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Climate Tipping and Economic Growth:

Precautionary Saving and the Social Cost of Carbon[§]

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Abstract

The optimal reaction to a pending productivity shock of which the expected arrival time increases with global warming is to accumulate more precautionary capital to smooth consumption and to levy a carbon tax, proportional to the marginal hazard of a catastrophe, to curb the risk of climate change. The carbon tax holds down the stock of greenhouse gases, so that the risk of catastrophe decreases and less precautionary saving is needed. We also allow for conventional marginal climate damages and decompose the optimal carbon tax in two catastrophe components and a conventional Pigouvian component. Further, the productivity catastrophe is compared with recoverable catastrophes and with a catastrophe shock to the temperature response. Finally, the trade-off between adaptation capital and capital used for production is analyzed.

Key words: non-marginal climate shock, tipping point, precaution, economic growth, risk avoidance, social cost of carbon, adaptation capital.

JEL codes: D81, H20, O40, Q31, Q38.

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1. **Introduction**

Global warming has been called the greatest externality for mankind (Stern, 2007). Most climate integrated assessment studies therefore call for a global price of carbon, realized via a global market for tradable CO2 permits or a global carbon tax, which corresponds to the present value of all future marginal damages arising from emitting one ton of carbon (e.g., Nordhaus, 2008; Stern, 2007; Golosov et al., 2014). Although most integrated assessment studies acknowledge that there may be catastrophic damages associated with global warming, the problem is typically resolved by introducing convexity in the function relating damages to global warming rather than facing the fact that climate change can manifest itself as a non-marginal shock and that it is uncertain when this will happen and how large the catastrophe will be. In fact, policy has to deal with the possibility of large, abrupt and persistent changes in the climate system, called regime shifts in the ecological literature (e.g. Biggs et al., 2012). A point where such a regime shift occurs is called a tipping point. The idea that the prime role of climate policy is to deal with a small risk of abrupt and often irreversible climate disasters and tipping points at high temperatures rather than internalizing smooth global warming damages at low and moderate temperatures is gaining attraction (e.g., Lenton and Ciscar, 2013; Kopits et al., 2013; Pindyck, 2013). It implies that policy has to deal with sudden shifts in damage that can, for example, manifest itself as shocks to the productivity of the economic system or as a sudden destruction of the capital stock. Such shocks to total factor productivity may be the result of flooding of cities, sudden increased occurrence of storms and droughts, abrupt desertification of agricultural land, or reversal of the Gulf Stream.

Another example of a regime shift that can occur with climate change is a sudden acceleration of global warming associated with a sudden reduction in cooling when ice sheets (e.g., Greenland) melt away (the ice-albedo effect).[1] A related example is when a rise in sea temperatures and sea levels triggers the sudden release of methane buried in sea-beds and permafrost (e.g., from the tundra in the Arctic, mostly Eastern Siberia).[2] Since methane is itself a powerful (albeit shorter lived) greenhouse gas, this methane release will increase global warming and set in motion further destabilization. Although these catastrophes can be viewed as introducing positive feedback in the carbon cycle, they can also be seen as a sudden increase in the climate sensitivity or a sudden increase in the stock of greenhouse gases in the atmosphere.

[1] The reason is that water and earth reflect less solar radiation than ice and absorb more heat. The warming up causes more ice to melt and sets in motion even more global warming. The positive feedback acts more quickly over the oceans than over land, because sea ice can melt faster than continental ice sheets. Similar effects can occur with the demise of rain forests as plants have lower reflectivity than soil, which causes less transpiration.

Uncertainty about the time and the size of the catastrophe is thus a key aspect of climate change. Weitzman (2009) draws attention to the fundamental uncertainty at the upper end of the probability distribution of possible increases in temperature and corresponding damage. The consequences for policy of a fat tail instead of a thin tail can be dramatic. Golosov et al. (2013) determine the rule for the optimal carbon tax by using a weighted average of low and high damages and implement this rule in an integrated assessment model. This paper takes a different route and uses a hazard rate to model the uncertainty on when the regime shift takes place. This is similar in spirit to the approach taken by Barro (2013) who is concerned with the optimal investment needed to curb the probability of an environmental disaster occurring. Our approach is concerned with the optimal price of carbon to curb the risk of a climate catastrophe occurring, which exploits the fact that the hazard of a catastrophe increases with the stock of atmospheric carbon.

The basis of our analysis is a standard Ramsey growth model where due to climate change a regime shift can occur. For expository purposes, we initially focus at the possibility that a sudden drop in total factor productivity can occur. A hazard rate which is independent of temperature or the stock of atmospheric carbon only affects the golden rule of capital accumulation so that the economy aims for a higher steady-state stock of capital than in the absence of a possible shock. The intuition is that consumption smoothing over the whole time horizon requires precautionary saving before the event takes place. However, fossil fuels are an input to production leading to the accumulation of greenhouse gases in the atmosphere. In general, the hazard rate or the probability of climate change depends on the stock of greenhouse gases so that the potential regime shift also requires the reduction of emissions in order to decrease the probability of a regime shift leading to a negative shock to productivity. An important part of our analysis is to show the effect of an increasing marginal hazard as a function of the stock of atmospheric carbon.

Accumulation of greenhouse gases is a global externality that has to be corrected in a market economy by a tax or tradable permit system (preceded by a voluntary agreement between states since it is a global problem). If the precautionary saving is ignored by the market, a capital subsidy can be used. The problem will be analysed as a social planner’s problem. The shadow value of the stock of accumulated greenhouse gases can be identified as the social cost of carbon or the optimal carbon tax that is needed in a market economy.

The main contribution of our paper is to derive explicit expressions for the precautionary return on capital and the optimal carbon tax which show the effects of an impending

[2] When sea temperatures rise above a certain level (a tipping point), bleaching of coral reefs occurs as well when these systems shift from a coral dominated state to an algae dominated state (Hughes et al., 2003). This suddenly destroys fish populations and touristical assets.

catastrophic shock to productivity where the expected time of this shock is brought forward with global warming. Furthermore, time paths are presented of consumption and capital both before and after the catastrophe based on an illustrative calibration of our Ramsey model of the global economy with catastrophic climate shocks. We subsequently extend our analysis in three directions.

First, we allow for gradual marginal climate damages familiar from the integrated climate assessment literature (e.g., Tol, 2002; Nordhaus, 2008; Stern, 2007; Golosov et al., 2014). The optimal carbon tax then consists of a catastrophe component and a more conventional component internalizing marginal climate damages. The catastrophe component consists of a risk-averting part to reflect that a higher carbon tax curbs carbon emissions and thus reduces the risk of catastrophe and a raising-the-stakes part which corresponds to the expected present value of future increases in after-calamity welfare from curbing carbon emissions with one unit.

Second, we contrast a regime shift resulting from a total factor productivity catastrophe with that of a climate catastrophe that speeds up the climate sensitivity. More action must thus be taken to overcome the problem of runaway global warming. With this catastrophe, global warming after the catastrophe becomes more severe whilst for the productivity shock the opposite happened as the economy emits less carbon when it is operating at a lower level of activity. We also consider recoverable catastrophes such as a sudden destruction of part of the capital stock or a sudden increase in the carbon stock, where again global warming brings forward the date of the occurrence of such catastrophes. One way of interpreting the latter shock is a sudden eruption of greenhouse gases from the oceans.

Third, we allow for adaptation capital to directly mitigate the effects of climate catastrophe. The best response is to have an optimal trade-off between adaptation capital to soften the impact of the calamity and precautionary saving to better smooth consumption before and after the catastrophe. The share of adaptation capital in total capital increases if the potential return after the event is larger than the loss in productivity before the disaster.

Two other papers have analyzed the effect of tipping points in versions of the well-known DICE model, an integrated assessment model for climate change (Nordhaus, 2008). Cai et al. (2012) and Lontzek et al. (2014) use a hazard rate for an upward shift in the damage function and calculate the consequences for the optimal carbon tax. Lemoine and Traeger (2013) consider a temperature threshold that is uncertain and add learning by formulating a hazard rate that is zero at temperature levels that have proven to be safe. A third paper by Smulders et al. (2014) has also emphasized the need for precautionary saving to deal with an impending climate disaster, but their analysis uses a constant hazard rate whereas our analysis highlights the importance of temperature-dependent hazard rates for climate policy in a Ramsey growth framework. We derive explicit expressions for the precautionary return and the optimal carbon tax, illustrate the results in a simple integrated assessment model of growth and climate change for the global economy, and compare how the policy actions required by our analysis differ from those in more conventional analyses of climate change.

Section 2 presents the basic model for economic growth with tipping points. Section 3 develops the case for a constant hazard rate where only precautionary saving matters. Section 4 analyses the case for stock-dependent hazard rates where a combination of precautionary saving and a carbon tax is required. Section 5 shows the effects of a change in the elasticity of intertemporal substitution. Section 6 connects the carbon tax that is needed in case of a potential regime shift to the standard Pigouvian tax based on internalizing marginal climate damages. Section 7 contrasts regime shifts resulting from catastrophes that destroy part of the total factor productivity with other regime shifts and recoverable catastrophes that destroy part of the capital stock or give a sudden boost to the stock of atmospheric carbon. Section 8 considers adaptation capital and section 9 concludes.

2. **A Ramsey Growth Model with Climate Tipping Points**

Consider a continuous-time Ramsey growth model with a constant population. Fossil fuel E is an input into the production process and has constant marginal cost $d > 0$. It is assumed that the fossil fuel is in abundant supply which is not unrealistic given the abundance of coal and the advent of huge new reserves of unconventional gas and oil. There is also a carbon-free imperfect substitute for fossil fuel R, renewable energy, with constant marginal cost $c > 0$. The capital stock is denoted by K. We assume that capital, fossil fuel and the renewable substitute are cooperative factors of production. Total factor productivity is A before the regime shift and drops to $(1-\pi)A < A$ afterwards, where $0 < \pi < 1$ is the size of the climate disaster. Utility is denoted by U, consumption by C, the production function by A times F before and $(1-\pi)A$ times F after the regime switch, the depreciation rate of capital by $\delta > 0$ and the pure rate of time preference by $\rho > 0$. For expositional purposes, we abstract from population growth and technical progress.

The use of fossil fuels leads to emissions of carbon dioxide. We measure fossil fuel use in GtC. The fraction of emissions that does not return quickly to the surface of the earth is denoted by $\psi > 0$. Although about a fifth of carbon emissions remain in the atmosphere for thousands of years or even forever (Golosov et al., 2014; Gerlagh and Liski, 2012), we suppose that all of it decays eventually and returns to the surface of the earth. The stock of atmospheric carbon P decays naturally at the rate $\gamma > 0$ (typically about 1/300). To focus on the points we wish to illustrate, we abstract from carbon capture and carbon sequestration, learning-by-doing in the renewable sector, and other forms of technical progress.

We assume that the potential drop in total factor productivity is known but that it is not known *when* the climate regime shift will take place. We will model the event uncertainty with a hazard rate *h* that gives the conditional probability of the tipping point *T*. Formally,

$$h(t) = \lim_{\Delta t \to 0} \frac{\Pr[T \in (t, t+\Delta t) \,|\, T \notin (0,t)]}{\Delta t}, \tag{2.1}$$

so that $h(t)\Delta t$ approximates the probability that the regime shift takes place between t and $t + \Delta t$, and not before t. For a constant hazard rate h, the probability distribution for the time T has the exponential density function $f(t) = he^{-ht}$, with mean $1/h$, and thus the cumulative density function $F(t) = 1 - e^{-ht}$ (a Poisson process). It follows that the probability of "survival" is given by e^{-ht}. If the hazard rate h is not constant, ht is replaced by $\int_0^t h(s)ds$.

A large stock of atmospheric carbon P increases the probability of climate change so that the hazard rate h depends on the stock of atmospheric carbon P, hence we have $h(t) = H\left(P(t)\right)$ with $H'(P) > 0$. If this stock P increases over time, the expected duration before the regime shift occurs, $1/H(P)$, decreases over time. It follows that failing climate policy makes the shock to productivity more imminent. The power of this type of modelling is that it captures part of the essence of the climate problem whilst the stochastic dynamic optimization problem can be transformed into a deterministic one (Clarke and Reed, 1994). This facilitates a tractable derivation of the optimal carbon tax under regime switches (cf., Tsur and Zemel, 1996; Naevdal, 2006; Polasky et al., 2011; de Zeeuw and Zemel, 2012; Smulders et al., 2014); van der Ploeg, 2014).

The problem for the social planner is to choose consumption, fossil fuel use and renewable use in order to maximize the expected value of social welfare

$$\max_{C,E,R} \mathrm{E}\left[\int_0^\infty e^{-\rho t} U(C(t))dt\right] \tag{2.2}$$

subject to the accumulation of capital and greenhouse gases

$$\begin{aligned} &\dot{K}(t) = \tilde{A}(t)F\left(K(t), E(t), R(t)\right) - dE(t) - cR(t) - C(t) - \delta K(t), \\ &\dot{P}(t) = \psi E(t) - \gamma P(t), \quad K(0) = K_0, \quad P(0) = P_0, \end{aligned} \tag{2.3}$$

with total factor productivity given by

$$\tilde{A}(t) = A, \quad 0 \le t < T, \qquad \tilde{A}(t) = (1-\pi)A < A, \quad t \ge T, \quad 0 < \pi < 1, \tag{2.4}$$

where the tipping point *T* is driven by the hazard rate *H*(*P*). We analyze the consequences of a constant hazard rate in section 3 and of a stock-dependent hazard rate in section 4.

