The Beijer Institute
of Ecological Economics

Environment, Uncertainty, and Option Values

Karl-Göran Mäler. 2002.

Environment, Uncertainty, and Option Values

Karl-Göran Mäler

Beijer Institute

Box 50005, 104 05 Stockholm

Fax: 46-8-15 24 64

E-mail: Karl@beijer.kva.se

August 6, 2002

Contents

1 Introduction

It is trivial to note that the future is uncertain. It is, however, far from trivial to analyze that uncertainty. The environmental field, in particular, is permeated by uncertainty. Besides usual economic uncertainties, we have extra problems from the major uncertainties characterizing our knowledge of environmental processes. We

quite often do not simply know the long run consequences of interventions in the environment. For example, for many new chemicals, we do not know whether they are carcinogenic or not. Our models of ecosystems dynamics are far from precise. Moreover, future preferences for environmental services may be uncertain, which means that future benefits from nature preservation today are uncertain. These topics will be addressed in this chapter. In the next section, we will look at an essentially static framework to look at the role of risk aversion in valuing uncertain environmental benefits. The main tool is the use of quadratic approximations of the von Neuman - Morgenstern utility functions, and the main result is that the benefits from environmental policy reforms depend on risk aversion as measured by the Arrow-Pratt measure of absolute risk aversion and on the variance and covariance of the distributions of preferences and the supply of environmental quality and the wealth (or income) of the individuals. When aggregating the benefits over the whole population of households in the economy, some risks will be highly correlated and it is therefore impossible to bring down the cost of risk bearing by pooling risks. On the other hand, it will of course be possible to reduce the cost of risk bearing by diversification (something which is not studied in this chapter). One of the issues of whether the difference between total benefits and expected benefits (a difference that was called option value in the earlier literature[1]) is positive and negative can be much better understand from the point of view of the covariances between environmental uncertainty and preference uncertainty.

The theory of assets with uncertain returns is very developed within the corporate finance theory. In particular, the capital budgeting decision is from a formal point of view quite similar to the analysis offered in this note. However, the main and important issue is that for many natural resource assets, there do not exist risk markets which can be used for pooling and sharing risk. Therefore, no models similar to the CAPM can be developed for those assets. However, much of the analytical framework from that literature can be used for an analysis of decision making on the use of environmental resources when the consequences are risky.

The third part of the chapter looks at the case when information on he state of the world is coming forward with time. Thus, it may be socially profitable to postpone a decision until we know more about the costs and benefits. This field of analysis was opened up by Arrow and Fisher (1974) and Henry (1974) and has been much advanced through contributions by Fisher & Hanemann (1986) ,?, ?. More recently, the analysis has been applied to a firm's investment decision (?). Their tools differ from the tools used by the earlier writers in that they base the analysis on stochastic differential equations. ? and ? use the same tools for analysing environmental problems. Here we will follow the old tradition.

In the discussion of time resolution of uncertainty, we will abstain from looking at cases when information can be obtained by investing in searching for it. Obviously, one should take into account that human activities can affect the flow of new information, but that has to wait til later.

2 Decision Making and Risk, a Static Framework

2.1 The basic framework

As is traditional, we assume that uncertainty can be described by a set of events or states of the world. Each event contains all the information relevant for decision making and the uncertainty consists of not knowing which event that will occur. The particular features characterizing an event we are interested in are income

---

[1] Cicchetti & A.M. Freeman III (1971), Bohm. (1975), Schmalensee (1972), J.V. Krutilla & Russel (1972)

W, supply of environmental resources Q and preferences as measured by utility functions U(W, Q). Note that this means that the individual is not certain what his preferences will be, unless he knows the event that will occur. We will in this section mainly use the indirect utility function, and moreover, we will not, except in the last section consider price uncertainty. One case in which price uncertainty may be quite important is the uncertainty about future interest rates. We will therefore come back to an analysis of that case later. Finally, we will assume that Q is one-dimensional. Generalizations to the case when Q has many but finite dimensions are straight forward.

Let us start by considering one arbitrary individual. For him each event i will describe his income W_i, the supply of environmental services Q_i, and utility function U_i. Let there be a probability measure λ_i over the set of events. The preferences of the individual can then be represented by a von Neuman Morgenstern expected utility.

$$\int U^i(W_i, Q_i) d\lambda_i = EU^i(W_i, Q_i) \tag{1}$$

This means that we simultaneously will study both what has been called supply uncertainty, that is the uncertainty about Q, and demand uncertainty, i e uncertainty about the preferences. As usual, we will assume that the individual is risk averse. This is in the simplest case defined as the case when the individual refuses to accept a fair bet, i.e. a bet with expectation zero. It is then easy to prove that risk aversion is equivalent to a concave utility functions. However, in our case the utility functions U_i vary from event to event and the situation is slightly more complicated. In view of the confusion about the appropriate definition of risk aversion in this situation, it may be worth while to give a brief analysis of the problem.

2.1.1 Risk Aversion

In discussing risk aversion, the focus is on the income or wealth variable, so we assume *for this discussion only*, that the environmental quality variable is constant over all possible events. Then we can as well for simplicity suppress that variable. Assume now the individual has to choose between accepting a fair lottery Y with the price Y_i if event i occurs. The conventional definition of risk aversion is now that if E(Y)$\leq$ 0, i.e. if the expected value of the lottery is not positive, then a risk avert individual would not accept a lottery ticket. Based on this definition Schmalensee found that risk aversion means that the marginal utility of income must be constant over all events. However, in view of the fact that income W varies over events, a risk averter may very well accept a lottery with negative expected value if the lottery is negatively correlated with income. In this case will the lottery act as an insurance. Thus we should look at the total lottery W + Y = Z. Assume then that

$$E(Z) = \overline{Z} \tag{2}$$

If

$$E(U(Z)) \leq E(U(Z)), \tag{3}$$

then we may, for the moment, say that the individual is risk averse (note that the right hand side is different from $U(\overline{Z})$, since U is dependent on which event that will occur).

3

Obviously, if the utility functions are independent of states, then risk version would be equivalent to a concave utility function. However, with state dependent utility functions this is no longer so. The reason is that a positive or negative correlation between Z and the marginal utility of income now becomes an important factor. With positive correlation it may happen that

$$E(U(Z)) > EU(\overline{Z}).\tag{4}$$

However, this possibility of correlation between marginal utility of income and income has hardly anything to do with common sense interpretation of risk aversion. Therefore, our definition of risk aversion is modified as follows.

Definition 1 *An individual is risk averse if*

$$E(U(Z) - U(\overline{Z})) \le 0$$

for all distributions of Z and U provided that they are independently distributed.

It is now easy to see that the state contingent utility function of a risk averse individual must be concave in income and conversely, concave state contingent utility functions imply risk aversion.

In fact, with U and Z independently distributed, the definition can be written as a repeated expectation

$$E_U E_Z(U(Z) - U(\overline{Z})) \le 0$$

for all distributions of Z.

