinson (1996) suggested the use of the rule we have obtained in the text for practical purposes, but offered no proof that the suggestion is valid. Serageldin (1995) has reported empirical work done at the World Bank on the use of the rule. See also World Bank (1996).

For an account of this, see Dasgupta and Heal (1979, ch. 7). The problem is deeper than was recognized in that work, since substitutability involves substitutability not merely in production, but also in consumption. On this see Dasgupta, Levin, Lubchenco and Mäler (1999).

the change in the value of net aggregate investment is positive (negative).

For making intertemporal welfare comparisons it is customary to compare NNP over time at constant prices. Proposition 5 says that this is not a correct procedure unless the economy is stationary (i.e.  $dp_t/dt = dq_t/dt = dr_t/dt = 0$ ). We conclude that intertemporal NNP comparisons are far less informative about changes in social welfare over time than is commonly believed. Indeed, they would be highly misleading indicators if relative prices were changing significantly. Note that this is consistent with our finding in Section 5 (Proposition 1), where we showed NNP provides a valid measure of the impact on social well-being of elementary policy reforms.

## 7.2 Comparisons of Social Welfare Across Space

In both popular and academic writings cross-country comparisons of GNP per head are today a commonplace method for comparing well-being across countries. The analysis in Section 7.1 suggests not only that this practice is wrong, but also that replacing GNP by NNP would not rescue matters. So the question is what index should be used instead? We look into this.

It is simplest to consider a continuum of closed economies, parametrized by  $x$  (a scalar). We may interpret differences among economies in terms of differences in initial endowments, or behavioural characteristics, or the resource allocation mechanisms guiding them. But in order to make meaningful comparisons of social well-being, we must be able to ascribe the same value-function to all countries, that is, the same utility function  $U(\cdot)$  and the same  $d$ .

Consider a date when the cross-country comparisons are to be made. To keep the notation simple, we drop the time subscript. Let  $H_x$  be the Hamiltonian in country  $x$  and  $V_x$  the value function there. Recall equation (15). In the present case it reads as  $H_x = dV_x$ . An argument identical to the one establishing equation (29) then yields

$$d[p_x dK_x/dx + q_x dZ_x/dx + r_x dS_x/dx + \partial V_x/\partial x] = U_C dC_x/dx + U_L dL_x/dx + dI_x/dx + \partial H_x/\partial x, \quad (30)$$

where  $I_x$  is net aggregate investment in country  $x$ .

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We assume a continuum of economies in order to make use of the calculus. It simplifies the computations. The analysis that follows can be easily adapted for the case where there is a discrete number of economies.

For tractability, the interesting special case to consider is  $\partial V_x / \partial x = \partial H_x / \partial x = 0$ . From the LHS of equation (30) we conclude:

Proposition 6: Social well-being in a country is higher (lower) than in any of its immediate neighbours if in the aggregate it is wealthier (less wealthy).

Proposition 6 formalizes the insight in Samuelson (1961) that in making welfare comparisons across countries, one should compare their wealths. It corresponds to Proposition 4.

An equivalent indicator for making welfare comparisons can be obtained from the RHS of equation (30):

Proposition 7: Social well-being in a country is higher (lower) than in any of its immediate neighbours if the value of the difference in the flow of consumption services between them plus the difference in the value of aggregate net investment between them is positive (negative).

Notice that the recommendation in Proposition 7 (which corresponds to Proposition 5) would not amount to NNP comparisons across countries unless local accounting prices were the same (i.e.  $dp_x/dx = dq_x/dx = dr_x/dx = 0$ ). We conclude that cross-country comparisons of NNP tell us nothing about differences in social well-being excepting under empirically uninteresting circumstances.

Equation (30) is exact, but the pair of (linear) indicators we have obtained in Propositions 6 and 7 serve their purpose accurately only when  $\partial V_x / \partial x = 0$ . We believe this to be a strong condition. If, as we suspect is the case,  $\partial V_x / \partial x$  is not even approximately zero, there are no linear indices to be had for making cross-country welfare comparisons.