3. Constant Hazard of a Productivity Catastrophe

3a. After the climate regime shift

The problem can be solved by backward induction. After the climate regime shift, the problem is a standard Ramsey growth model with total factor productivity $(1-\pi)A$. We want to focus on the effect of a disastrous shock to total factor productivity, but return to a discussion of different types of regime shifts and disasters in section 7. We solve the problem with the Hamilton-Jacobi-Bellman equation, because this is also a convenient technique to solve the problem before the regime shift and we need the after-event value function. We define the net output function which gives maximum output *net* of input costs and capital depreciation as an increasing function of the capital stock and a decreasing function of the size of the climate disaster

(3.1)
$$Y(K,\pi) \equiv \underset{E,R}{\mathrm{Max}} \left[(1-\pi)AF(K,E,R) - dE - cR - \delta K \right],$$
$$Y_K = (1-\pi)AF_K - \delta > 0, \quad Y_\pi = -AF < 0.$$

For brevity sake, we have suppressed the arguments *A*, *c* and *d* in the net output function. However, $Y_A = (1-\pi)F > 0$ and the optimal choice of fossil-fuel use and renewable use follow from the usual marginal productivity conditions, with $E^A = -Y_d > 0$ and $R^A = -Y_c > 0$, where after-calamity values are denoted with superscript *A*. Fossil-fuel use and renewable use increase in the capital stock *K* and decrease in the size of the disaster π and the own price.

The current-value Hamilton-Jacobi-Bellman equation in the value function V^A is thus

(3.2)
$$\rho V^A(K,\pi) = \underset{C}{\mathrm{Max}} \left[U(C) + V_K^A(K,\pi) \left\{ Y(K,\pi) - C \right\} \right].$$

Optimality requires that the marginal utility of consumption equals marginal welfare:

(3.3)
$$U'(C^A) = V_K^A(K,\pi).$$

Equation (3.3) can be solved for the following policy rule for the rate of consumption:

(3.4)
$$C^A = C^A(K,\pi), \qquad C_K^A = V_{KK}^A / U'' > 0, \quad C_\pi^A = V_{K\pi}^A / U'' < 0.$$

Consumption increases with the capital stock, since the value function is concave in capital and utility is concave in consumption. It decreases with the size of the climate disaster π because a bigger disaster boosts the marginal value of capital V_K^A which requires a corresponding boost to the marginal utility of consumption $U'(C^A)$ and thus necessitates a drop in consumption.

Differentiating (3.2) with respect to K and using (3.4) yields an ordinary differential equation for consumption after the regime shift C^A as a function of K that also depends on π:

$$(3.5) \qquad \left[Y(K,\pi)-C^A(K,\pi)\right]C_K^A(K,\pi)=\sigma\left[Y_K(K,\pi)-\rho\right]C^A(K,\pi),$$

where $\sigma \equiv -U'/CU''>0$ is the elasticity of intertemporal substitution. The coefficient of relative risk aversion and the coefficient of intergenerational inequality aversion are both equal to $1/\sigma$. The value function is obtained from the HJB equation (3.2):

$$(3.6) \qquad V^A(K,\pi)=\frac{U\left(C^A(K,\pi)\right)+U'\left(C^A(K,\pi)\right)\left[Y(K,\pi)-C^A(K,\pi)\right]}{\rho}.$$

The differential equation (3.5) as function of K can also be written as a saddle-point system of differential equations in C and K as functions of time. This yields the Keynes-Ramsey rule, which gives the growth rate of consumption as the rate of intertemporal substitution times the difference between the marginal net product of capital and the pure rate of time preference, and the dynamics of capital accumulation:

$$(3.7) \qquad \begin{aligned} &\dot{C}^A(t)=\sigma\left[Y_K\left(K(t),\pi\right)-\rho\right]C^A(t), \\ &\dot{K}(t)=Y\left(K(t),\pi\right)-C^A(t), \quad K(T)=K_T. \end{aligned}$$

The steady state $K^{A*}(\pi)$ with $C^{A*}=Y(K^{A*},\pi)$ of this system is determined by the modified golden rule of capital accumulation $Y_K(K^{A*},\pi)=\rho.$ The steady-state capital stock is low if the size of the disaster is large (and the discount rate is high). As far as the transient phase is concerned, the capital stock is predetermined at time T, but the rate of consumption jumps down at that point of time to place the economy on the stable manifold, $C^A(T+)=C^A\left(K(T+),\pi\right)$. The optimal path along the stable manifold of the system (3.7) towards the steady state is $C^A(t)=C^A\left(K(t),\pi\right), t\geq T.$[3] The direction of the stable manifold in the steady state can be derived from (3.5) with l'Hôpital's rule:

$$(3.8) \qquad C_K^A(K^{A^*},\pi)=\lim_{K\to K^{A^*}}\frac{\sigma\left[Y_K(K,\pi)-\rho\right]C_K^A(K,\pi)+\sigma Y_{KK}(K,\pi)C^A(K,\pi)}{Y_K(K,\pi)-C_K^A(K,\pi)},$$

[3] Backward integration of the dynamical system from the neighbourhood of the steady state until the path reaches the given value $K(T)$ is more stable than forward integration starting with the given value for $K(T)$ and a guessed value for $C^A(T+)$.

This yields the quadratic equation $z^2 - \rho z + \sigma Y_{KK}(K^{A^*}, \pi)C^{A^*} = 0$ in terms of the slope of the manifold in the steady state, $z \equiv C_K^A(K^{A^*}, \pi)$. The positive solution to this quadratic is the slope of the stable manifold in the steady state:

$$z = \frac{\rho}{2} + \frac{1}{2}\sqrt{\rho^2 - 4\sigma Y_{KK}(K^{A^*}, \pi)C^{A^*}} > \rho > 0. \tag{3.9}$$

In the sequel of the paper we will use the log-linear approximation of the stable manifold,

$$C^A = C^A(K, \pi) \cong Y\left(K^{A^*}(\pi), \pi\right)\left(\frac{K}{K^{A^*}(\pi)}\right)^{\phi}, \quad \phi \equiv z\frac{K^{A^*}(\pi)}{Y\left(K^{A^*}(\pi), \pi\right)} > 0, \tag{3.10}$$

because it gives an explicit expression for the value function (3.6) which we need in the remainder of the analysis. This log-linear approximation turns out to be very accurate in our simulations.

3b. Before the climate regime shift

The question is how the prospect of a climate regime shift affects the optimal growth path before this regime shift actually occurs. For a constant (exogenous) hazard rate *h* the accumulation of greenhouse gases does not matter. The problem becomes

$$\max_{C,E,R} \mathrm{E}\left[\int_0^T e^{-\rho t}U\left(C(t)\right)dt + e^{-rT}V^A\left(K(T), \pi\right)\right] \tag{3.11}$$

subject to the accumulation of the capital stock *K*. The optimality conditions for fossil-fuel and renewable use and the net output function are the same as in section 3a but with total factor productivity equal to *A*, or π = 0. We denote before-event values with a superscript *B*. The current-value Hamilton-Jacobi-Bellman (HJB) equation in the value function $V^B(K, \pi)$ is

$$\rho V^B(K, \pi) = \underset{C}{\mathrm{Max}}\left[U(C) + V_K^B(K, \pi)\left\{Y(K, 0) - C\right\} - h\left\{V^B(K, \pi) - V^A(K, \pi)\right\}\right] \tag{3.12}$$

with the optimality condition

$$U'(C^B) = V_K^B(K, \pi). \tag{3.13}$$

Effectively, the stochastic finite-horizon dynamic optimization problem (3.11) is transformed into the infinite-horizon deterministic problem (3.12) with damage function $D(K, \pi) \equiv h\left[V^B(K, \pi) - V^A(K, \pi)\right]$. For the hazard rate *h* = 0, the standard Ramsey growth model results, of course, but with total factor productivity equal to *A*. We call this the *naive* solution because the potential regime shift is ignored. In general, the last term in (3.12) captures the expected fall in welfare resulting from an impending climate calamity occurring at some uncertain date in the future.

We get the policy rule for the rate of consumption before the regime shift $C^B = C^B(K,\pi)$ from (3.12) and (3.13): the corresponding ordinary differential equation is

$$\text{(3.14)} \qquad \left[Y(K,0) - C^B(K,\pi)\right]C_K^B(K,\pi) = \sigma\left[Y_K(K,0) - \rho + \theta(K,\pi)\right]C^B(K,\pi),$$

where

$$\text{(3.15)} \qquad \theta(K,\pi) \equiv h\left[\frac{V_K^A(K,\pi)}{U'\left(C^B(K,\pi)\right)} - 1\right] > 0.$$

Using (3.4) and the concavity of the utility function $U(.)$, it is easy to see that the value of $\theta(K,\pi)$ is positive because consumption jumps down after the regime shift has occurred and total factor productivity has deteriorated. At this point some conclusions can be drawn.

The only difference with the naive solution, where the potential regime shift is ignored, is that the modified golden rule of capital accumulation has become $Y_K(K^{B^*},0) = \rho - \theta(K^{B*},\pi)$ which gives the steady state $K^{B*}(\pi)$ with $C^{B^*} = Y^B(K^{B*})$. Note that it is a *target* steady state, because after the regime shift has occurred the system will move to the after-event steady state K^{A*}. Since $\theta > 0$, the targeted steady state K^{B*} is higher than the naive steady state K^*, which follows from $Y_K^B(K^*,0) = \rho$. More capital must be accumulated in the presence of a potential regime shift to be prepared for the shock and smooth consumption over time. We refer to θ as a *precautionary return* of capital accumulation, which is larger for a larger hazard rate *h* and for a larger drop in consumption from $C^B(T)$ to $C^A(T)$ at the time of the climate calamity. If the market ignores the potential regime shift and does not prepare for it, the government must implement a capital subsidy equal to $\theta(K,\pi)$ to ensure the socially optimal amount of precautionary saving.

The solution to the whole problem has two possible patterns. Either the capital stock $K(T)$ at time T of the regime shift is still below the after-event steady state K^{A*} in which case the capital stock accumulates further towards K^{A*}, or this $K(T)$ is above K^{A*} in which case the capital stock moves down towards K^{A*}.

Note that a potential regime shift leads to the opposite conclusion as compared to a doomsday scenario in which all value is destroyed: $V^A(K) = 0$. In the doomsday case the hazard rate *h* is added to the discount rate ρ and the precautionary return becomes negative, $\theta = -h < 0$, which implies that before the catastrophe consumption is higher and capital accumulation is lower than in the naive solution (cf. Polasky et al., 2011).[4] But if there is still life after the shock and the climate catastrophe does not lead to complete destruction of the economy, discounting increases but this effect is more than offset by a precautionary argument, as can be seen from (3.15), so that $\theta > 0$.

3c. Illustrative simulations of naïve and rational scenarios with a constant hazard rate

We calibrate the parameters of our model to the world economy of 2010 using data from the BP Statistical Review and the World Bank Development Indicators (see appendix A). We use a utility function $U(.)$ with a constant elasticity of intertemporal substitution σ = 0.5 and a pure rate of time preference ρ = 0.014, and a Cobb-Douglas production function with capital share of 30 percent, energy share of 6.5 percent and depreciation rate of δ = 0.05 for the capital stock. The world economy starts in 2010 with a capital stock of 200 trillion dollar.

We consider a catastrophic shock of a 20 percent drop in total factor productivity (π = 0.2) which has an expected time of arrival of 40 years, which corresponds to a constant hazard rate of h = 0.025. [5] In response to this impending catastrophe, we distinguish two scenarios: one where the world is naïve and ignores the catastrophe altogether (the naïve outcome), and one where the world responds rationally to the impending catastrophe whose time of occurrence is uncertain (the rational outcome). The simulation outcomes for these scenarios are given by the solid and dashed lines in figure 1 for when the regime shift takes place in 2050, and by the dotted lines for when the catastrophe has not occurred yet.

The steady states for the capital stock corresponding to the naive outcome, the regime before the climate calamity, and the regime after the climate calamity are $K^* = 392, K^{B^*} = 473$ and $K^{A^*} = 276$ trillion dollar, respectively. The steady-state levels for the rate of consumption and the stock of carbon in the atmosphere for these three outcomes are $C^* = 58.6, C^{B^*} = 59.4$ and $C^{A^*} = 41.3$ trillion dollar, respectively, and $P^* = 1731$, $P^{B^*} = 1838$ and $P^{A^*} = 1218$ GtC, respectively. The steady-state precautionary return is $\theta(K^{B^*}, \pi) = 0.76$ percent per annum.