A necessary and sufficient condition for this is that for all states

$$E_Z(U(Z) - U(\overline{Z})) \le Z$$

for all states. Thus we have athe following theorem:

Theorem 2 *An individual is risk averse if and only if his utility function is concave in all events.*

In what follows, we will generally assume that risk aversion characterizes the behavior of the individual and thus we will be using concave utility functions.[2]

We will in the next sections use the expected utility representation of preferences developed above in analyzing some environmental quality decision situation.

2.2 The value of changes in risk

Suppose that the different states are characterized by

- i) utility functions U^i

- ii) wealth W_i

- iii) environmental quality Q_i

  Moreover, there is a probability measure λ_i over the different states. Then, as we have seen, preferences can be represented by the expected utility

$$E_\lambda U^i(W_i, Q_i)\tag{5}$$

---

[2]Note that is the same result as Bohm (Bohm. 1975) claimed, although the motivation is different.

where E_λ indicates for which distribution the expectation is computed. Let us now consider the case when it is possible to change the probability distribution to λ' by f .e. environmental policy. Then expected utility is

$$E_{\lambda'}U^i(W_i, Q_i)$$

Different welfare measures for the change in probability distribution can now be defined. The compensating variation CV and the equivalent variation EV are defined by

$$E_\lambda U^i(W_i, Q_i) = E_{\lambda'}U^i(W_i - CV, Q_i) \tag{6}$$

$$E_\lambda U^i(W_i + EV, Q_i) = E_{\lambda'}U^i(W_i, Q_i)$$

These measures have the usual interpretations. CV is the amount that can be taken away from the individual when the probability distribution changes. If $CV > 0$, then the change in the distribution has increased the expected utility. EV is the amount that would increase expected utility with the same amount as would the change in the probability distribution. Both EV and CV are correct representations of the underlying preference structure and the choice between them is mainly a matter of convenience.

Another alternative representation of the environmental change is possible, which implies the one just given, but is more convenient in some applications. Instead of representing the environmental change as a change in the probability distribution, one can look at it as a change in the characterization of each state.

Assume then that each state is characterized by environmental quality Q_i so that expected utility is

$$\int U^i(W_i, Q_i)d\lambda_i \tag{7}$$

Assume now that the environmental quality changes in each state by ΔQ_i so that

$$Q_i' = Q_i + \Delta Q_i \tag{8}$$

The expected utility is now

$$\int U^i(W_i, Q_i')d\lambda_i \tag{9}$$

It can be proved (see appendix 1) that provided certain conditions hold there is a probability distribution λ' so that

$$\int U^i(W_i, Q_i + \Delta Q_i)d\lambda_i = \int U^i(W_i, Q_i)d\lambda_i$$

With this new representation CV and EV are defined by

$$E_\lambda U^i(W_i, Q_i) = E_\lambda U^i(W_i - CV, Q_i') \tag{10}$$

5

and

$$E_\lambda U^i(W_i + EV, Q_i) = E_\lambda U(W, Q_i')$$ (11)

The interpretation is exactly the same as the one given above. We will in the sequel only study CV. Define the compensating variation CVi contingent upon the occurrence of state i as

$$U^i(W_i, Q_i) = U^i(W_i - C_i.Q_i')$$ (12)

We will now try to relate CV to the expected value of CV_i. We have from the definitions

$$E_\lambda U^i(W_i - CV, Q_i') = E_\lambda U^i(W_i, Q_i) = E_\lambda U^i(W_i - CV_i, Q_i')$$ (13)

By making a quadratic expansion around $\overline{W} - CV, and\ Q'$, where $\overline{W} = E_\lambda W_i$ and subscripts denote partial derivatives, we have

$$E_\lambda \left\{ U^i + (W_i - \overline{W})U_W^i + \frac{1}{2}(W_i - \overline{W})^2 U_{WW}^i \right\} =$$

$$E_\lambda \left\{ U^i + (W_i - \overline{W} - CV_i + CV)U_W^i + \frac{1}{2}(W_i - \overline{W} - CV_i + CV)^2 U_{WW}^i \right\}$$ (14)

Define

- $\overline{CV} = E_\lambda CV_i$
- var $(CV_i) = E_\lambda(CV_i - \overline{CV})^2$
- $\overline{U}_W = E_\lambda U_W^i(\overline{W} - CV, Q_i')$
- cov $(CV_i, U_W^i) = E_\lambda \left\{ (CV_i - \overline{CV})(U_W^i - \overline{U_W}) \right\}$
- cov $(CV_i, W_i) = E_\lambda \left\{ (CV_i - \overline{CV})(W_i - \overline{W}) \right\}$

Let us for simplicity assume that $(CV_i - \overline{CV})$ is so small that $(CV_i - \overline{CV})^2$ can be neglected in expression containing $(CV_i - \overline{CV})$, (otherwise we have to solve quadratic equation yielding complicated messy formulas but essentially the same qualitative conclusions). Then we obtain the desired relation between the expected state contingent compensated variations and the compensated variation CV:

$$CV = \overline{CV} + \frac{cov(CV_i, U_W^i)}{\overline{U}_W} -$$

$$-\frac{1}{2} E_\lambda \left\{ \frac{U_{WW}^i}{\overline{U}_W} \left[(CV_i - \overline{CV})^2 + 2(CV_i - \overline{CV})(W_i - \overline{W}) \right] \right\}$$

. If we can assume that U_{WW} is state independent, the factor $-\frac{U_{WW}^i}{\overline{U}_W}$ can be identified as the Arrow-Pratt measure R of absolute risk aversion. Then

$$CV = \overline{CV} + \frac{cov(CV_i, U_W^i)}{\overline{U}_W} + \frac{1}{2}R\left\{ var(CV_i) + 2cov(CV_i, W_i) \right\}$$ (15)

This is our basic expression for the value of the change in environmental quality. It is based on a set of not too restrictive assumptions.

2.3 Aggregation over individuals

In general, we are not interested, however, in the individual compensating variation but in the aggregate over the relevant population. Let the population be represented by the set H and denote variables for the individual with a superscript h.

It may happen that the individual state contingent values CV_i^h are dependent on the size of the population. If, for example, the total benefits are independent of the population size, then the individual benefits will in average decrease with increasing size. Even if the benefits are of a public goods nature, congestion may decrease the individual benefits if the number of users increases. Only when we have a pure public good without congestion will the individual compensating variations be independent of the size of population sharing the benefits.

Let therefore H be the measure of the set H (this double use of the letter H will not cause any confusion) and assume that CV_i^h is a function of H $CV_i^h(H)$ In general we would expect this to be a decreasing function, but with positive externalities it may in fact be increasing.