## 8. Technological Change and Growth Accounting

How should NNP be computed in the presence of technical change? Note first that resource augmentation,  $N$ , in equation (4) could itself be regarded as a form of technical progress. This said, it must also be granted that the growth and decay of knowledge involve wider considerations. For example, it has been customary in the economics literature to regard technical progress as shifts in production functions. In what follows we explore this route by introducing technical progress in the production of the final good in the model of

The condition requires that the same resource allocation mechanism prevails in all countries. The condition is strong.

### Section 3.

We need to extend our notation. Denote by  $E_{1t}$  and  $E_{2t}$  expenditures on resource augmentation and on generalized research and development (R & D), respectively. Now define  $Z_{1t}$  and  $Z_{2t}$  by the equations

$$dZ_{1t}/dt = E_{1t}, \quad (31)$$

$$\text{and} \quad dZ_{2t}/dt = E_{2t}. \quad (32)$$

$Z_1$  and  $Z_2$  can be thought of as two types of knowledge. Denote the resource augmentation function as  $N(E_1, Z_1, S)$  and imagine that output of the produced consumption good at  $t$  can be expressed as

$$Y_t = e^{\gamma t} Q(Z_{2t}) F(K_t, L_t, R_t), \quad (33)$$

where  $\gamma \geq 0$  and  $Q'(Z_2) \geq 0$ . Technical progress in the production of the final good appears here as the term  $e^{\gamma t} Q(Z_{2t})$ . It combines exogenous factors ( $\gamma$ ) with endogenous ones ( $Z_2$ ).

Let consumption be the numeraire,  $u_1$  and  $u_2$  the local accounting prices of  $Z_1$  and  $Z_2$ , respectively, and let the remaining local accounting prices be denoted as in Section 5. Retracing the arguments leading to (23b), it is a simple matter to conclude that NNP reads as:

$$\pi_t = C_t - n_t L_t + m_t dK_t/dt + u_{1t} dZ_{1t}/dt + u_{2t} dZ_{2t}/dt + v_t dS_t/dt. \quad (34)$$

Similarly one can confirm that the discussion in Section 5 on the evaluation of policy reform remains unchanged in the presence of technical change.

The question remains: what factors contribute to changes in GNP over time? To see what the answer could be, consider that GNP in our model economy is given by (33). Differentiating both sides of equation (33) with respect to  $t$ , re-arranging terms, and dropping the time subscript from variables for the sake of notational simplicity, we obtain the growth accounting identity as

$$(dY/dt)/Y \equiv \gamma + (Q'(Z_2)dZ_2/dt)/Q(Z_2) + (F_K dK/dt)/F + (F_L dL/dt)/F + (F_R dR/dt)/F. \quad (35)$$

The sum of the first two terms on the RHS of equation (35) measures the percentage rate of change in "total factor productivity", while the remaining terms together represent the contributions of changes in the "factors of production" to the percentage rate of change in GNP. Since  $\gamma$  is an exogenous factor, it is unexplained within the model. For this reason it is called the "residual". When it is not zero,  $\gamma$  could well be the most important determinant of  $\partial V_t / \partial t$ .

In a famous article, Solow (1957) used a reduced-form of the production function in (33) to estimate the contribution of changes in the factors of production to growth of non-farm GNP per "man-hour" in the US economy over the period 1909-1949, and discovered that it was a mere 12 percent of the average annual rate of growth. In other words, 88 percent of the growth was attributable to the residual. (Solow's estimate of  $\alpha$  was 1.5 percent per year.) A significant empirical literature since then has shown that when K is better measured (e.g. by accounting for changes in the utilization of capacity and changes in what is embodied in capital; see footnote 28 below) and when account is taken of human-capital formation, the residual is small for the non-farm sector in the US economy.

This is congenial to intuition. We doubt if it is prudent to postulate everlasting increases in total factor productivity, let alone in per capita output. To do so would be to place an enormous burden of proof on an experience which is not much more than a few hundred years old. Extrapolation into the past is a sobering exercise: over the long haul of time (say, a few thousand years), the residual has been not much more than zero.