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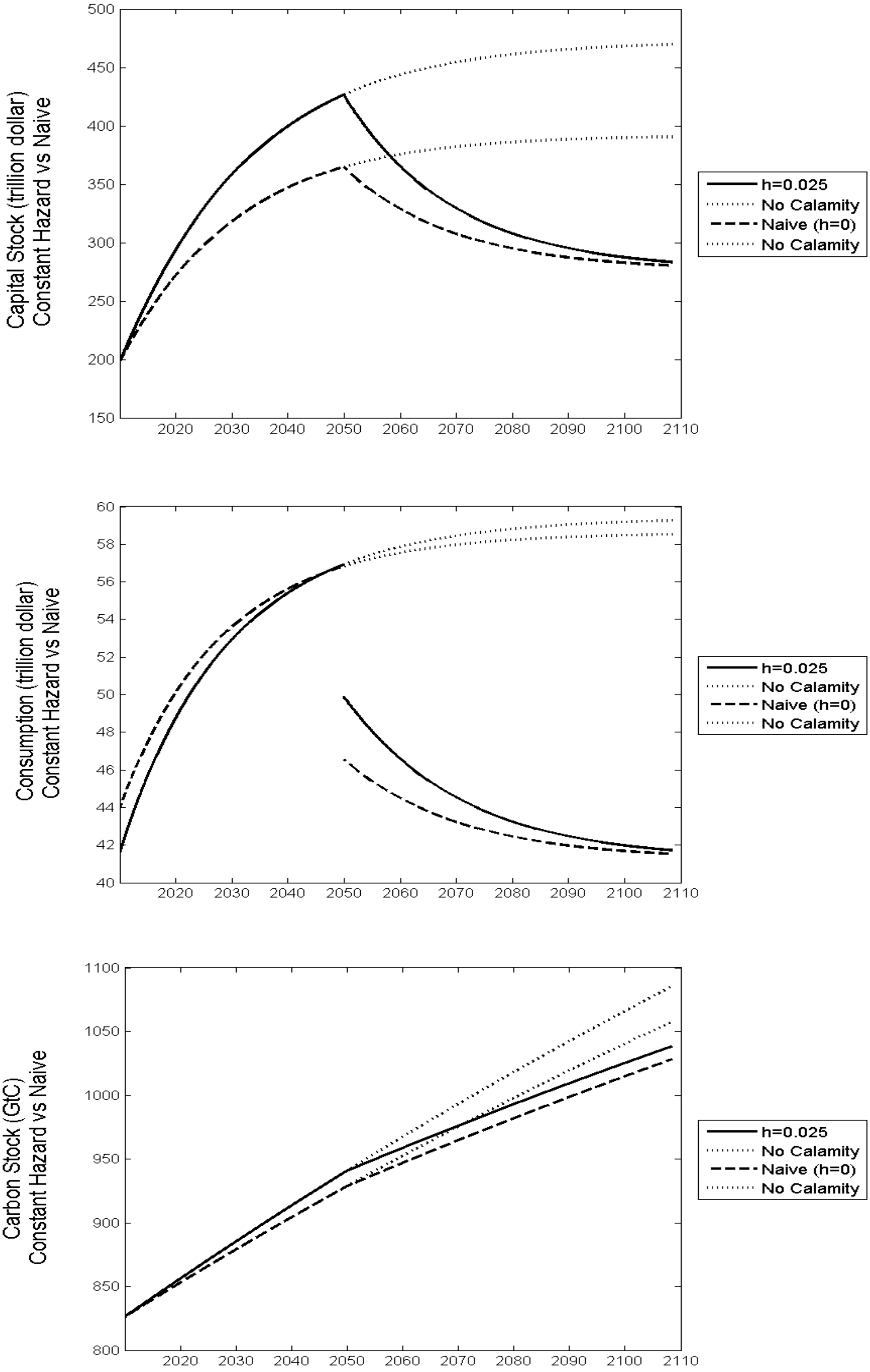
[4] This effect reminds one of Rowan Atkinson's 1983 film *Dead on Time*, where Bernard Fripp has thirty minutes left to live and sets on a frantic mission to live life to the full whilst it lasts.

[5] Barro (2013) has a sample of 158 rare macroeconomic disasters defined as those leading to at least a drop of 10% in GDP. The mean disaster of this sample is 20.7% of GDP, but note that no climate disasters have been recorded yet that exceed the 10% limit. Calibration of climate shocks is thus very speculative. Purely for illustrative purposes we focus at a shock of 20% in total factor productivity.

Figure 1: Naive and Rational Outcomes with Constant Hazard



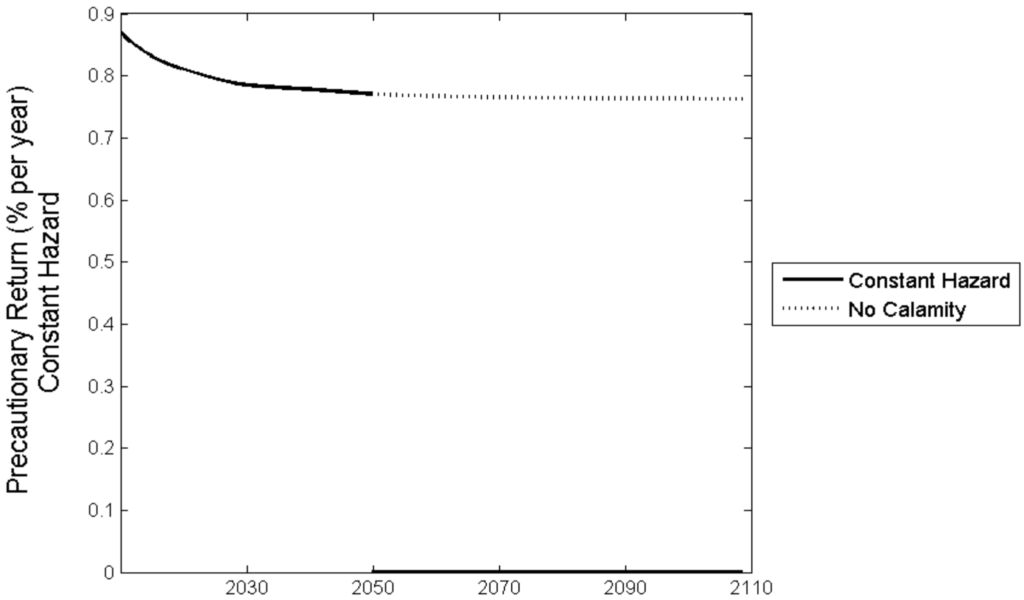


Figure 1 shows the optimal paths of capital, consumption, pollution and the precautionary return for these two scenarios.[6] Since the regime shift occurs sufficiently late, the capital stock starts to fall after the catastrophe (as $K^B(T)$ is larger than K^{A*}).[7] Due to the precautionary saving in the rational outcome, the drop in consumption is mitigated compared with that in the naive outcome. This is achieved at the expense of less consumption especially in the short run when most precautionary saving takes place as can be seen from the gradual decline in the precautionary return on capital from 0.87 to 0.76 percent per year.

After the calamity fossil fuel use is less and thus the stock of carbon rises less rapidly than before. Due to precautionary capital accumulation, fossil fuel use increases before the calamity and thus the final stock of carbon in the atmosphere is higher than in the naive outcome. This worsens global warming which brings forward the expected date of the catastrophe, but this effect is not captured with a constant hazard rate *h*. We therefore allow in section 4 for a hazard of a regime shift which increases with the carbon stock. With a catastrophic shock to productivity, economic activity after the regime shift will be less and thus fossil fuel, the final stock of carbon in the atmosphere and global warming will be less. We demonstrate in section 7 that this is not the case with a catastrophic shock to the temperature response.

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[6] We solve the dynamic system for *C* and *K* backwards in time. In case the regime shift occurs at time *T*, the paths before the regime shift and after the shock are connected by $K^B(T) = K^A(T)$.

[7] Precautionary saving resulting from regime shifts differs from that what occurs if mistakenly it is assumed that the catastrophe occurs with certainty after 40 years (the certainty-equivalent outcome). Then there is no jump in consumption if the calamity strikes at time $1/h = 40$. If it strikes before (say, at time 25), precautionary saving has been insufficient so that consumption jumps downwards at the time of the calamity. If the calamity strikes later than time $1/h$, consumption jumps upwards.

4. Hazard of a Climate Calamity Increases with the Stock of Atmospheric Carbon

With a hazard rate that increases with the carbon stock, the analysis after the regime shift is the same as in section 3 whilst the problem before the regime shift is still given by (3.11) but accumulation of carbon in the atmosphere plays a role now via the hazard function, $H(P)$. The value function V^B is thus a function of the capital stock *K and* the stock of atmospheric carbon *P*. The current-value HJB equation in the value function $V^B(K,P)$ becomes[8]

(4.1)
$$\rho V^B(K,P) = \underset{C,E,R}{\text{Max}}\Big[U(C) - H(P)\big\{V^B(K,P) - V^A(K)\big\} \\ V_K^B(K,P)\big\{AF(K,E,R) - dE - cR - C - \delta K\big\} + V_P^B(K,P)(\psi E - \gamma P)\Big],$$

with the optimality conditions

(4.2)
$$U'(C^B) = V_K^B(K,P), \qquad AF_E(K,E^B,R^B) = d + \tau, \qquad AF_R(K,E^B,R^B) = c,$$

with

(4.3)
$$\tau = \psi \frac{-V_P^B(K,P)}{V_K^B(K,P)}.$$

Here τ is identified as the *social cost of carbon*, which corresponds to the marginal disvalue of emissions $-V_P^B$ times the fraction of emissions ψ that does not return quickly to the surface of the earth, converted from utility units to final goods units by dividing by the marginal utility of consumption V_K^B. Note that problem (4.1) is, as before, equivalent to a deterministic problem with damage function $D(K,P) \equiv H(P)\big[V^B(K,P) - V^A(K)\big]$.

With τ as the additional cost of fossil fuel input, we define $Y^B(.)$ as the function giving maximum level of output *net* of total input costs and capital depreciation:

(4.4)
$$Y^B(K,\tau) \equiv AF(K,E^B,R^B) - (d+\tau)E^B - cR^B - \delta K, \qquad Y_\tau^B = -E^B < 0.$$

Differentiating the HJB equation (4.1) with respect to *K* and to *P*, using (4.2), yields a set of differential equations for V_K^B and V_P^B as functions of time (the Pontryagin conditions). This leads to (omitting the dependence on time *t*):

(4.5)
$$-\dot{V}_K^B = \big[Y_K^B(K,\tau) - \rho - H(P)\big]V_K^B + H(P)V_K^A(K), \\ \dot{V}_P^B = \big[\rho + \gamma + h(P)\big]V_P^B + H'(P)\big[V^B(K,P) - V^A(K)\big].$$

Using the first part of (4.5) and (4.2), we get the modified Keynes-Ramsey rule:

---

[8] For simplicity, we suppress the dependence of the value functions on π from now on.

(4.6) $$\dot{C}^B = \sigma\left[Y_K^B(K,\tau)+\theta-\rho\right]C^B, \qquad \theta \equiv H(P)\left[\frac{V_K^A(K)}{U'(C^B)}-1\right].$$

The growth rate of consumption is thus proportional to the marginal net product of capital *plus* the precautionary return θ *minus* the pure rate of time preference ρ.

Using (4.3) and (4.5), we get a differential equation for the social cost of carbon τ:

(4.7) $$\dot{\tau} = \left[Y_K^B(K,\tau)+\gamma+H(P)+\theta\right]\tau - \frac{\psi H'(P)\left[V^B(K,P)-V^A(K)\right]}{U'(C^B)}.$$

If the hazard rate h is constant, i.e., $H'(P)=0$ and thus $\tau = 0$, the solution boils down to the solution of section 3b where only precautionary saving matters. If the hazard rate h depends on the stock of carbon in the atmosphere P with $H'(P)>0$, the social cost of carbon and the carbon tax τ is positive so that production substitutes away from fossil fuel use.

The accumulation of capital and the stock of atmospheric carbon can be written as

(4.8) $$\begin{aligned}\dot{K} &= Y^B(K,\tau)-\tau Y_\tau^B(K,\tau)-C^B, \qquad K(0)=K_0,\\ \dot{P} &= -\psi Y_\tau^B(K,\tau)-\gamma P, \qquad P(0)=P_0.\end{aligned}$$

In case of the decentralized market economy, the second term on the right-hand side of the capital accumulation equation gives the lump-sum rebates of the carbon tax revenues.

The steady state of the system (4.6)–(4.8) is determined by the modified golden rule of capital accumulation $Y_K^B(K^{B*},\tau^{B*})=\rho-\theta^{B*}$ with the steady-state precautionary return on capital $\theta^{B*}=H(P^{B*})\left[\frac{V_K^A(K^{B*})}{U'(C^{B*})}-1\right]$, the steady-state social cost of carbon

(4.9) $$\tau^{B*} = \frac{\psi H'(P^{B*})\left[V^B(K^{B*},P^{B*})-V^A(K^{B*})\right]}{\left[\rho+\gamma+H(P^{B*})\right]U'(C^{B*})} = \frac{\psi H'(P^{B*})\left[U(C^{B*})-\rho V^A(K^{B*})\right]}{\left[\rho+H(P^{B*})\right]\left[\rho+\gamma+H(P^{B*})\right]U'(C^{B*})},$$

the steady-state level of consumption $C^{B*}=Y^B(K^{B*},\tau^{B*})-\tau^{B*}Y_\tau^B(K^{B^*},\tau^{B*})$ and the steady-state stock of atmospheric carbon $P^{B*}=-\psi Y_\tau^B(K^{B*},\tau^{B*})/\gamma$. As in section 3, this defines a *target* steady state, because after the regime shift has occurred the system will move to the after-calamity steady state K^{A*}. The steady-state social cost of carbon τ^* depends on the gap between the before-disaster and after-disaster values, both evaluated at the targeted steady state. The precautionary return θ^* pushes up the steady state capital stock K^{B*} to be prepared for a possible shock but the social cost of carbon τ^{B*} pushes it down again to reduce the risk.