The total compensating variation is now (CV will now denote the total compensating variation over the whole population and similarly for $\overline{CV}$)

$$CV \quad = \quad \int_H CV^h dh = \tag{16}$$

$$= \quad \int_H \overline{CV}^h dh + \int_H \frac{cov(CV_i^h, U_W^{ih})}{\overline{U}_W^h} dh + \tag{17}$$

$$+\frac{1}{2} \int_H R \left\{ var(CV_i^h) + 2cov(CV_i^h, W_i^h) \right\} dh \tag{18}$$

Let us investigate the three terms in this expression. The first is

$$\int_H \overline{CV}^h dh = \overline{CV}$$

which obviously is the aggregated expected state contingent benefits. The second term is

$$\int_H \frac{cov(CV_i^h, U_W^{ih})}{\overline{U}_W^h} dh$$

The reason for this term is of course that the marginal utility of income is contingent on the state, and the utility value of the monetary benefits CV_i^h depends on the marginal utility of income. If CV_i^h and U_i^h are independently distributed for each individual h, then this term will vanish. We will in the next section see some examples where such an independence assumption may be reasonable.

Let

- $v^{ih} = \frac{U^{hi}}{\overline{U}_W^h}$

- $\overline{CV}_i = \int_H CV_i^h d$

- $\overline{v}^i = \int_H v^{ih} dh$

and note that $\int v^{ih} d\lambda_i = 1$.

Then the second term can be written

$$\int_H \frac{cov(CV_i^h, U_W^{ih})}{\overline{U}_W^h} dh = \int_H \int (CV_i^h - \overline{CV}^h)(v^{ih} - 1)d\lambda_i dh =$$

$$= \int \int CV_i^h v^{ih} d\lambda_i dh - H\overline{CV} =$$

$$= \int \int (CV_i^h - \overline{CV}^h)(v^{ih} - \overline{v}^i)d\lambda_i dh + \int \overline{CV}_i(\overline{v}^i - 1)d\lambda_i \qquad (19)$$

The mean value theorem gives an i' such that the last term becomes

$$\overline{CV}_i(\overline{v}^{i'} - 1)$$

If $\overline{CV}_i$ and $\overline{v}_i$ are distributed sufficiently symmetrically,

$$\overline{v}^{i'} \sim 1$$

and the expression in 19 becomes

$$H\ cov(CV_i, v^i)$$

This covariance term characterizes for each state the covariance between the benefits and the marginal utility of income over different individuals. If it can be assumed that different individuals are independent of each other in this respect, the covariance becomes zero and the term

$$\int \int cov(CV_i^h v^{ih})d\lambda_i dh$$

vanishes although the covariance for each individual may be different from zero.

On the other hand, if there is a nonzero covariance over individuals, the term cannot be neglected. This may happen if the variations in benefits and marginal utility of income have a common cause, for example random changes in a certain price. We will look into this later.

There remains the third term,

$$\frac{1}{2}\int R^h(var(CV_i^h) + 2cov(CV_i^h, W_i^h))dh \qquad (20)$$

Obviously, this term represents the cost of bearing the risk of variations in CV_i^h. The following factors influence the size and sign of this term, the covariance between CV_i^h and W_i^h and the variance of CV_i^h, the degree of risk aversion and the joint distribution of these variables over states and individuals.

The next section will be devoted to a discussion of the cost of risk bearing and mechanisms for risk sharing.

2.4 The cost of risk bearing

Assume that utility functions are identical in all states. As in the previous section, the cost for individual h of bearing the risk is

$$-\frac{1}{2}\frac{U^h_{WW}}{\overline{U}^h_W}\left\{var(CV^h_i)+2cov(CV^h_i,W^h_i)\right\} \tag{21}$$

When will this cost be positive, negative or zero?

i) $R^h = -\frac{U^h_{WW}}{\overline{U}^h_W} = 0$, that if the individual is risk neutral, the cost of risk bearing is zero. However, we will assume this is not the case, that is R>0.

ii) If the bracket $var(CV^h_i)+2cov(CV^h_i,W^h_i)$ then the cost is also zero. This can happen if the environmental change generates benefits that are essentially independent of income. If the bracket is negative, then the environmental mental change will act as an insurance and $CV^h < \overline{CV}^h$ and vice versa if the bracket is positive.

Let us now go back to the aggregate cost of risk bearing and consider different mechanisms for allocating the risk.

One mechanism would be to have no risk sharing at all. In that case the social cost of bearing the risk is

$$\frac{1}{2}\int_H R^h\left\{var\ CV^h_i+2cov(CV^h_i,W^h_i)\right\}dh \tag{22}$$

If all individuals have the same absolute risk aversion and if the joint distribution of benefits and wealth is the product of distributions over individuals and states resp., i.e. if for all h CV^h_i and W^h_i are independent random variables, the aggregate cost is

$$\frac{1}{2}R\left\{\overline{CV}_i+2cov\int_H cov(CV^h_i,W^h_i)dh\right\}$$

Depending on the distribution of covariance over the individuals, this aggregate cost may be positive, zero or negative.

B. If the risks are "individual risks" in Malinvaud's meaning[3], the aggregate cost above will be approximately zero because the covariance term vanishes. Thus, society should behave in a risk neutral manner, and could achieve that by implementing an insurance scheme. An insurance scheme which is actuarially fair can be described as a measurable function x^h such that $E_\lambda x^h = 0$. If each consumer chooses the insurance that is best for her, we will have as a result that $\partial U_i/\partial W^h$ is equal across all states. The gain from the optimal insurance $x\overline{x}^h$ for individual h is given by

$$E_\lambda U^h(W^h+\overline{x}^h,Q')-E_\lambda U^h(W^h,Q')\approx\frac{1}{2}R^h var\ (x^h)\overline{U}^h_W \tag{23}$$

Thus the variance term in the expression for CV in 22 corresponds to the premium an individual is willing to pay for an insurance that will eliminate the uncertainty. However, such a complete insurance system seems unrealistic in view of the serious problems of moral hazard due to the stochastic nature of preferences. Only if the uncertainty of the preferences is due to an objectively measurable variable can the moral hazard problem be overcome. We will in the next section find a case where such an insurance scheme exists, although not completely.

---

[3] Malinvaud (1972)

C. Let us now assume that individual benefits are highly correlated. Suppose for simplicity that the set H is finite
H = $\{1, 2, ..., H\}$ and that individual benefits CV_i^h are constrained by

$$\sum_{h=1}^{H} CV_i^h = Y_i(H) \tag{24}$$

where $Y_i(H)$ is the total benefits in state i. We will see that the assumption that Y_i depends on H is crucial for the results we will derive.

Suppose total welfare can be written[4]

$$\int \sum_{h=1}^{H} \beta^h U^h(W_i^h + CV_i^h, Q_i) d\lambda_i \tag{25}$$

Assuming that lump sum transfers are feasible in each state, the optimal allocation of the total benefits in each state among the individuals is given by

$$\max \sum_{h=1}^{H} \beta^h U^h(W_i^h + CV_i^h, Q_i) \tag{26}$$

s.t.

$$\sum_{h=1}^{H} CV_i^h = Y_i(H)$$

The necessary conditions are

$$\beta^h U_W^h - \mu = 0 \tag{27}$$

where μ is a Lagrange multiplier.