It is in any case hard to believe that serendipity, unbacked by R&D effort and investment in physical capital (learning by doing), can be a continual source of productivity growth. A positive value of  $\alpha$  would imply that the economy is guaranteed a "free lunch" forever. To be sure, such an assumption would guarantee that growth in aggregate consumption was sustainable. In fact, that would be its attraction: it would enable us to assume away problems of environmental and resource scarcities. But there are no theoretical or empirical grounds for presuming that it is a reasonable assumption. At this point in our understanding of the process by which discoveries are made, it makes greater sense to set  $\alpha = 0$  in (33), (which would imply that  $\partial V_t / \partial t = 0$ ). This thought is reinforced by the

Solow assumed in particular that  $Q'(Z_2) = 0$ .

Jorgenson (1995) contains a masterly account of this complex literature.

Lau (1996) reports on a series of studies that have specified the aggregate production function to be of the form  $Y_t = F(A_t K_t^a H_t^{(1-a)}, L_t)$ , where K is physical capital, H is human capital, A is the augmentation factor of the composite capital, L is the number of labour-hours, and  $0 < a < 1$ . The studies have uncovered that, since the end of the Second World-War, the contribution of technical progress (i.e. the percentage rate of change in  $A_t$ ) to growth in  $Y_t$  in today's newly industrialized countries has been negligible. He also reports that, if new knowledge is taken to be embodied in new capital-equipment, the contribution of growth in the value of  $A_t$  to growth in  $Y_t$  among western industrialized economies has been a mere 10 percent, that of growth in physical capital some 75 percent, while the contributions of growth in human capital and labour-hours have each been some 7 percent. Lau also notes that the studies are silent on whether technical

observation that most environmental resources go unrecorded in growth accounting. The implication is obvious: when we regress growth in GNP on growth in inputs which exclude the use of environmental resources, we obtain too high an estimate of  $\alpha$  if in fact the use of such resources has been growing. In adopting this position, we are not suggesting that there is no such thing as technical change; what we are suggesting is that of the first two terms on the right-hand-side of equation (35), it is the second term which is significant. It denotes the contribution of technical change to productivity growth.

Productivity growth in equation (35) is productivity growth in GNP. It has often been suggested that we should instead be interested in productivity growth in NNP, as defined in equation (34). For example, in their important early work on Indonesia, Repetto *et al.* (1989) showed that if one were to include deforestation, soil erosion, and the depreciation of oil reserves in the country's national accounts, Indonesia's rate of growth in NNP during the 1980s would be half the estimated growth rate of her GNP. And there are other environmental and natural resources that Repetto *et al.* did not consider.

In Section 7.1 it was shown that NNP comparisons across time tell us nothing about changes in social well-being unless an economy is in a steady state. It was also shown that we should ask instead if, in the aggregate, net investment is positive. It is possible for an economy's GNP (per head) to increase over a period of time even while, in the aggregate, net investment (per head) is negative. We know of no evidence that in recent years this has not been experienced in a number of countries.

## 9. Conclusions

This paper has been about NNP. We have been concerned with what NNP means, what it should include, what it offers us and, therefore, why we may be interested in it. We have shown that while NNP, properly defined, can be used as a gauge for evaluating economic policy, it should not be used in any of its more customary roles. For example, it was shown in Sections 7.1 and 7.2 that comparisons of NNP across time, across countries, and (by similar reasoning) across groups, are not equivalent to what they are widely thought to be equivalent to, namely, comparisons of social well-being across time, across countries, and

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progress in Western industrialized economies has been exogenous or the fruit of expenditures on research and development.

Serageldin (1995) contains a report on the beginnings of this research programme.

across groups. So in Section 7 we also developed such indices as would be appropriate for making comparisons of social well-being (Propositions 4-7). In particular, we showed that in cases where the resource allocation mechanism is independent of calendar time, social well-being increases (decreases) over a brief interval of time if during the interval the value of the change in the flow of consumption services plus the change in the value of investment is positive (negative). We also showed that, equivalently, social well-being is an increasing function of time if, and only if, net investment in the economy's capital assets is positive; that is, if the economy grows wealthier over time. Neither rule amounts to NNP comparisons. A corresponding pair of results was obtained for cross-country comparisons of social well-being.