The differential equations (4.6)-(4.8) correspond to a saddle-point system with K and P as the predetermined variables and C and τ as the non-predetermined variables with the transversality condition that the system converges to the target steady state.[9]

4a. The precautionary return on capital and the social cost of carbon

The precautionary return on capital is given by the expression in (4.6). As discussed in section 3, it is proportional to the hazard of a climate calamity and through this channel increases with temperature and the stock of atmospheric carbon. This precautionary return, if necessary forced upon the market by a capital subsidy, induces precautionary saving which softens the blow to consumption if the climate calamity strikes. In the presence of climate tipping points, the social cost of carbon depends not only on the hazard rate itself but also on the *marginal* hazard of a climate calamity occurring. The rationale for the social cost of carbon is thus very different from the one that is calculated in most integrated assessment models, as the present value of marginal climate damages (see section 6 below). Integration of (4.7) gives the social cost of carbon as the present value of the expected losses in welfare that will occur when the climate disaster strikes at some future date:

$$
\begin{aligned}
\tau(t) &= \psi\int_t^\infty e^{-\int_t^s r^Y(s')ds'} \frac{H'(P(s))\left[V^B(K(s),P(s))-V^A(K(s))\right]}{U'(C^B(s))}ds \\
&= \frac{\psi\int_t^\infty e^{-\int_t^s r^U(s')ds'} H'(P(s))\left[V^B(K(s),P(s))-V^A(K(s))\right]ds}{U'(C^B(t))},
\end{aligned}
\tag{4.10}
$$

where $r^Y \equiv Y_K^B(K,\tau)+\gamma+H(P)+\theta$ denotes the social rate of interest for discounting utility in final goods units and $r^U \equiv \rho+\gamma+H(P)$ the social rate of interest for discounting utility. Note that the discount rates used to discount these expected drops in welfare resulting from a climate calamity include the rate of decay of atmospheric carbon γ and the hazard rate itself. Also, the rate used to discount final goods units includes the precautionary return on capital whereas the rate used to discount utility units does not. It follows from (4.10) that the social cost of carbon is large if the drops in future welfare from climate calamities and the marginal hazard are large. The *convexity* of the hazard function thus pushes up the social cost of carbon whilst the *level* of the hazard rate depresses it (via the higher discount rate to be used).

[9] Backward integration from the neighbourhood of the steady state or forward integration, guessing $C^B(0)$ and $\tau(0)$, are difficult because of the instability of small numerical mistakes. In the numerical illustration, we therefore reduce the dimension by fixing the precautionary return θ and the social cost of carbon τ at their steady-state levels, and apply the same algorithm as in section 3b.

4b. Illustrative simulations with hazard rates that rise with global warming

The difference with section 3b is that instead of a constant hazard rate of h = 0.025, we assume that at the initial carbon stock, P_0 = 826 GtC, the hazard rate is $H(826) = 0.025$ and that it increases to $H(1652) = 0.067$ when the carbon stock is 1652 GtC. So as the stock of atmospheric carbon doubles and global warming increases by an additional 3 degrees Celsius (given a climate sensitivity of 3), we assume that the average time it takes for the climate calamity to occur drops from 40 to 15 years. We will calibrate both a linear hazard function and a quadratic hazard function to these two points:

(4.11) $$H_1(P) = 0.025 + 5.04\times10^{-5}(P-826), \qquad H_2(P) = 0.025 + 6.11\times10^{-8}(P-826)^2.$$

Both hazard functions are depicted in figure 2. At higher stocks of atmospheric carbon the quadratic hazard function leads to higher hazard rates than the linear one. For example, if the carbon stock quadruples to 3304 GtC (and global warming increases with an additional 6 degrees Celsius) the hazard rate increases to 40 percent per annum with the quadratic and 15 percent per annum with the linear hazard function.

Figure 2: Different Specifications for the Hazard Function

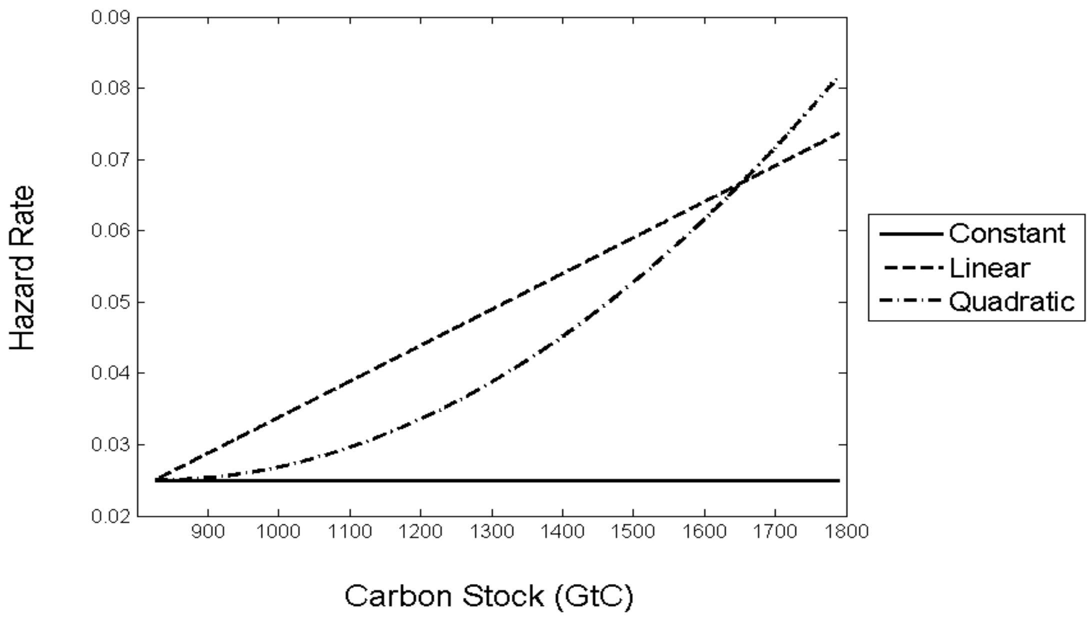


Table 1 reports the steady states for the different scenarios. As we have seen in section 3b, a constant hazard rate pushes up the capital stock to enable smoothing of consumption in the event of a sudden drop in factor productivity, but it also boosts fossil fuel use and the stock of carbon in the atmosphere. However, if the risk of catastrophe increases with global warming, this induces a target carbon tax of 22 US$/tCO2 in case of a linear and 57 $/tCO2 in case of a quadratic hazard function to curb carbon emissions and thus the risk of disaster.

Table 1: After-Disaster, Naive and Before-Disaster Target Steady states (20% TFP shock)

	After disaster	Naive solution	Constant hazard	Linear hazard	Quadratic hazard	EIS = 0.8
Capital stock (T $)	276	392	472	530	486	436
Consumption (T $)	41.3	58.6	59.4	59.6	59.2	58.9
Fossil fuel use (GtC/year)	7.3	10.4	11.0	9.7	7.7	7.7
Renewable use (million GBTU/year)	8.2	11.7	12.4	12.7	12.2	11.8
Carbon stock (GtC)	1218	1731	1838	1623	1281	1279
Precautionary return (%/year)	0	0	0.76	1.24	0.99	0.57
Social cost of carbon ($/tCO2)	0	0	0	22.4	56.9	51.0

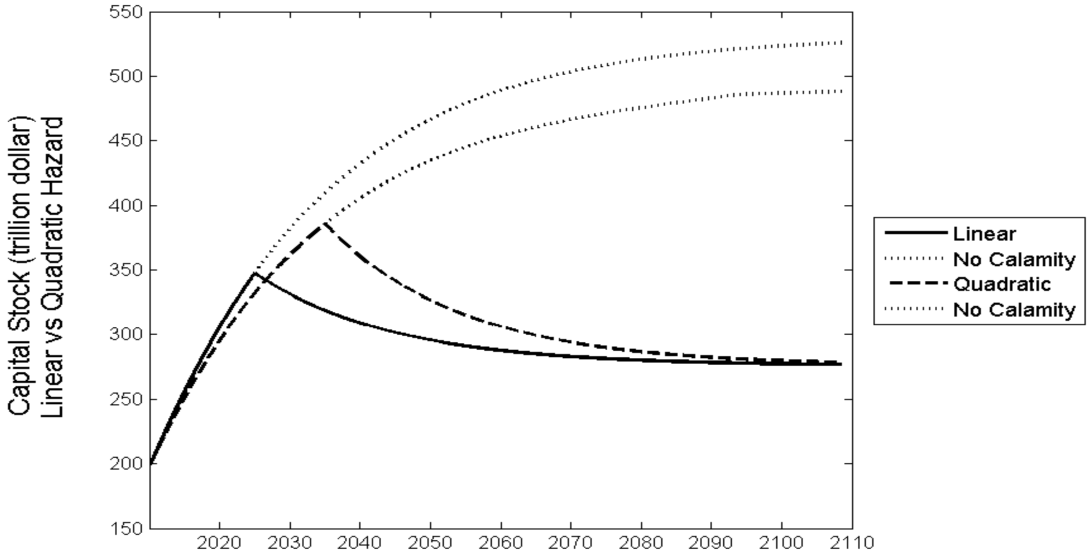
The quadratic hazard function has a higher target carbon tax and thus manages to reduce the target carbon stock substantially below the level of that with the linear hazard function (1281 instead of 1623 GtC). As a result, the corresponding hazard rate under the quadratic hazard function is lower (3.8 instead of 6.5 percent per annum) and thus the target precautionary return on capital accumulation is lower under the quadratic than under the linear hazard function (0.99 versus 1.24 percent per annum). Hence, the target before-catastrophe steady-state capital stock is only 486 trillion dollars whilst under the linear hazard function it is 530 trillion dollars. In both cases the target precautionary return on capital accumulation is higher than with a constant hazard rate (0.76 percent per annum) and hence, the target capital stock is higher. The reason is that the higher probability of a calamity increases the target level of precautionary saving.

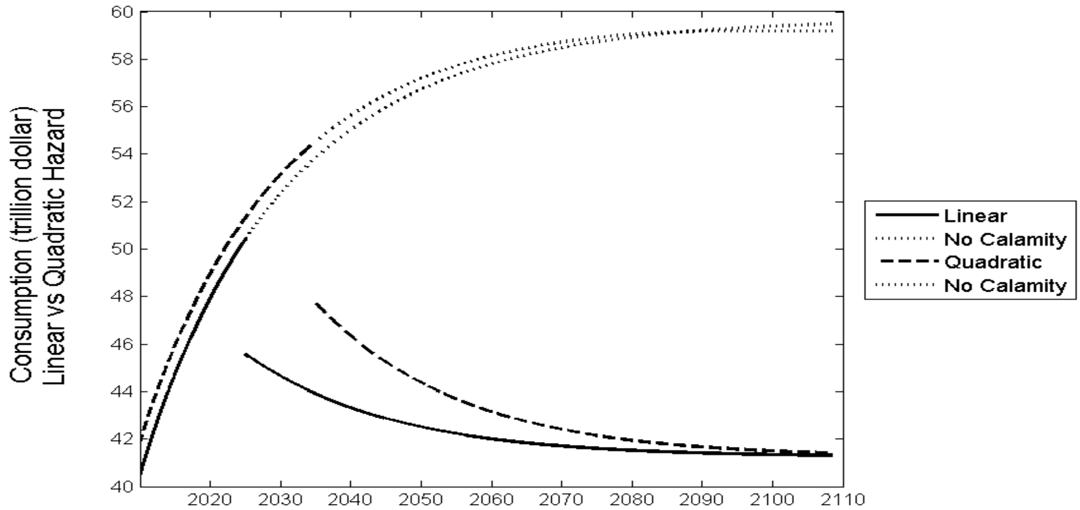
With the quadratic hazard function the marginal hazard rate is slightly larger at the target level of the carbon stock than with the linear hazard function ($H_2'(1281) = 5.56\times10^{-5} > H_1'(1623) = 5.04\times10^{-5}$).[10] This partly explains why the corresponding carbon tax is higher for the quadratic hazard function. Another reason is that the hazard rate itself is lower with the quadratic hazard function and thus the discount rate used to calculate the present value of the drop in value after a calamity (see (3.10)) is lower and consequently the target level of the carbon tax is higher.