The maximum value of the objective function is denoted $V(Y_i(H))$.

The maximum of the objective functions in the original problem can now be written

$$\max \int V(Y_i(H) d\lambda_i \tag{28}$$

Let the Arrow-Pratt measure of absolute risk aversion R_v be defined as

$$R_V = -\frac{V"}{V'} \tag{29}$$

We know that V' $= \mu$ and thus

$$V" = \frac{d\mu}{dY}$$

---

[4]This is not mecessary for the analysis. Any assumption that keeps the individual small compared to the total income would give the same result, f.e. if the total benefits are distributed so that $CV_i^h = \beta^h CV_i$ would yield the same result.

Differentiation of $\beta^h U_W^h = 0$ yields

$$V" = \frac{U_W^h}{\sum_{h=1}^H \frac{1}{\beta^h}}. \tag{30}$$

Thus

$$R_V = -\frac{U_{WW}^h}{\beta^h \sum_j \frac{1}{\beta^j} U_W^j}$$

Let $\frac{1}{\beta^k} U_W^h = \min_j \frac{1}{\beta^j} U_W^k$. Then

$$R_V \leq -\frac{U_{WW}^h}{\beta^h H \frac{1}{\beta^k} U_W^k} = \frac{1}{H}\frac{\beta^k}{\beta^h}\left[-\frac{U_{WW}^h}{U_W^k}\right] \text{ for all h} \tag{31}$$

Choose h = k and note that the individual measure of absolute risk aversion is R_U

$$R_U = -\frac{U"}{U'} \tag{32}$$

and so

$$R_V \leq \frac{1}{H}\frac{1}{R_k} \tag{33}$$

With increasing size of the population the social risk aversion will therefor go to zero.

The social cost of risk bearing is now

$$\frac{1}{2}\frac{1}{H}R_U \left\{var(Y_i(H)) + 2cov(Y_i(H), W_i(H))\right\} \tag{34}$$

where $W_i(H) = \sum_{h=1}^H W_i(h)$, i.e. national income.
i) If the environmental asset is a pure public good[5] without congestion

$$Y_i(H) = HY_i \tag{35}$$

and the social cost of risk bearing is

$$R_U \left\{H \ var(Y_i) + 2cov(Y_i, W_i)\right\}$$

In this case, an increase in population will increase the total cost of risk bearing.
ii) If the asset generates purely private benefits

$$Y_i(H) = Y_i \tag{36}$$

---

[5] This case has been analysed within a different approach by Fisher (1973)

and the cost of risk bearing is

$$\frac{1}{H} R_U \left\{ H \ var(Y_i) + 2cov(Y_i, W_i) \right\}$$

First, if Y_i and W_i are independent, it follows that the cost of risk tends to zero when the number of individuals sharing the risk increases[6]. If $W_i(H)$ is independent of H, then the cost of risk bearing also tends to zero with the number of individuals sharing the risk. If there is decreasing marginal returns with respect to H, so that $W_i(H)$ is decreasing, the same result obtains.

iii) In the general case, if

$$\frac{Y_i(H)}{\sqrt{H}} \ and \ \frac{W_i(H)}{\sqrt{H}}$$

go to zero with increasing H, the cost of risk bearing will be smaller the larger the population is. The importance of this is obvious.

Even if insurance markets cannot work because of the correlation of the risks individuals are bearing, it is thus possible to reduce that cost in certain cases by letting more people bear the risk.

2.4.1 Option prices and option values

Let us apply the theory developed in the last sections to the valuation of a natural assets. Consider the example provided by Schmalensee, i.e. the possible development of Yellowstone National Park which would irreversibly destroy its unique features. The environmental variable Q can in this case assume two values, Q' corresponding to preservation of the National Park and Q" corresponding to irreversible destruction. Uncertainty comes partly from income uncertainty $\{W_i^h\}$ and partly from preference or utility uncertainty $\{U_i^h\}$.

CV^h is defined from

$$EU_i^h(W_i^h - CV^h, Q) = EU_i^h(W_i^h, Q'') \tag{37}$$

It is known as the option price, i.e. the price the individual is willing to pay for keeping the option of going to Yellowstone in the future. CV_i^h is defined from

$$U_i^h(W_i^h - CV_i^h, Q') = U_i^h(W_i^h, Q'') \tag{38}$$

From the previous sections 15we know that

$$CV^h = \qquad \overline{CV}^h + \frac{cov(CV_i^h, U_{i,W}^h)}{\overline{U}_W^h} + \tag{39}$$

$$+ \frac{1}{2} R^h \left\{ var(CV_i^h) + 2cov(CV_i^h, W_i^h) \right\} \tag{40}$$

The difference between CV^h and $\overline{CV}^h$ is known as the option value OV^h

$$OV^h = CV^h - \overline{CV}^h \tag{41}$$

---

[6] This result was derived by Arrow and Lind Arrow & Lind (1972)

and a substantial discussion has taken place in the literature whether the option value is positive or negative[7]. It is clear from the formula above that the option value may be of either sign. However, it is at least possible to outline the factors influencing the size and sign of the option value.

The first term

$$\frac{cov(CV_i^h, U_{i,W}^h)}{\overline{U}_W^h}$$

reflects the collinearity between CV_i^h and U_i^h. If the natural asset is not considered of high importance by the individual (i.e. it would occupy a big share of his budget if would have to pay for it), it is hard to see why random variations in the marginal utility of income should be of importance. Thus, this term should be small. Moreover, as we saw in the previous section, if we aggregate over individuals and if CV_i^h, and $U_{i,W}^h$ are distributed independently for each state i over individuals, the aggregate will be close to zero.

Thus, there seems to be reason to assume that this term is negligible.

The last term

$$\frac{1}{2} R^h \left\{ var(CV_i^h) + 2cov(CV_i^h, W_i^h) \right\}$$

is on the other hand more interesting. Obviously, it is impossible to say anything in general about the sign and size of this term. However, for some particular cases, some conclusions may be drawn.

i) If the uncertainty about future preferences is in a sense genuine, a knowledge about future income would not increase our ability to predict future benefits. Thus CV_i^h and W_i^h will not be correlated for any individual and the term simplifies to

$$\frac{1}{2} R^h \, var(CV_i^h)$$

which obvious is positive. In this case risk aversion will imply a positive option value. This is probably the case that corresponds most closely to the problem discussed by Weisbrod and Cicchetti-Freeman[8].

ii) The uncertainty about future preferences may be due to uncertainty about some economic variable not explicitly in the utility function. If the natural asset is a recreational facility and if high future oil prices shifts recreational demand from foreign facilities to the domestic asset and vice versa, it is reasonable to assume that CV_i^h and W_i^h are negatively correlated. Thus the risk premium term for the individual in that case

$$\frac{1}{2} R^h \left\{ var(CV_i^h) + 2cov(CV_i^h, W_i^h) \right\}$$

may have either sign depending on whether the second term dominates the first or not. It may therefore happen that the individual has a positive option value. However, from the point of view of society, if the recreational asset is such that $\overline{CV}_i$ increases less than in proportion with the size of the population, the term $\frac{1}{H}\overline{CV}$ will be small and the covariance term will dominate.