The recent theoretical literature on the intertemporal welfare economics of NNP (as summarized, say, in Heal, 1998) has focussed on economies pursuing optimal policies. Our analysis has included not only such economies, but also those where the government is capable of engaging only in policy reforms. The apparatus we have developed is valid even when technological and ecological transformation possibilities are non-convex.

Green NNP has widely been interpreted as constant-equivalent consumption. In Section 7 it was shown that, excepting for the uninteresting case where  $U$  is linear in consumption (or else homogeneous of degree less than one), this interpretation is simply false. What is true is that the Hamiltonian which equals constant-equivalent utility. However, since the Hamiltonian is typically a non-linear function of consumption and leisure, it is of little practical use.

In developing the concept of NNP we have made use of a series of models of increasing generality. However, of necessity even the most general of the models had important features missing. We comment on a few of them. Readers can easily fill in the details.

(1) Problems associated with intragenerational distribution have been ignored. However, it is theoretically a simple matter to include them. The way to do it would be to enlarge the set of commodities so as to distinguish a good consumed or supplied by one person from that same good consumed or supplied by another person. This means, for example, that a piece of clothing worn by a poor person should be regarded as a different commodity from that same type of clothing worn by a rich person. Such commodities are called "named goods" (Hahn, 1971). Accounting prices of named goods would typically

depend on the names attached to them. With this re-interpretation of goods and services, the results we have obtained continue to hold.

(2) Environmental externalities can be incorporated by a device identical to (1) above. To describe who is affected, in which manner, and by whose actions involves the use of named goods and services. It follows that accounting prices would be "named", so as to distinguish private costs from social costs and private benefits from social benefits. Indeed, Pigouvian taxes and subsidies on externalities can be computed on the basis of named accounting prices (Dasgupta and Heal, 1979, ch. 3; Mäler, 1991; Freeman, 1992).

(3) Uncertainty has been avoided here. Assume then that social well-being at date  $t=0$  is the expected value of the present discounted flow of utility. The natural move would be to make use of the idea of contingent goods, and therefore of contingent accounting prices. Our analysis would then go through.

(4) The discussion has been restricted to closed economies. However, the analysis can be extended to an economy that trades with the rest of the world. Dasgupta, Kriström and Mäler (1995) and Sefton and Weale (1996) contain an account of this.

(5) Human capital has been absent from our discussion. Analytically it is not difficult to include it. Human capital can be thought of as another form of capital. So net investment in it would be included in NNP (see Dasgupta, Kriström and Mäler, 1995, for a formulation). However, unlike physical capital, human capital is non-transferable. So they should be regarded as named goods.

(6) The models studied here have not included demographic change. It is customary in growth accounting to regard changes in population over time as exogenously given. However, in many societies parents regard children as both an end in themselves and a means to other things (e.g. income security). So population needs to be regarded as a stock whose movements over time are, at least in part, endogenously determined.

The problem is that our current understanding of the determinants of fertility behaviour is weak. Moreover, serious problems arise when one comes to construct intergenerational welfare economics in such a world. There is no received theory. Population

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We are assuming in this example that income or wealth mal-distribution is the cause of concern. Dasgupta, Marglin and Sen (1972) suggested the use of income distributional weights as a rough-and-ready way to capture such concern. The Bergson social welfare function was designed precisely to incorporate these considerations.

ethics is an underdeveloped field of inquiry. For the moment it would seem reasonable to conduct such analyses as we have conducted conditional on specified demographic movements. This has been our approach here.

Finally, it is as well to re-stress that this paper been about conceptual matters only. Our findings imply that the estimation of accounting prices should now be a priority. This said, it must be acknowledged that estimating the accounting prices of certain categories of resources will prove to be impossible. So no single index could suffice. But this means that tradeoffs would have to be made explicitly (e.g. how much biodiversity should be permitted to be destroyed for the sake of so many dollars of aggregate income?). These are hard choices, even tragic choices. But we believe they are unavoidable.

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See Dasgupta (1998) for a discussion of some of the more transparent problems that arise when one thinks about the concept of optimum population.

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