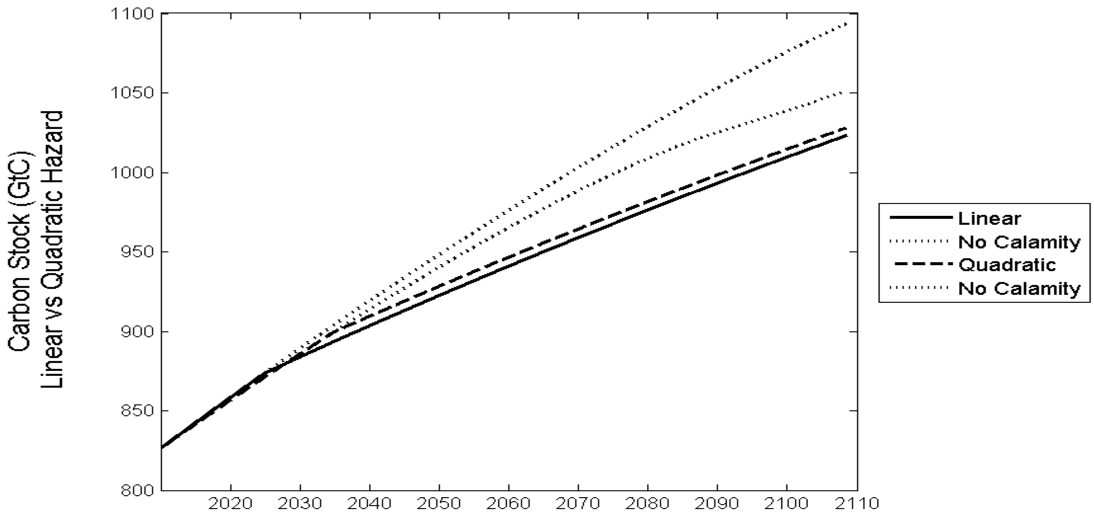
The optimal time paths of capital, consumption and the accumulated carbon stock for the linear and quadratic hazard functions are plotted in figure 3 and have the same pattern as in figure 1. However, with an increasing stock of greenhouse gases the hazard rates go up and therefore it is to be expected that the regime shifts occur earlier. This is especially the case for the linear hazard rate and therefore precautionary saving is higher in order to mitigate the

[10] With a 30% shock to total factor productivity the marginal hazard for the quadratic hazard function is smaller. Still, the optimal target carbon tax is bigger due to the lower hazard rate and thus lower discount rate used to discount expected climate damages.

Figure 3: Rational Outcomes with Linear and Quadratic Hazard Functions







effect of the shock. It follows that consumption is lower in the beginning and only catches up if the regime shift happens to occur late. In case of a quadratic hazard function, the hazard rate goes up more slowly in the relevant range. Moreover, the high carbon tax keeps the stock of carbon in the atmosphere down. It is to be expected that the regime shift occurs later than for the linear hazard rate and precautionary saving is not as high. If the linear hazard function is the more realistic one, substantial precautionary saving and a moderate carbon tax are required. If the quadratic hazard function is the more realistic one, precautionary saving is lower[11] but a higher carbon tax is required. For both hazard functions fossil fuel use is less after than before the calamity as a result of a lower level of economic activity (even though the carbon tax has been abolished) and thus accumulation of carbon in the atmosphere occurs less rapidly than before the calamity.

5. **Aversion to Risk and Inequality**

Much attention has been given to the role of time preference for precautionary climate policy. It has been argued that this should not be deduced from market outcomes but should be the result of ethical considerations (e.g., Stern, 2007) or prudence considerations (e.g., Kimball, 1990; Gollier, 2012). We have added to this debate another rationale for precaution which results from the need to be better prepared when a large-scale climate shock with an uncertain arrival time eventually hits the world.

An important other parameter indicating society's attitude to intergenerational welfare comparisons is the coefficient of relative intergenerational inequality aversion (*CRIA*) which has been set equal to $1/\sigma = 2$ in the simulations. In fact, this key parameter also corresponds to the coefficient of relative risk aversion (*CRRA*). A higher elasticity of substitution σ corresponds to both a lower *CRIA* and a lower *CRRA* and therefore has two effects. A higher elasticity of substitution σ induces a lower precautionary return θ (as in equations (4.6) and (3.4)) and thus less precautionary saving.[12] Effectively, society has a lower *CRIA* and is therefore less willing to sacrifice consumption and accumulate precautionary capital in order to be prepared for the eventual shock. As a result of using less capital in the production process, there will be less use of fossil fuel and thus carbon emissions will be lower. This yields a lower social cost of carbon. Furthermore, a lower *CRRA* implies that society is less willing to avert risk of a climate catastrophe and this depresses the social cost of carbon and the carbon tax τ too. It is not clear what the net effect on the stock of atmospheric carbon is, since a lower *CRIA* induces less precautionary capital accumulation and thus less fossil fuel

[11] For much more convex hazard functions precautionary saving can be swamped completely, so that capital is lower than under the naïve outcome to curb fossil fuel emissions and global warming.
[12] This effect does not depend on the coefficient of relative prudence, $1 + 1/\sigma$, as in Kimball (1990).

use and carbon emissions whereas a lower *CRRA* reduces the carbon tax which means that carbon emissions are reduced less and the carbon stock will be higher.

To illustrate these arguments, we increase the elasticity of intertemporal substitution from σ = 0.5 to σ = 0.8, and thus reduce the *CRIA* and the *CRRA* from 2 to 1.25: see last column of table 1. As a result of the lower *CRIA*, we see substantial reductions in the precautionary return (from 1.24 to 0.73 percent per annum under the linear hazard function and from 0.99 to 0.57 percent per annum under the quadratic hazard function). This results in substantial drops in the target before-calamity capital stocks from 530 to 462 trillion dollars under the linear hazard and from 486 to 436 trillion dollars for the quadratic hazard function. As a result of the lower *CRRA*, we see only moderate reductions in the carbon tax (from 22.4 to 22.0 \$/tCO2 under the linear and from 56.9 to 51.0 \$/tCO2 under the quadratic hazard function). The target stock of carbon in the atmosphere is curbed from 1623 to 1557 GtC under the linear hazard function and from 1281 to 1279 GtC under the quadratic hazard function. Less intergenerational inequality aversion curbs the target rate of consumption, but only by a very modest amount under each of the two hazard functions.

We have focused in sections 4 and 5 on the carbon tax that is required in case of a potential regime shift with a catastrophic shock to the total factor productivity of the economy. In section 6 we show that this cost is pushed up even further in case we also take the regular marginal damages into account that prevail in the integrated assessment literature (e.g. Tol, 2002; Nordhaus, 2008; Stern, 2007; Golosov et al., 2014).

6. Direct effect of Global Warming on Total Factor Productivity

So far, we have assumed that climate change abruptly destroys part of total factor productivity at some point in time but ignored that it decreases *A* directly and in a gradual fashion. Suppose therefore that *A* decreases with global mean temperature *Temp*, so that $A = A(Temp)$ with $A_{Temp} < 0$ and $A_{TempTemp} < 0$.[13] Nordhaus (2008) has $A = \frac{1}{1+0.00284Temp^2}$ in his DICE-2007 model, which implies climate damages of 1.7% of GDP when global warming is 2.5 degrees Celsius (i.e., *A* = 0.983). Global mean temperature increases less than proportionally with the stock of carbon in the atmosphere, which is often described by $Temp = \chi \ln(P / P_{PI}) / \ln(2),$ where χ is the climate sensitivity (the increase in global mean temperature resulting from a doubling of the atmospheric carbon stock *P*) and P_{PI} = 596.4 GtC denotes the pre-industrial stock of carbon in the atmosphere. We can write total factor

[13] We abstract from the lag between doubling the carbon stock and the rise in temperature (≈ 70 years).

productivity as a decreasing function of the carbon stock, $A(P) = A\left(\chi \ln(P/P_{PI})/\ln(2)\right)$. Golosov et al. (2014) employ a climate sensitivity of χ = 3 (cf., IPPC, 2007) and argue that the Nordhaus (2008) damages are well approximated by the reduced-form specification

$$A(P) = e^{-\xi(P-\bar{P})}\bar{A}, \tag{6.1}$$

where ξ = 0.02379 US\$/tC is the damage coefficient, $\bar{P}$ is the 2010 stock of carbon in the atmosphere, and $\bar{A}$ is total factor productivity in 2010 before future climate damages.

It follows that the after-calamity value function depends not only on the capital stock, but also on temperature or the stock of carbon in the atmosphere, $V^A(K,P,\pi)$. The social cost of carbon or carbon tax after the calamity $\tau^A(K,P,\pi) \equiv -V_P^A(K,P,\pi)/U'(C) > 0$ will thus be positive to internalize these gradual climate damages and depends on both the capital stock and the stock of atmospheric carbon. The policy rule for consumption after the calamity $C^A = C^A(K,P,\pi)$ now depends on the stock of atmospheric carbon as well as the stock of capital. Hence, the expression for the after-calamity value function (3.6) changes to

$$V^A(K,P,\pi) = \frac{U\left(C^A(K,P,\pi)\right)}{\rho} + \frac{U'\left(C^A(K,P,\pi)\right)\left[Y^A(K,P,\pi) - C^A(K,P,\pi) + \gamma\tau^A(K,P,\pi)P/\psi\right]}{\rho}. \tag{6.2}$$

As a consequence, the second part of (4.5) and (4.7) become:

$$\dot{V}_P^B = \left[\rho + \gamma + H(P)\right]V_P^B + H'(P)(V^B - V^A) - A'FV_K^B - H(P)V_P^A, \tag{6.3}$$

$$\dot{\tau} = \left[Y_K^B(K,\tau) + \gamma + H(P) + \theta\right]\tau - \frac{\psi}{U'(C^B)}\left[H'(P)(V^B - V^A) - A'FV_K^B - H(P)V_P^A\right]. \tag{6.4}$$

6a. Decomposing the social cost of carbon

Integrating (6.4), we can write the social cost of carbon as the sum of three terms:

$$\tau(t) = \underbrace{\frac{\psi}{U'\left(C^B(t)\right)}\int_t^\infty e^{-\int_t^s r^U(s')ds'} H'\left(P(s)\right)\left[V^B\left(K(s),P(s)\right) - V^A\left(K(s),P(s)\right)\right]ds}_{\text{risk-averting part of SCC}} + \underbrace{\xi\psi\int_t^\infty e^{-\int_t^s r^Y(s')ds'} AF(s)ds}_{\text{conventional Pigouvian part of SCC}} + \underbrace{\psi\int_t^\infty e^{-\int_t^s r^Y(s')ds'} H\left(P(s)\right)\left[\frac{-V_P^A\left(K(s),P(s)\right)}{U'\left(C(s)\right)}\right]ds}_{\text{raising-the stakes part of SCC}}. \tag{6.5}$$

The first term in (6.5) is the *risk-averting* component of the social cost of carbon discussed in section 4 and corresponds to the present value of all expected potential losses that may happen when a climate catastrophe strikes at some unknown point in the future. It is designed to curb the risk of a climate calamity. The second term in (6.5) is the *marginal-damages* component of the social cost of carbon. It is closely related to the conventional Pigouvian social cost of carbon and corresponds to the present value of marginal global warming damages resulting from the gradual deteriorating effect of temperature on economic production. The only difference is that the discount rate is the usual sum of the rate of interest and the rate of decay of atmospheric carbon *plus* the hazard of the catastrophe occurring *plus* the precautionary return.[14] The third term in (6.5) is the *raising-the-stakes* component of the social cost of carbon which gives the expected present value of all future increases in after-calamity welfare from curbing carbon emissions with one unit.

We have seen in section 3b that a potential climate regime shift increases discounting, with higher consumption, but also induces precaution, with higher investment, and the net effect is precautionary saving. Higher discounting also depresses the social cost of carbon but the additional components, induced by the potential climate regime shift, raise it above the conventional Pigouvian level. The risk-averting component in (6.5) is driven by the gap between the values before and after the non-marginal shock and the raising-the-stakes component is driven by the higher marginal cost of atmospheric carbon after the catastrophe.

With catastrophic damages resulting from incipient tipping points and marginal climate damages, the target before-calamity steady-state carbon tax is given by,

(6.6) $$\tau^{B*} = \tau^{B*}_{\text{conventional}} + \tau^{B*}_{\text{risk-averting}} + \tau^{B*}_{\text{raising-the-stakes}},$$

where

(6.7) $$\begin{aligned} \tau^{B*}_{\text{conventional}} &\equiv \frac{\xi\psi A(P^{B*})F(K^{B*},\tau^{B*})}{\rho+\gamma+H(P^{B*})}, \\ \tau^{B*}_{\text{risk-averting}} &\equiv \frac{\psi H'(P^{B*})\left[U(C^{B*})-\rho V^{A}(K^{B*},P^{B*})\right]}{\left[\rho+\gamma+H(P^{B*})\right]\left[\rho+H(P^{B*})\right]U'(C^{B*})}, \\ \tau^{B*}_{\text{raising-the-stakes}} &\equiv \frac{H(P^{B*})\tau^{A}(K^{B*},P^{B*})U'\left(C^{A}(K^{B*},P^{B*})\right)}{\left[\rho+\gamma+H(P^{B*})\right]U'(C^{B*})} \end{aligned}$$

[14] With the negative exponential specification of climate damages, logarithmic utility, Cobb-Douglas production, and 100% depreciation of capital each period, the conventional expression for the social cost of carbon in a discrete-time Ramsey model with no climate catastrophes is proportional to GDP or gross output, i.e., $\tau = \psi\xi AF/(\rho+\gamma)$ (cf. Golosov et al., 2014). The ratio of the social cost of carbon to GDP is proportional to the damage coefficient ξ and inversely proportional to the sum of the discount rate ρ and the rate of decay of atmospheric carbon γ.

and $A(P)F(K,\tau)$ is the maximum level of gross output if fossil fuel and renewable use are at their optimal levels. As before, the first part of this expression is driven by the marginal climate damages specified in (6.2), the second part curbs the risk of a catastrophe, and the third part is the raising-the-stakes term.