---

[7]See Bohm. (1975), Cicchetti & A.M. Freeman III (1971), Schmalensee (1972), J.V. Krutilla & Russel (1972), and Bishop (1982)

[8]Weisbrod (1964), J.V. Krutilla & Russel (1972), Cicchetti & A.M. Freeman III (1971)

Thus, even if the individual has a positive option value, society may, in spite of this, have a negative option value. The reason for this is that the risk component corresponding to the variance can be better shared through society and the remaining risk component is essentially an insurance against the future oil prices. In particular, this implies that investments in this facility should be discounted with a negative risk premium. Of course, if the facility is a pure public good with no congestion, then this result does not hold, and the risk term for society is simply the individual risk premium multiplied with the size of the population.

2.4.2 Health effects

We will now consider the situation when Q_i represent health effects from pollution. Assume a pollution control policy has been proposed which would reduce the risk of getting

pollution induced health effects. As we have seen previously this change in risk can be represented by a change in Q. We assume that preferences are state independent.

The willingness to pay of the individual for the proposed policy is then

$$CV^h = \overline{CV}^h + \frac{1}{2} R^h \left\{ var(CV_i^h) + 2cov(CV_i^h, W_i^h) \right\}$$

if the state dependent benefits CV_i^h exist. Obviously, we are now analyzing a problem so delicate that the expected utility representation may not be valid. Assume for example, that Q=0 represents a state in which the individual contracts lung cancer and that Q = 1 represent a state in which the individual stays healthy. Our utility representation implies that there is a CV_i^h, so that

$$U^h(W_i^h + CV_i^h, 0) = U^h(W_i^h, 1)$$

However, there may not exist any compensation for contracting lung cancer and since state 0, has a positive probability, it follows that $\overline{CV}^h = \infty$! On the other hand, we know from everyday experience that people are willing to take risks involving contracting lung cancer (smokers for example) which contradicts the hypothesis that $CV_i^h = \infty$. Thus, a linear utility may not be the best way of analyzing the economic measurement of health effects. More important is that the quadratic approximation used to derive the formula above hardly is valid if the state contingent benefits vary substantially between different states. In spite of this we will, continue to use the framework developed in previous sections.

The health effects CV_i^h represent individual risks in the sense used previously. Thus, there is scope for an insurance system and if the population is large enough, the law of large numbers will guarantee each individual $\overline{CV}^h$ independent of state. Thus, in the aggregate, the risk premium will vanish and $CV = \int_H \overline{CV}^h dh$. This presumes, however, that there do exist complete insurance markets. In most western states well developed insurance companies are operating, offering customers protection for hospital expenses, medicine, doctors' fees, and income losses. Some of them also offer compensation for psychic suffering from the disease. However, hardly all the cost to the individual can be insured, so let us write

$$CV_i^h = X_i^h + Y_i^h$$

where X_i^h is the insured part and Y_i^h the uninsurable part. Then

14

$$CV = \int_H \overline{CV}^h dh + \frac{1}{2} \int_H R^h \{var(Y_i^h) + cov(Y^h, W_i^h)\} dh \qquad (42)$$

If the insurance covers income losses, one would not expect any correlation between Y_i^h and W_i^h and thus

$$
\begin{aligned}
CV &= \int_H \overline{CV}^h dh + \int_H R^h var(Y_i^h) dh = \\
&= \int_H \overline{X}^h dh + \int_H \overline{Y}^h dh + \int_H R^h var(Y_i^h) dh
\end{aligned}
$$

X_i^h corresponds to those benefits that are easily measured and therefore easily insured. In most cases X_i^h will stand for financial costs, i.e. expenditures on medical care and earning
losses.
As we see from 43

$$\int_H X_i^h dh$$

is an underestimate of CV as the term $\int_H \overline{Y}^h dh + \int_H R^h var(Y_i^h) dh$ i.e. the expected cost from suffering and the associated risk premium is not included.

2.4.3 Price uncertainty

Let us introduce the interest rate r as a variable in the indirect utility function and let us also assume that r is a stochastic variable. We will also neglect the many consumers case and instead assume that the utility represents social utility. The utility function is now

$$U_i(W_i, r, Q_i)$$

where i as previously represent the state of the world. We can now go through the same kind of exercise as we have done in previous sections in order to derive approximations to the true CV. The result is under essentially the same assumptions as before

$$CV = \overline{CV} + \frac{cov(CV_i, U_{i,W})}{\overline{U}_W} + \frac{1}{2}R\{var(CV_i) + 2cov(W_i, CV_i)\} - \qquad (43)$$

$$-\frac{U_{r,W}}{\overline{U}_W} cov(r, CV_i) \qquad (44)$$

where $U_{r,W} = \partial^2 U/\partial r \partial W$ has been assumed state independent. We will assume that $U_{r,W} < 0$, that is an increase in the price r will reduce the marginal utility of wealth. It is interesting to note that even if the individual is risk neutral so that the utility function is linear in wealth, the last term may be different from zero. In the simple case when preferences are state independent and represent risk neutral behavior we would have

$$CV = \overline{CV} - \frac{U_{r,W}}{\overline{U}_W} cov(r_i, CV_i) \qquad (45)$$

Thus the possible correlation between on the one hand the state contingent benefits CVi and the price (or the interest rate) will create a difference between the expected benefits $\overline{CV}$ and the true benefits CV. Is such a correlation to be expected? Should not the covariance between state contingent compensation variations and the interest rate be zero in general? The interest rate will either reflect the desired trade off between consumption in different time periods or the future marginal productivity of capital. In the latter case, the interpretation is that the interest rate gives an indication of the necessary capital investment today in order to increase consumption with one unit in a future time period. But the future marginal productivity of capital will in general be influenced of the availability of natural resources. An increase in the availability of resources will in general increase the expected future productivity and thereby the interest rate. If we are analyzing a project with large environmental consequences, not only will the cost be large and state contingent benefits small because of that, but the interest rate will be small in the states that correspond to small benefits. Thus we should expect a negative covariance between CV_i and r_i. This means that, even in the absence of risk aversion and with state independent preferences, will the true benefit be smaller than the expected compensating variation. The covariance between interest rate and the benefits will thus act as if the decision maker is risk avert. Another way of seeing that is to look at the following formula for the present value of a current project with future environmental consequences.

$$NB_0 = C_0 - \frac{C_1}{1 + r}$$

where C_O is the present benefit, C_1 is the future cost from environmental degradation, r is the interest rate, and NB_0 is the present value of the net benefits. Assume C_1 and r are stochastic variables and assume that the decision maker is risk neutral. The standard procedure is to calculate the number

$$C_0 - \frac{\overline{C}_1}{1 + \overline{r}} \tag{46}$$

where $\overline{C}_1$ and $\overline{r}$ are the expected values of C_1 and r. The expected value of NB_0 is, however,

$$E(NB_0 = C_0 - E\left\{\frac{C_1}{1 + r}\right\} \approx C_0 - E\left\{C_1(1 - r)\right\} \approx C_0 + \frac{\overline{C}_1}{1 + \overline{r}} - cov(r, C_1) \tag{47}$$

Once again, we see that the covariance between r and C_1 will affect the expected net benefits and with a positive correlation between r and C_1, the expected net benefit will be less that what would be calculated with the conventional practice.