6b. Illustrative calculations for different sizes of productivity catastrophes[15]

The first three columns of table 2 present the steady-state values of the after-calamity and before-calamity outcomes for both a linear and a quadratic hazard function and can be compared to the corresponding outcomes in table 1 to show the effects of adding the usual marginal climate damages. As a result of the additional marginal damages, after the calamity there is a conventional carbon tax of 11 \$/tCO2 which curbs the stock of carbon in the atmosphere from 1502 to 1107 GtC. This has a slightly depressing effect on capital and consumption after the calamity. Before the disaster strikes, there is a much bigger carbon tax for the linear hazard function (55 instead of 22 \$/tCO2) than for the quadratic hazard function (71 instead of 57 \$/tCO2), as compared with the situation that there are no marginal climate damages. This before-calamity carbon tax is in both cases dominated by the catastrophe components, especially the risk-averting component. As a consequence of introducing marginal climate damages, the carbon stock is much more reduced with the linear hazard function (from 1623 to 1287 GtC) than with the quadratic hazard function where global warming has already been curbed without taking account of marginal damages as in table 1 (from 1281 to 1161 GtC). The differences between the effects of the two hazard functions thus become smaller. Internalizing marginal as well as catastrophic climate damages curbs global warming and thus lessens the need for precautionary capital accumulation, especially for the linear hazard function (from 530 to 492 trillion dollars for the linear hazard function and from 486 to 465 trillion dollars for the quadratic hazard function). This reflects that the precautionary return, which to a large extent is driven by the hazard rate and the carbon stock, drops by much more with the linear hazard function (from 1.23 to 1.10 percent per annum) than with the quadratic hazard function (from 0.99 to 0.90 percent per annum). The convex hazard function has a lower marginal hazard rate now at the target level of the carbon stock (4.1×10^{-5} for the quadratic hazard and 5.0×10^{-5} for the linear hazard function) but the risk-averting component of the carbon tax is still higher because the hazard rate is lower. It is, however, not so much higher now because the levels of the hazard rates differ less between the linear hazard function and the quadratic hazard function.

[15] Appendix 2 develops the after-calamity policy rules for consumption and the carbon tax as functions of the capital stock and the stock of atmospheric carbon. These approximations are log-linearizations around the steady state and in that sense extensions to higher dimensions of the procedure in section 3a.

Table 2: Target Steady States with Catastrophic and Marginal Climate Damages

	Naïve solution	20% shock in TFP			10% shock in TFP		
		After shock	Linear	Quadratic	After shock	Linear	Quadratic
Capital stock (T $)	378	271	492	465 (426)	323	431	421
Consumption (T $)	57.1	40.8	58.3	58.2 (58.0)	48.7	57.8	57.8
Carbon stock (GtC)	1502	1107	1287	1161 (1119)	1303	1425	1320
Temperature (^{o} Celsius)	4.00	2.68	3.33	2.88 (2.72)	3.38	3.77	3.44
Precautionary return (%/year)	0	0	1.10	0.90 (0.56)	0	0.57	0.49
Social cost of carbon ($/GtCO2)	15.4	11.0	54.8	71.2 (73.1)	13.2	29.8	41.5
marginal	15.4	11.0	4.3	5.7 (8.5)	13.2	3.8	4.7
risk averting	0	0	35.0	51.9 (54.8)	0	12.4	24.2
raising stakes	0	0	15.4	13.7 (9.8)	0	13.7	12.5

Note: In brackets are results for when the hazard rates are halved for each level of *P*.

The remaining columns of table 2 show the consequences for these outcomes of a catastrophic drop of 10 percent in total factor productivity. Since there is more economic activity and more carbon emissions after the calamity than with the 20 percent disaster, the after-calamity social cost of carbon jumps up from 11 to 13 $/tCO2. Before the calamity strikes, both risk-averting components of the social cost of carbon fall relative to the outcome with a 20 percent calamity but the biggest falls occur for the risk-averting components (from 35 to 12 $/tCO2 for the linear and from 52 to 24 $/tCO2 for the quadratic hazard function). Although there is more economic activity with a 10% shock after the calamity, the initial capital stock is lower because there is less precautionary capital accumulation. The precautionary return drops more or less in line with the drop in total factor productivity, so that less precautionary capital accumulation occurs before the calamity. Despite the lower initial capital stock after the calamity, the lower structural drop of only 10% in total factor productivity leads to bigger carbon stocks and more global warming.

6c. Sensitivity with respect to the size of the hazard rates

There is lots of uncertainty about the exact specification of the hazard rate for a total factor productivity catastrophe, so it is good to examine the sensitivity with respect to the hazard rate. The figures in brackets in table 2 report for a quadratic hazard function the effects of lowering the base level of the hazard rate from 0.025 to 0.0125 (i.e., increasing the average time before the catastrophe strikes from 40 to 80 years) and adjusting the hazard function so that a doubling of the atmospheric carbon stock reduces the average time it takes for the climate calamity to occur to 30 years. The lower hazard of a catastrophe lowers precautionary saving, but increases the social cost of carbon by a small amount. The marginal hazard rates are lower but the levels of the hazard rates are lower as well and the last effect dominates so that the social cost of carbon increases, as we have seen before. With a linear hazard function (not reported in table 2) there is still substantial precautionary capital accumulation after halving the hazard rates (437 T$). The social cost of carbon is a little higher for the linear hazard function (77.2 $/tCO2) than for the quadratic hazard function, mainly due to the risk-averting component as the marginal hazard rate for the linear hazard function is higher than for the quadratic hazard function ($2.52\times 10^{-5} > 1.78\times 10^{-5}$). The interplay between the marginal hazard rate and the level of the hazard rate can work out in different ways but the message is the same: a big carbon tax is needed to curb the risk of a climate catastrophe.

7. Carbon and Capital Catastrophes

Here we first consider an alternative regime shift with climate tipping points and then discuss other types of economic catastrophes which correspond to recoverable shocks.

7a. Catastrophic change in the climate sensitivity

The catastrophe discussed so far was an economic catastrophe which was triggered by global warming. It is of interest to consider catastrophes which directly increase global warming due to an abrupt positive feedback in the carbon cycle. A simple shortcut to modeling this is to have a hazard of a sudden increase in the climate sensitivity χ.[16] In that case, after the disaster the stock of atmospheric carbon and temperature will increase and this will lead to a gradual erosion of total factor productivity. The damages (6.1) and the climate sensitivity of 3 used in (6.1) imply that global warming decreases total factor productivity according to:

(7.1) $$A\left(Temp\right) = \bar{A}\exp\left[-\xi\left(2^{Temp/3}P_{PI} - \bar{P}\right)\right].$$

Equation (7.1) is our primitive. A catastrophic increase in the climate sensitivity from 3 to χ increases the temperature response and thus, for a given stock of atmospheric carbon, increases post-calamity climate damages. This can be seen from the reduced form obtained by substituting the temperature response into (7.1):

(7.2) $$A(P) = \bar{A}\exp\left[-\xi\left(\left(P / P_{PI}\right)^{\frac{\chi}{3}} P_{PI} - \bar{P}\right)\right],$$

A higher climate sensitivity $\chi > 3$ increases, for a given carbon stock, the temperature response but also pushes up damages ($A'(P) = -\xi(\chi/3)\left(P/P_{PI}\right)^{(\chi-3)/3} A(P) < -\xi A(P) < 0$). Figure 4 plots (7.2) and indicates how much stronger total factor productivity decreases with the stock of atmospheric carbon, after a shock to climate sensitivity.

[16] An alternative is to have a sudden and abrupt increase in the damage parameter ξ. Yet another alternative is to allow for sudden and abrupt positive feedback by changing the system dynamics of the carbon cycle as discussed in Lemoine and Traeger (2014) and van der Ploeg (2014).

Figure 4: Effects of Climate Sensitivity on Global Warming Damages

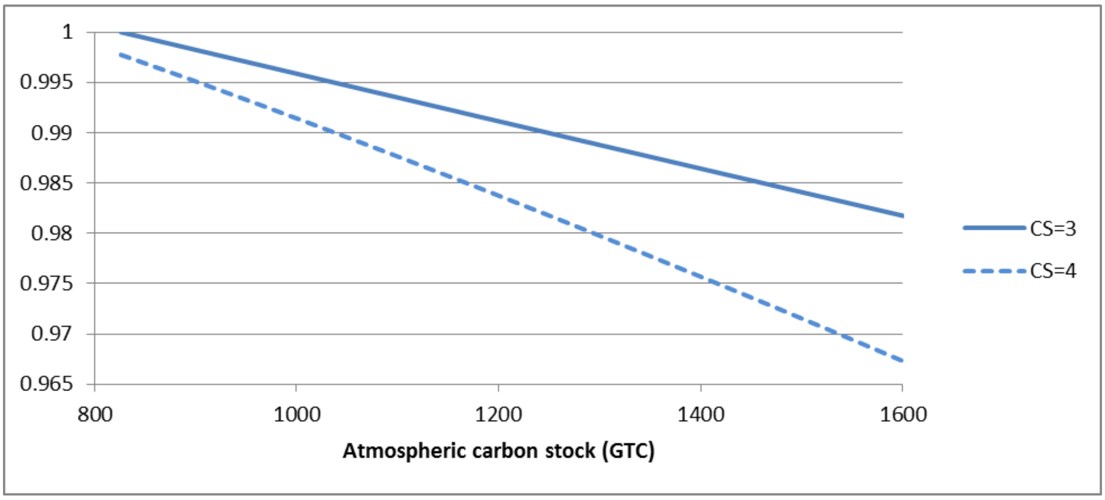


Figure 4 shows that a bigger climate sensitivity of 4 (dashed lines) than our benchmark of 3 (solid lines) leads, for a given carbon stock, to bigger damages, with the hazard function unaltered. A sudden increase in the climate sensitivity thus leads to a tougher policy challenge. A doubling of the initial carbon stocks induces a 2.0 percent drop in total factor productivity if the climate sensitivity is 3 and a 3.4 percent drop if the climate sensitivity is 4.

Table 3 shows the rational before-calamity and after-calamity responses when the climate sensitivity is not 3 but 4, damages are given by (7.2), there are no factor productivity shocks, and the expected time of the sudden increase in the climate sensitivity increases with global warming in the usual way.[17] The after-calamity response requires a much higher carbon tax than the naïve solution (27 instead of 16 $/tCO2). Before the calamity also a substantial target carbon tax is required (27 $/tCO2). The level is not different from the after-calamity carbon tax, but there is a shift from marginal to raising-the-stakes damages.

Table 3: Target Steady States for Capital and Carbon Catastrophes

(Quadratic Hazard Functions)

	Naïve solution	*CS* jumps from 3 to 4		20% jump in *P*	20% drop in *K*
		After calamity	Before calamity		
Capital stock (T $)	378	372	382	381	433
Consumption (T $)	57.1	56.3	57.3	57.1	57.6
Carbon stock (GtC)	1502	1374	1400	1490	1534
Temperature (^{o} Celsius)	4.00	4.82	3.69	3.96	4.09
Precautionary return (%/year)	0	0	0.05	0.03	0.57
Social cost of carbon ($/GtCO2)	15.4	26.7	26.5	16.9	18.5
marginal	15.4	26.7	4.1	3.8	3.8
risk averting	0	0	2.2	1.4	2.5
raising stakes	0	0	20.2	11.7	12.2

[17] Responses with a linear hazard function hardly differ from those with a quadratic hazard function.

Putting up more carbon in the atmosphere means that, once the calamity has struck, the damages are much higher and thus welfare after the calamity is much lower. To correct for this, the raising-the-stakes component of the carbon tax is substantial. This in itself reduces fossil fuel emissions and thus lessens the need for correction of marginal global warming damages. Before the calamity a small precautionary return is required and as a consequence the target capital stock is a little higher than in the naïve outcome (despite the higher carbon tax). The precautionary return is only small because the shock to total factor productivity via the shock to climate sensitivity is not so big. Interestingly, just as with the total factor productivity shocks discussed in section 6, the stock of atmospheric carbon decreases after the calamity. But the difference is that the after-calamity temperature is now higher instead of lower than with the total factor productivity shock presented in table 2.

7b. Recoverable capital and carbon catastrophes

So far, we have considered catastrophic drops in total factor productivity or the climate sensitivity whose expected date of arrival increases with global warming. These are both examples of regime shifts. These should be contrasted with *recoverable* catastrophes, which may lead to a sudden destruction of part of the stock of capital used in the production process or a sudden release of carbon in the atmosphere. These catastrophes are not regime shifts, since the economy eventually recovers from them and thus the steady state of the after-calamity regime is unaffected. With the capital-stock disaster, the after-calamity system is the same as the before-calamity system but the capital stock is set back to a lower level of accumulation whilst with the total factor productivity shock the after-calamity system never recovers. With a hazard of a sudden increase in the stock of atmospheric carbon resulting from a sudden eruption of greenhouse gases, there will be a temporary increase in global warming with the gradual damages (6.3) or (7.1) that follow from that. Both these recoverable shocks are temporary and thus do not affect the after-calamity steady state.

Table 3 also shows the effects of a hazard of a 20% increase in the stock of atmospheric carbon. Again, there is almost no precautionary saving with almost no effect on the rate of consumption and the carbon tax is small (17 \$/tCO2) as it is a recoverable catastrophe rather than a permanent regime shift. The stock of atmospheric carbon and global warming are reduced somewhat below the naïve outcome (1490 instead of 1502 GtC). Both carbon and climate sensitivity catastrophes are less imminent than a productivity catastrophe resulting from, say, a reversal of the Gulf Stream (e.g., Lenton and Ciscar, 2013). Lowering the hazard rates by a factor 5 to 20 necessitates a lower social cost of carbon.

Finally, table 3 shows the effects of a hazard of a 20% destruction of the capital stock. Now there is a sizeable precautionary return (0.57% per year) and substantial capital accumulation to be prepared for a sudden disaster (433 instead of 378 trillion dollars). Since this goes together with more fossil fuel use, there is despite a higher carbon tax (18.5 instead of 15.4 \$/tCO2) *more* global warming than in the naïve outcome (1534 GtC).

Before the calamity strikes, the risk-averting components of the carbon tax are tiny as the damages done by the catastrophe are temporary. Comparing with the naïve outcome, the raising-the-stakes component takes over most of the marginal damages of the carbon tax, as we have also seen for the shock in climate sensitivity. A difference between the two types of catastrophes is that there is substantial precautionary capital accumulation before the calamity strikes with an impending capital disaster but unsurprisingly hardly any for an impending carbon stock disaster.

8. Investment in Adaptation Capital

Our analysis shows that the risk of a climate tipping point induces precautionary capital accumulation so as to be prepared when the shock eventually hits. Unfortunately, this leads to burning more fossil fuel and a higher risk of a regime shift. Alternatively, one could invest in adaptation capital L (e.g., dykes and other water defences) as well as in productive capital. Adaptation capital L reduces the shock π in total factor productivity, so that $\pi = \Pi(L)$ with $\Pi'(L) < 0$. Suppose that both types of capital are *ex ante* perfect substitutes and that they both have the same depreciation rate, δ. We assume a "putty-clay" technology. Hence, from the tipping point onwards, L is constant as there is no need to cover for depreciation and L cannot be turned *ex post* into productive capital.[18] To keep matters simple, let us abstract from marginal climate damages as in sections 6-7, so that the after-calamity function is not a function of the carbon stock. Hence, the rule for optimal consumption and the value function after the event become functions of, on the one hand, productive capital $K - L$, and, on the other hand, adaptation capital L, i.e., $C^A(K - L, \Pi(L))$ and $V^A(K - L, \Pi(L))$.

The right-hand side of the current-value HJB equation (3.12) in the value function V^B before the event then changes to

$$(8.1)\ \Omega \equiv \underset{C,L}{\text{Max}} \left[U(C) + V_K^B(K,P)\left\{ Y^B(K-L,P) - C \right\} - H(P)\left\{ V^B(K,P) - V^A\left(K-L,\Pi(L)\right) \right\} \right],$$

with the condition for the optimal stock of adaptation capital,

$$(8.2)\qquad Y_{K-L}^B(K-L,P) = \frac{H(P)\left[V_\pi^A\left(K-L,\Pi(L)\right)\Pi'(L) - V_{K-L}^A\left(K-L,\Pi(L)\right) \right]}{V_K^B(K,P)}.$$

We define $\hat{V}_L^A(K,L) \equiv V_\pi^A\left(K-L,\Pi(L)\right)\Pi'(L) - V_{K-L}^A\left(K-L,\Pi(L)\right) > 0$ and suppose $\hat{V}_L^A > 0,$ so that the total marginal return on adaptation capital is decreasing in *L* and increasing in *K*, i.e. $V_{LL}^A < 0$ and $V_{LK}^A > 0.$ [19] The share of adaptation capital *L* is zero if the net effect of a lower shock Π(*L*) and a lower productive capital *K* - *L* after the event cannot be positive. Otherwise, condition (8.2) requires that the expected marginal return on adaptation capital, converted into final goods units, must be equal to the marginal productivity of capital used in production. In that case, the optimal amount of adaptation capital increases with both the aggregate capital stock and the atmospheric carbon stock or global warming:

(8.3) $$L = L(P,K) \text{ with } L_P > 0 \text{ and } L_K > 0.$$

If the hazard rate is invariant to global warming, equation (8.3) becomes $L = L(h,K)$ with $L_h > 0$ and $L_K > 0.$ We have seen in section 3b that due to precautionary saving an increase in the hazard rate *h* induces a higher level of aggregate capital *K*. Here we see that a bigger risk of catastrophe leads to a higher level of adaptation capital *L*. In general, the hazard rate depends on global warming in which case the optimal level of adaptation capital and the aggregate capital stock will increase with the stock of carbon in the atmosphere.

9. **Conclusion**

Climate change will probably manifest itself in the future as a regime shift in the climate system resulting from a climate tipping point (Lenton and Ciscar, 2013) and this means that the potential shock should be an important driver of the social cost of carbon. It has also been argued that spending money now to slow global warming should not be conceptualized primarily as being about optimal consumption smoothing so much as an issue about how much insurance to buy to offset the small chance of a ruinous catastrophe (Weitzman, 2007). This paper takes up these challenges and considers economic growth with a potential regime shift which results in a non-marginal shock to the total factor productivity of the economic system. It is shown that this has important implications for optimal policy.

The most striking result is that both precautionary saving and a carbon tax are needed. Precautionary saving may be picked up by the market but if not, a capital subsidy is needed. More capital is required to be prepared for the shock and to smooth consumption over time

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[18] An alternative is not to make the putty-clay assumption and suppose that *L* is malleable after the disaster in which case one has $C^A(K, \Pi(L))$ and $V^A(K, \Pi(L))$.

[19] The condition (8.2) follows from $\Omega_L = -V_K^B(K)Y_{K-L}^B(K-L) + H(P)\hat{V}_L^A(K,L) = 0.$ The second-order condition for a maximum is $\Omega_{LL} = V_K^B Y_{K-L,K-L}^B + H(P)\hat{V}_{LL}^A < 0.$ Since $\Omega_{LP} = H'(P)\hat{V}_L^A > 0,$ we have $L_P = -\Omega_{LP}/\Omega_{LL} > 0.$ Since $\Omega_{LK} = -V_K^B Y_{K-L,K-L}^B + H(P)\hat{V}_{LK}^A - V_{KK}^B Y_{K-L}^B > 0,$ so $L_K = -\Omega_{LK}/\Omega_{LL} > 0.$

and less fossil fuel use is required to lower the risk of climate change. In combination with a standard Pigouvian tax, this carbon tax becomes even higher because the regime shift also increases the marginal cost of fossil fuel emissions after the shock. It may be useful to specifically invest in adaptation capital lowering the shock in case it occurs, if these benefits outweigh the loss of productive capital before and after the shock. A higher elasticity of substitution or a lower aversion to risk and to intergenerational inequality lowers both this carbon tax and precautionary saving.

Regime shifts are characterized in our analysis as structural shocks to total factor productivity or to climate sensitivity. We show that the effects on optimal policy are much larger than in case of shocks to the capital stock or the stock of atmospheric carbon. The latter shocks only set back growth temporarily and do not change the steady state of the economy. As a consequence, there is less need for optimal policy to curb the risk of tipping.

Another important implication for optimal policy is that a constant marginal hazard rate requires a balanced package of precautionary saving and a carbon tax. However, for an increasing marginal hazard rate a higher carbon tax is needed so that the stock of greenhouse gases is kept down and precautionary saving can be more modest. This implies that if the idea is that climate change may still be far away but that it may be approaching more rapidly if the stock of greenhouse gases increases, optimal climate policy requires a high carbon tax.

Our results illustrate the importance of looking at the effects of catastrophes on climate policy, and especially the need for both a carbon tax to curb the risk of such hazards occurring and precautionary capital accumulation to be better prepared when such disasters strike. Our illustrative calculations indicate that the resulting catastrophe components and especially the risk-averting component of the carbon tax are substantial compared with the usual carbon tax calculated by Nordhaus (2008) and Golosov et al. (2014) based on only marginal damages.

Our conclusion is that destruction of non-recoverable factors of production by climate catastrophes requires urgent action both to mitigate the risks of such actions occurring and to be better prepared for them. Our calculations suggest that conventional marginal global warming damages necessitate a global carbon tax of 15 \$/tCO2.[20] Allowing for an impending negative shock of 20% to total factor productivity with an expected arrival in 40 years and falling to 15 years as the carbon stock doubles boosts this figure to 55 or 71 \$/tCO2 for a linear or quadratic hazard function, respectively. In addition, a precautionary return of about 1% per annum induces capital accumulation of 30% and 23%, respectively. A -10% shock halves the precautionary returns and requires a global carbon tax of 30 to 40 \$/tCO2. Halving the hazard rates for any given carbon stock hardly changes the required carbon tax, but does diminish the precautionary return considerably to 0.16% per annum. Catastrophic shocks to recoverable factors of production such as the capital stock are only temporary and thus require much less action. On the other hand, catastrophes that unleash hitherto dormant positive feedback loops in the carbon cycle may need much more risk-averting action to prevent runaway global warming from occurring.

We believe catastrophes offer a better narrative and dialogue with policy makers to convince them of the need to implement ambitious climate policy. However, as made clear by Lenton and Ciscar (2013) and others, we need much more research and dialogue with climate scientists to obtain better information on the different types of productivity, capital, carbon and climate sensitivity catastrophes that can occur and the different hazard and marginal hazard rates of such disasters occurring. We also need to be more precise about how long it will take before the impact of the catastrophe is fully felt and adopt our analysis for this purpose (cf., Lontzek et al., 2014). For example, drops in total factor productivity resulting from reversal of the Atlantic Meriodonal Overturning Circulation (think of the adverse economic impact of reversal of the Gulf Stream for Northern Europe) may take a century, full disappearance of the Greenland or the West Antarctic Ice Sheet may take as long as three centuries, and release of permafrost may take close to a century. Also, desertification of agricultural land may take many decades before its full impact is felt. Other damages such as flooding of cities or city conglomerates and increased occurrence of storms and droughts may intensify more gradually with global warming and need not be modelled as regime shifts.

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[20] This is not too different from the latest ballpark measure of 12 $/tCO2 (including uncertainty, equity weighting and risk aversion) given by Nordhaus (2014).

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Appendix 1: Calibration

We use a utility function U with constant elasticity of intertemporal substitution σ = 0.5, a pure rate of time preference ρ = 0.014, output AF before and $(1-\pi)AF$ after the disaster, the Cobb-Douglas production function $F(K,E,R) = K^{\alpha}(E^{\omega}R^{(1-\omega)})^{\beta}$, and depreciation rate δ = 0.05. Table A1 presents the parameters and initial values of our model that are calibrated to figures for the world economy for the year 2010. Data sources are the BP Statistical Review and the World Bank Development Indicators.

Table A1: Calibration of the Ramsey Model with a Climate Tipping Point

Pure rate of time preference, ρ	0.014
Elasticity of substitution, σ	0.5 (and 0.8)
Depreciation rate of capital, δ	0.05
Share of capital in value added, α	0.3
Share of fossil fuel in value added, $\beta\omega$	0.0626
Share of fossil fuel in energy, ω	0.9614
Share of energy in value added, β	0.0651
Share of labour in value added, $1 - \alpha - \beta$	0.6349
Total factor productivity, A	11.9762
Eventual climate shock, π	0.2 (and 0.1)
Fraction of carbon staying up, ψ	0.5
Natural decay of carbon, γ	0.003
Cost of fossil fuel, d	9 US \$/million BTU = 504 US \$/tC
Cost of renewable, c	18 US\$/million BTU
Initial level of GDP, Y_0	63 trillion US\$
Initial level of capital, K_0	200 trillion US\$
Initial fossil fuel use, E_0	468.3 million GBTU = 8.3 GtC
Initial renewable use, R_0	9.4 million GBTU
Initial stock of carbon, P_0	826 GtC = 388 ppm by vol. CO2
Equilibrium climate sensitivity, χ	3 (and 4)

We set the 2010 capital stock to 200 trillion US\$ which is below the steady-state level of 295 trillion US\$ that is consistent with the 2010 figure for world GDP of 63 trillion US \$ (i.e., $63/(\rho+\delta)$) to reflect that the global economy is still catching up. Bio-fuels production in 2010 was 1.45% of total oil production. Counting also nuclear and other renewable sources of energy might triple this figure. Hydro-electricity production is tiny. We thus take 2% for the share of renewable energy (9.4 million GBTU) in total fossil fuel energy output (468.3 million GBTU) in 2010.

Prices of oil, natural gas and coal are 14, 6.5 and 4 US$, respectively, per million BTU and we set the average cost of fossil fuel equal to 9 US$ per million BTU. We set the cost of the renewables twice as high. Hence, the budget share ω of fossil fuel in total energy is (9×468.3)/(9×468.3+18×9.4) = 0.9614. The share of fossil fuel in GDP is $\beta\omega/(1-\beta)$ = (9×468.3)/63000 = 0.0669, so that β = 0.0651 is the share of total energy in gross output and $\omega\beta$ = 0.0626 the share of fossil fuel in gross output. We use a share of capital α in gross output of 0.3 so that the share of labour in value added is 0.6349. We calibrate total factor productivity A to match 2010 world GDP, so $(1-\beta)\times 63\times 200^{-\alpha}\times 468.3^{-\beta\omega}\times 9.4^{-\beta(1-\omega)}$ = 11.9762.