3   Intertemporal resolution of uncertainty

We will look at the following situation. Assume a project is designed that will include the use of a particular environmental resource over several time periods. However, the value of this resource in the future time periods is not known with certainty. There is a possibility that information on these values may be generated over time. What is the appropriate decision making framework. In most analyses

Figure 1: Decision tree when no information is forthcoming

of uncertainty of future environmental resource use, the information structure has been very simple. All information is contained in an a priori probability distribution and no more information will be available at a later stage. For some applications, this may be a realistic assumption. If the decision to use a natural asset in a special way can be completely reversed at a later stage (without any costs), the prospect of getting more information in the future does not have to be included in the analysis. In this case there are no essential intertemporal connections. In the other extreme when the decision is completely irreversible, the prospect of better information may be quite important and we will turn to a discussion of that situation.

The pioneering analysis of this problem was given by Arrow & Fisher (1974) and Henry (1974) , who showed that the prospect of being better informed at a later stage meant fewer irreversible decisions, given some assumptions on the nature of costs and benefits. We will prove their assertions but using a different approach. Most of the following analysis is based on Marshak & Miyasawa (1968) and ?. However, Hanemann (1989) has been most influential.

3.1   The Arrow Fisher analysis

The easiest way to present the Arrow Fisher analysis is perhaps in terms of decision tree. Assume a decision maker has to decide whether to develop a piece of land or preserve it. That is represented by the two branches d_1 and p_1 in the figure below.The development will be irreversible so if $d_1$has been chosen in the first period, the decision maker has to continue with having the land developed in the second period, represented by the branch d_2 . On the other hand, if she has choosen preservation in the first period, that is if she has followed the branch p_1. After the first period, she has the choice of either developing the land - d_2 - or continue preserving it -p_2. However, although the benefits from developing the land is known at the initial node, the benefits from preservation is not. They can be high - B_{ph} - with probability q_1 or low -B_{pl} - with probability q_2. The benefits from developing the land in the first period is $B_d(1)$ and in the second period $B_d(2)$. We will assume that $B_{ph} > B_d(2) > B_{pl}$.

17

Figure 2: New information is forthcoming

If she choses d_1 in the first period, the expected benefits are $B_d(1) + B_d(2)$. If on the other hand she choses preservation in the first period but development in the second period, the expected benefits are $B_d(2)$ which is less than the benefits from developing in the first period. Finally, if she choses preservation in both periods, the expected benefits are $q_1 B_{ph} + q_2 B_{pl}$. Thus she will , if she maximizes the expected value, chose p_1 if

$$B_d(1) + B_d(2) < q_1 B_{ph} + q_2 B_{pl} \qquad (48)$$

and vice versa. Note, that in this case, no information is forthcoming between period 1 and period 2 concerning the benefits from preservation. Let us now look at the case when information is forthcoming before a decision is being made for the second period.

As in the case above, if development is undertaken in the first period, then the expected benefits are $B_d(1) + B_d(2)$. However, if preservation is undertaken in the first period, the information on whether the preservation benefits are high or low will change the decision for the second period. Assume that the decisionmaker gets the signal in the beginning of the second period that the preservation benefits will be high, then she will chose continued preservation as $B_{ph} > B_d(2)$. Thus the benefits from preservation in both periods are B_{ph} in this case. If the signal is that the benefits are low, then she will chose development in the second period and the benefits are $B_d(2)$. Expected benefits, as seen from the original node is then $q_1 B_{ph} + q_2 B_d(2)$. Preservation in the first period will now take place if

$$B_d(1) + B_d(2) < q_1 B_{ph} + q_2 B_d(2) \qquad (49)$$

If preservation in the first case is chosen when no new information is forthcoming, that is, if $B_d(1) + B_d(2) < q_1 B_{ph} + q_2 B_{pl}$, it follows from $B_d(2) > B_{pl}$, that $B_d(1) + B_d(2) < q_1 B_{ph} + q_2 B_d(2)$, and preservation in the first period will also be chosen when new information is forthcoming in the beginning of the second period.

Assume now that development is undertaken in the first period when new information is forthcoming, that is, assume that $B_d(1) + B_d(2) > q_1 B_{ph} + q_2 B_d(2)$.

Then it follows that $B_d(1) + B_d(2) > q_1 B_{ph} + q_2 B_{pl}$, that is development will also be undertaken.

Furthermore, as $q_1 B_{ph} + q_2 B_d(2) > q_1 B_{ph} + q_2 B_{pl}$, for some values of the benefits, preservation may be chosen when information is forthcoming but not when it is not forthcoming. Finally, the effectrive use of forthcoming information on the future benefits will, in the case of preservation, increase the value of the net benefits from $q_1 B_{ph} + q_2 B_{pl}$ to $q_1 B_{ph} + q_2 B_d(2)$. This increase in the objective function is known as the quasi option value, or simpy option value (note that this option value is different from the one discussed in the first part of this chapter), and it can also be interpreted as the value of the forthcoming information. Summarising, we have

- if preservation in the first period is chosen when no information is forthcoming, it will also be chosen when such information is forthcoming

- if development in first period is chosen when information is forthcoming, it will also be chosen when no such information is forthcoming

- there are cases when preservation will be chosen when information is forthcoming, but not when it is not forthcoming

- the possibility of using forthcoming information creates a quasi option value.

The general conclusion is that the combination of irreversibility and the possibility new information will favour preservation. This is sometimes called the *irreversibility effect*. Moreover, any decision rule that is based on a mechanical use of expected benefits is bound to be erronous in a number of cases.

3.2 Research agenda

The simple Arrow-Fisher analysis is, because of its simplicity, quite revealing, but it also conceals many important issues, which need to be understood in order for the irreversibility effect to be useful. We will therefore discuss and make some attempts to analyse the following features.

- generalisations to general benefit functions

- sharper definitions of the meaning of forthcoming information

- active search for new information instead of passively waiting for it to occur

- not complete irreversibility but a change can be reversed at a cost

- irreversibility is unknown

- option value as a risk premium in the sense of the first part of the chapter, when there are irreversible changes

  We will try to address these issues in the sequel.