To convert we use a conversion factor for fossil fuel of 1 GtC = 56 million GBTU. We measure fossil fuel in GtC, so the emission-input ratio equals one. Hence, the market price can be expressed as 504 US$ per ton of carbon. Other conversion factors are: 1 ppm by volume CO2 = 2.13 GtC and 1 kg carbon = 44/12 = 3.67 kg CO2. The fraction of carbon ψ that does not return quickly to the surface of the earth is 0.5.

Our production function gives the optimal choice of fossil-fuel use and renewable use

(A.1)
$$E = \frac{\beta\omega\tilde{A}F}{d+\tau}, \quad R = \frac{\beta(1-\omega)\tilde{A}F}{c},$$

and output *net* of total input costs and capital depreciation

(A.2)
$$Y(K,\tau) = (1-\beta)\left[\tilde{A}\beta^{\beta}\left(\frac{\omega}{d+\tau}\right)^{\beta\omega}\left(\frac{1-\omega}{c}\right)^{\beta(1-\omega)}\right]^{\frac{1}{1-\beta}} K^{\frac{\alpha}{1-\beta}} - \delta K.$$

The modified golden rule of capital accumulation $Y_K(K^*,\tau^*) = \rho - \theta^*$ yields the steady-state before-catastrophe and after-catastrophe capital stocks:

(A.3)
$$K^{B*} = \left[A\beta^{\beta}\left(\frac{\alpha}{\rho+\delta-\theta^{B*}}\right)^{1-\beta}\left(\frac{\omega}{d+\tau^{B*}}\right)^{\beta\omega}\left(\frac{1-\omega}{c}\right)^{\beta(1-\omega)}\right]^{\frac{1}{1-\alpha-\beta}},$$
$$K^{A*} = \left[(1-\pi)A\beta^{\beta}\left(\frac{\alpha}{\rho+\delta}\right)^{1-\beta}\left(\frac{\omega}{d+\tau^{A*}}\right)^{\beta\omega}\left(\frac{1-\omega}{c}\right)^{\beta(1-\omega)}\right]^{\frac{1}{1-\alpha-\beta}}.$$

where $\tilde{A} = (1-\pi)A$ and θ^* = 0 after the catastrophe, and $\tilde{A} = A$ (and τ^* = 0 in case of a constant hazard rate) before the catastrophe.

Appendix 2: Steady States, After-Calamity Policy Rules and Transient Paths

No marginal climate damages:

With the parameter values and functional forms discussed in appendix 1 and a 20% TFP shock but no marginal climate damages, the after-event steady state follows from (A.4) with $\theta^* = \tau^* = 0$: K^{A*} = 276 T US\$, C^{A*} = 41.3 T US\$. Equations (3.9) and (3.10) yield:

$$\text{(A4)} \qquad \phi = \left[\frac{1}{2}\rho + \frac{1}{2}\sqrt{\rho^2 + 4\sigma(\rho+\delta)\left(\frac{1-\alpha-\beta}{1-\beta}\right)\frac{C^{A*}}{K^{A*}}} \right] \frac{K^{A*}}{C^{A*}} = 0.4310,$$

so that C^A is approximated by $C^A(K) = 3.6589 K^{0.4310}$. This policy rule gives explicit expressions for the marginal value function $V_K^A(K) = U'\left(3.6589 K^{0.4310}\right)$ from (3.4) and for the value function $V^A(K)$ from (3.6). If the elasticity of intertemporal substitution is increased to $\sigma = 0.8$, the optimal path $C^{A}(K)$ can be approximated by $C^A(K) = 2.0808 K^{0.5314}$.

Besides (A3) for the capital stock (with $\pi = 0$) the steady-states of the other variables before the event follow from the following system of equations:

$$\text{(A5)} \qquad \theta^* = H(P^{B*})\left[\left(\frac{C^{B*}}{C^A(K^{B*})} \right)^{1/\sigma} - 1 \right], \quad \tau^* = \frac{\psi H'(P^{B*})(C^{B*})^{1/\sigma} \left[\frac{(C^{B*})^{(1-1/\sigma)}}{1-1/\sigma} - \rho V^A(K^{B*}) \right]}{\left[\rho + H(P^{B*})\right]\left[\rho + \gamma + H(P^{B*})\right]},$$

$$C^{B^*} = \left[\left(1 - \beta + \frac{\beta\omega\tau^*}{d+\tau^*} \right)\left(\frac{\delta + \rho - \theta^*}{\alpha} \right) - \delta \right] K^{B*}, \quad P^{B*} = \frac{\psi}{\gamma}\left(\frac{\beta\omega}{d+\tau^*} \right)\left(\frac{\delta+\rho-\theta^*}{\alpha} \right) K^{B*},$$

where $\tau^* = 0$ in case of a constant hazard rate h.

With marginal climate damages:

The after-calamity system for our functional forms (also allowing for general climate sensitivity χ, different from 3, and making use of expression (7.2)) is given by:

$$\text{(A6)} \qquad \begin{aligned} \dot{K} &= Y^A(K,P,\tau) + \tau E - C, \\ \dot{P} &= \psi E - \gamma P, \\ \dot{C} &= \sigma\left[A^A(P) F_K(K,E,R) - \delta - \rho \right] C, \\ \dot{\tau} &= \left[A^A(P) F_K(K,E,R) + \gamma - \delta \right]\tau - \psi \xi^* A^A(P) F(K,E,R), \end{aligned}$$

where $\xi^* \equiv \xi \frac{\chi}{3} (P/P_{PI})^{\frac{\chi-3}{3}}$, $A^A(P) = (1-\pi)\bar{A}\exp\left[-\xi\left((P/P_{PI})^{\frac{\chi}{3}} P_{PI} - \bar{P} \right) \right]$,

$E=\dfrac{\beta\omega A^A(P)F}{d+\tau}$ and $Y^A(K,P,\tau)$ with $Y_K^A=\rho$, $Y_P^A=-\xi^* A^A F$ and $Y_\tau^A=-E^A$, where all terms are evaluated at the after-calamity steady state. Define the vector $x\equiv(K,P,C,\tau)'$, $\Gamma_1\equiv\dfrac{1-\alpha-\beta}{1-\beta}$ and $\Gamma_2\equiv\xi^*\psi-\alpha\dfrac{\tau}{K}$. We thus obtain the linearized after-calamity system $\dot{x}\cong\Upsilon(x-x^{A*})$ with state-transition matrix:

$$\text{(A7)}\quad \Upsilon=\begin{pmatrix} \rho+\tau E_K & -\xi^*\left(A^AF+\dfrac{\tau E}{1-\beta}\right) & -1 & \tau E_\tau \\ \psi E_K & -\dfrac{\psi\xi^* E}{1-\beta}-\gamma & 0 & \psi E_\tau \\ -\sigma\dfrac{C}{K}(\rho+\delta)\Gamma_1 & -\dfrac{\sigma\xi^* C(\rho+\delta)}{1-\beta} & 0 & -\dfrac{\beta\omega\sigma C(\rho+\delta)}{(1-\beta)(d+\tau)} \\ -\left(\dfrac{\tau}{K}+\Gamma_2\right)(\rho+\delta) & \left[\dfrac{\Gamma_2}{1-\beta}-\dfrac{\psi}{P}\left(\dfrac{\chi-3}{3}\right)\right]\xi^* A^AF & 0 & \rho+\gamma+\dfrac{EF\Gamma_2}{1-\beta} \end{pmatrix},$$

where we have used

$$E=\frac{\beta\omega}{d+\tau}\left[A^A(P)K^\alpha\beta^\beta\left(\frac{\omega}{d+\tau}\right)^{\beta\omega}\left(\frac{1-\omega}{c}\right)^{\beta(1-\omega)}\right]^{\frac{1}{1-\beta}},\ E_K=\frac{\alpha}{1-\beta}\frac{E}{K},$$

$$E_P=-\frac{\xi^*}{1-\beta}E,\ E_\tau=-\frac{1-\beta(1-\omega)}{1-\beta}\frac{E}{d+\tau},\ A^AF=\frac{Y^A(K,P,\tau)}{1-\beta}+\delta K$$

and $A^AF_K=\alpha\left[A^AK^{\alpha+\beta-1}\beta\left(\dfrac{\omega}{d+\tau}\right)^{\beta\omega}\left(\dfrac{1-\omega}{c}\right)^{\beta(1-\omega)}\right]^{\frac{1}{1-\beta}}$. Log-linearization gives more accurate results so we use $\dot{\tilde{x}}=\tilde{\Upsilon}\tilde{x}$ where $\tilde{x}\equiv(\tilde{x}_P',\tilde{x}_N')'$ with $\tilde{x}_p\equiv\left(\ln\left(\dfrac{K}{K^{A*}}\right),\ln\left(\dfrac{P}{P^{A*}}\right)\right)'$ and $\tilde{x}_n\equiv\left(\ln\left(\dfrac{C}{C^{A*}}\right),\ln\left(\dfrac{\tau}{\tau^{A*}}\right)\right)'$ the vectors of predetermined and non-predetermined variables, respectively, and $\tilde{\Upsilon}_{ij}\equiv\Upsilon_{ij}\dfrac{x_j^{A*}}{x_i^{A*}}$.

Let $\tilde{\Upsilon}M=M\Lambda$ where the diagonal matrix Λ has the eigenvalues of $\tilde{\Upsilon}$ in decreasing order along the diagonal and the columns of the matrix M contain the eigenvectors corresponding to each of the eigenvalues. Suppose that the eigenvectors span the whole space so that each $\tilde{x}$ can be written as a linear combination of the eigenvectors, hence $\tilde{x}=M\tilde{y}$. It follows that the canonical form is $\dot{\tilde{y}}=\Lambda\tilde{y}$. If the system displays saddlepath stability, the first two eigenvalues have positive real parts and the last two have negative real parts. We rule out explosive trajectories, so the first two elements of $\tilde{y}$ must be zero. Hence, $\tilde{x} = M\tilde{y}$ $= \begin{pmatrix} M_{pu} & M_{ps} \\ M_{nu} & M_{ns} \end{pmatrix} \begin{pmatrix} 0 \\ \tilde{y}_s \end{pmatrix} = \begin{pmatrix} M_{ps}\tilde{y}_s \\ M_{ns}\tilde{y}_s \end{pmatrix}$ so the stable manifold is $\tilde{x}_n = M_{ns}\tilde{y}_s = M_{ns}M_{ps}{}^{-1}\tilde{x}_p$.

Given initial values for the predetermined variables, the initial values for non-predetermined variables consumption and tax rate must be on the stable manifold: $\tilde{x}_n(0) = M_{ns}M_{ps}{}^{-1}\tilde{x}_p(0)$. From then on the economy stays on the stable manifold, so this manifold is an invariant subspace. We thus have the following time paths:

(A8) $$\tilde{x}_p(t) = M_{ps} \begin{pmatrix} e^{\lambda_3 t} & 0 \\ 0 & e^{\lambda_4 t} \end{pmatrix} M_{ps}{}^{-1}\tilde{x}_p(0), \qquad \tilde{x}_n(t) = M_{ns} \begin{pmatrix} e^{\lambda_3 t} & 0 \\ 0 & e^{\lambda_4 t} \end{pmatrix} M_{ps}{}^{-1}\tilde{x}_p(0).$$

For our benchmark parameter values, we get $\tilde{x}_n = \begin{pmatrix} 0.4313 & -0.02260 \\ 0.7258 & -0.01542 \end{pmatrix} \tilde{x}_p$ and thus:

(A9) $$C^A(K,P) = C^{A*}\left(\frac{K}{K^{A*}}\right)^{0.4313}\left(\frac{P}{P^{A*}}\right)^{-0.02260}, \quad \tau^A(K,P) = \tau^{A*}\left(\frac{K}{K^{A*}}\right)^{0.7258}\left(\frac{P}{P^{A*}}\right)^{-0.01542}.$$

The policy rule for after-calamity consumption reacts almost identically to changes in the capital stock as without marginal climate damages, but now also reacts negatively to the stock of atmospheric carbon to reflect the need to consume less when the stock of carbon is high. The after-calamity carbon tax reacts positively to the capital stock, since the economy is then better able to cope with such a tax. Since in the damages proposed by Golosov et al. (2014) the concavity of the temperature response to the stock of atmospheric carbon dominates the convexity of the function relating damages to temperature, climate damages are a concave function of the stock of atmospheric carbon. This explains why the after-calamity carbon tax reacts negatively to the stock of atmospheric carbon. However, with a climate sensitivity of 4 instead of 3, the after-calamity stable manifold is given by:

(A10) $$C^A(K,P) = C^{A*}\left(\frac{K}{K^{A*}}\right)^{0.43113}\left(\frac{P}{P^{A*}}\right)^{-0.048597}, \quad \tau^A(K,P) = \tau^{A*}\left(\frac{K}{K^{A*}}\right)^{0.72764}\left(\frac{P}{P^{A*}}\right)^{0.24346}.$$

The after-calamity carbon tax now responds positively to the stock of atmospheric carbon.