3.3 Baysean updating

We will consider the following decision-making situation. We are studying two time periods. In time period one a decision has to be made on variable X_1 and in time period two a decision on X_2.   X1 and X_2 can be thought of resource use. Once development has been made, it is impossible to restore the resource and therefore $X_1 \leq X_2$. The pay-off is given by the expected value of the utility function

$$U(X_1, X_2, Z) \tag{50}$$

19

where Z is a random variable.

When the decision is made in the first period, the only information on Z is an a priori probability distribution r. We assume that Z only can take a finite number of values, i. e. $Z=(Z_1, ..., Z_m)$ and the corresponding probabilities are $r = (r_1, ... , r_m)$.

After the decision on x_1 has been made, but before the decision on x_2 is made, the decision maker gets a signal Y. Y is a random variable taking the values $(Y_1, ... , Y_n)$ with probabilities $q = (q_1, ..., q_n)$.

If Y and Z are perfectly correlated, the decision maker knows with probability one which realization of Z that will occur, but if Y and Z are independent, the signal Y gives no information at all on Z.In the general case, we know that the decision maker will revise his probabilities according to Bayes' theorem[9] . Let be the conditional probability distribution of Z given the signal Y be

$$\pi_{i,j} = Pr(Z = Z_i \mid Y = Y_j) = \frac{\Pr(Y = y_j \mid Z = z_i)r_i}{\sum_{k=1}^{m} \Pr(Y = y_j \mid Z = z_k)}. \tag{51}$$

Let

$$\lambda_{j,i} = Pr(Y = Y_j \mid Z = Z_i) \tag{52}$$

be the likelihoods. Then

$$\pi_{j,i} = \frac{r_i \lambda_{i,j}}{\sum_k r_k \lambda_{k,j}} \tag{53}$$

Let $\Lambda = [\lambda_{j,i}]$ and $\Pi = [\pi_{i,j}]$

Then we also have (A^T means the transpose of A)

$$\Pi q = r \tag{54}$$

and

$$r^T \Lambda = q^T. \tag{55}$$

3.4   Information structures

The signal Y is given within a given information structure defined by $(Y_1, ..., Y_n)$ and the probabilities q and Λ. Another information structure, Y', is defined by $(Y_1', ..., Y_n')$, q', and Λ'. Obviously, if we have the same priors r, $\Pi q = \Pi' q' = r$ and $r^T \Lambda' = q'$. Our decision making problem can now be written

$$\max_{x_1} \sum_{j=1}^{m} q_j \max_{x_2} \sum_{i=1}^{n} \pi_{i,j} U(X_1, X_2, Z_i) \tag{56}$$

where X_1 and X_2 must of course be chosen within the feasibility sets. In analyzing this problem we follow Epstein (1980) and we will introduce a new notation.

Let $= \xi = [\xi_1, ..., \xi_n]$ such that $\xi_i \geq 0$ and $\sum_{i=1}^{n} \xi_i = 1$. Define

---

[9] See (?) for an extremely well-written presentation of Bayesian decision making. Raiffa & Schleifer (1961) gives a more advanced discussion

$$J(X_1, \xi) = \max_{x_2} \sum_i \xi_i U(X_1, X_2, Z_i) \tag{57}$$

J can be interpreted as the maximum expected utility from using X_1 in the first period, given the probability distribution ξ. We can then formulate the decision problem as follows:

$$\max_{X_1} \sum_j q_j J(X_1, \pi_j) \tag{58}$$

where π_j is the j:t column of Π. Following Blackwell (1951), and Marshak & Miyasawa (1968), let us define the concept of "more informative".

Consider two information schemes Y and Y' corresponding to the same prior probability distribution. The corresponding posterior probabilities are π and π' and the probabilities for the signals are q and q' respectively. Y is defined to be more informative than Y' if and only if

$$\sum_j q_j J(X_1, \pi_j) \geq \sum_j q'_j J(X_1, \pi'_{i,j}) \tag{59}$$

for all X_1, all utility functions U and all feasibility sets. Y more informative than Y' then means that independent of the initial choice of x_1 and the utility functions, Y will give a higher well-being, that is the signal Y enables us to achieve a higher well-being than signal Y'.

In order to get a feeling of the meaning of this definition, let us consider two extreme cases:

i) Y' means that $\pi'_{i,j} = r_i$ j $= 1, \ldots, $ m, that is the signal Y' does not carry any new information.

ii) Y" implies perfect information, that is m = n and

$$\pi"_{i,j} = \begin{cases} 0 \text{ if i} \neq j \\ 1 \text{ if i=j} \end{cases} \tag{60}$$

so that the signal reveals with certainty which z that will occur. Obviously q" = r. If Y is an arbitrary information structure our intuition requires that Y" is more informative than Y which in turn should be more informative than Y'. That this is, indeed, the case is easily proved.

We first have

$\pi"q" = \pi g = \pi'q' = r. \in$

Then, from the definition of J it follows that J is convex in ξ (maximum of a linear function in X).

Thus

$\sum_j q_j J(x_1, \pi_j) \geq J(x_1, \sum_j q_j \pi_j) = J(x_1, r) = J(x_1, \sum_j q'_j \pi'_j) = \sum_j q'_j J(x_1, \pi_j)$
for all U and x_1. Thus Y is more informative than Y'.

Note that in this case, we have

$\max_{x_1} \sum_j q_j \max_{x_2} \sum_i \pi'_i U(x_1, x_2, z_i) = \max_{x_1} \sum_j q_j \max_{x_2} \sum_i r_i U(x_1, x_2, z_i) =$
$\max_{x_1, x_2} \sum_i r_i U(x_1, x_2, z_i)$, that is we maximize the unconditional expectation of the benefits, and no adjustments need to be done with respect to forthcoming information.

We also have

$\sum_{j=1}^m q"_j \max_{x_2} \sum_i \pi"_{i,j} U(x_1, x_2, z_i) = \sum_j q"_j \max U(x_1, x_2, z_j) = \sum_i r_i \max_{x_2} U(x_1, x_2, z_i) = \sum_j q_j \sum_i \pi_{i,j} \max_{x_2} U(x_1, x_2, z_i) \leq \sum_j q_j \max_{x_2} \sum_i \pi_{i,j} U(x_1, x_2, z_i)$

for all U and x_1 and for all feasibility sets. Thus Y" is more informative than Y.

Another way of understanding the definition of being "more informative than" is to look at the case when

$$Z = Y + \delta,$$

where δ is the observation error Assume that δ is normally distributed with zero mean and is independent of Z. It is now easy to demonstrate that if we are comparing two distributions of δ, the one with smaller variance is more informative than the other(Bradford & Keleian 1977).

Marshak & Miyasawa (1968) proved the following characterization of Y being more informative than Y'.

Theorem 3 *Y is more informative than Y' if and only if for every convex function ρ on S^{m-1} $(S^{m-1} = \{\xi \in R^m; \xi_i \geq 0, \ \sum_i \xi_i = 1\})$*

$$\sum_j q_j \rho(\pi_j) \leq \sum_j q'_j \rho(\pi'_j) \tag{61}$$

.

The if part is easy to prove. We know that $J(x_1, \xi)$ is convex in ξ. If 61 is true, we necessarily must have $\sum_j q_j J(x_1, \pi_j) \leq \sum_j q'_j J(x_1, \pi'_j)$. The converse is much harder to prove and the reader is referred to Marshak & Miyasawa (1968).

3.5   Irreversibility

Using this characterization, **?** proved the following theorem:

Theorem 4 *Let X'_1 and $X"_1$ be the solutions of our problem when the information structures are Y' and Y" resp. Assume that Y' is more informative than Y". Assume that J(X_1, ξ) is concave and differentiable in X_1. If $\frac{\partial J}{\partial x_1}$ is concave (convex) in x_1 then $X'_1 \leq X"_1$ $(X'_1 \geq X"_1)$.*

The simple proof goes as follows:

Assume $J_{x_1} = \partial J / \partial x_1$ is convex in ξ, then, because Y' is more informative than Y" it follows from 61 and the fact that x_1 and x_2 maximize that

nce Jx1x1 < 0 it follows that Xi < x1. If J is concave, Jx is convex end x1 > xi.

We can now use Epstein's theorem to obtain a slight generalization of results obtained by Henry and Arrow and Fisher.

Theorem 5 *Assume either that i) X_1 and X_2 only can take on the values 0 and 1 and $X_1 \leq X$ or ii) that $\sum q_j J(X_1..)$ is linear in X_1 and $0 \leq X_1 \leq \ X_2 \leq 1$. Then if Y' is more informative than Y", it follows that if $X_1 = 0$ then $X_2 = 0$.*

(The first part of the theorem is already in Epstein and the second part corresponds to Arrow- Fisher's result.)

Proof. In both cases X'_1 and $X_1"$ are either 0 or 1. Suppose $X_1" = 0$ and hence

$$\sum_{j=1}^m q_j" J(0, \pi_j") > \sum_j q_j" J(1, \pi_j").$$

As Y' is more informative than Y", we have

$$\sum_{j=1}^{m} q_j' J(0, \pi_j') \geq \sum_j q_j" J(0, \pi").$$

Moreover, as the change is irreversible, that is $x_2 \geq x_1$,

$$\sum_j^m q_j" J(1, \pi_{ij}") = \sum_j \sum_i q_j" \pi_{ij}" U(1, 1, z_i) =$$

$$= \sum_i r_i U(1, 1, z_i) =$$

$$= \sum_j \sum_i q_j' \pi_{i,j}' U(1, 1, z_i) = \sum_j q_j' J(1, \pi_i')$$

Combining the above, we have

$$\sum_j q_j' J(0, \pi_j') > \sum_j q_j' J(1, \pi_j')$$

and $x_1' = 0$. ∎

Note that the inequality $x_1 \leq x_2$ reflects an irreversibility in the decision. If a resource use equal to x_1 has been decided, future resource use must be equal to or exceed this amount. Thus, if we are going to make an irreversible decision (building a hydropower plant, developing Yellowstone National Park to an industrial site etc.), then the prospect of getting more information in the future on costs and benefits will not increase the benefits of undertaking the development now.

However, this theorem was based on the assumption that the optimal value of x_1 is either 0 or 1. The theorem is not necessarily true if the assumptions yielding this are abandoned, unless other restrictions are introduced. Such restrictions apply to the net benefit or utility function. Assuming that the U function takes the special form

$$U(X_1, X_2, z_i) = U(X_1) + v(X_2, z_i) \tag{62}$$

where U and v are strict concave functions of x_1 and x_2 a corresponding result can be derived.

We are thus considering the problem

$$\max_{X_1} \{ U(X_1) + \sum_j \max_{X_2 \geq X_1} \sum_i \pi_{ij} v(X_2, z_i) \}$$

x_1 may be interpreted as the development in period 1, giving net benefits $U(x_1)$ and x_2 is the total development in the next period yielding the present value of the net benefits equal to $v(x_2, z_i)$ The irreversibility is expressed in the condition $x_2 \geq x_1$.

The J function becomes

$$J(X_1, \xi) = \max_{X_2 \geq X_1} \sum_i \xi_i v(X_2, z_i)$$

Assume that $X_2'(X_1)$ solves this maximum problem, where X_2' is continous and piecewise differentiable. Then

$$J(X_1, \xi) = \max_{X_2 \geq X_1} \sum_i \xi_i v(X_2'(X_1), z_i)).$$

Furthermore,

$$\frac{dX_2'}{dX_1} = \begin{cases} 0 \text{ if } X_2' > X_1 \\ 1 \text{ if } X_2' = X_1 \end{cases}$$

and therefore

$$\frac{\partial J}{\partial X_1} = \sum_i \frac{\partial v(X_2', z_i)}{\partial X_{2ii}} \frac{dX_2'}{dX_1} = \begin{cases} 0 \text{ if } X_2' > X_1 \\ \sum_i \xi_i \frac{\partial v}{\partial X_2'} \text{ if } X_2' = X_1 \end{cases} \tag{63}$$

Moreover, $X_2' > X_1$ if and only if

$$\sum_i \xi_i \frac{\partial v(X_2', z_i)}{\partial X_2} > 0$$

Thus

$$\frac{\partial J}{\partial X_1} = \min \left\{ 0, \ \sum_i \xi_i \frac{\partial v}{\partial X_2} \right\} \tag{64}$$

As both terms in he bracket are concave, it follows that $\frac{\partial J}{\partial X_1}$ is concave in ξ. Then it follows from Epstein's theorem that

$$X_1' \leq X_1\text{"}$$

We then have the following theorem

Theorem 6 *If $U(X_1, X_2, z_i) = U(X_1) + v(X_2, z_i)$ and $U(X_1)$ and $v(X_2, z_i)$ are concave functions of X_1, and X_2, respectively, and if the optimal value for X_1 with information structure Y' is X_1', and with information structure Y" is X_1", then*

$$X_1' \leq X_1\text{"}$$

Thus, a fairly general proposition has been established. If more information will be available in the future, "less" irreversible changes should be undertaken now.

3.6   Irreversibility at a cost

Let us now consider the case when it is possible to restore the development but at a cost. This means that we will replace the restriction $X_2 \geq X_1$ with a cost function for the measures that are necessary to make $X_2 < X_1$ feasible.

If the decision in the first period is X_1, assume it is possible to restore the resource in the second period to $X_2 < X_1$, but at a cost given by the cost function $c(X_1, X_2)$ defined by

$$c(X_1, X_2 = \begin{cases} 0 \text{ if } X_1 < X_2 \\ \gamma(X_1 - X_2) \text{ if } X_1 > X_2 \end{cases} \qquad (65)$$

with γ as a constant. Note that with this cost function, cost is a continuous function of the amount of restoration but the marginal cost is discontinuous at zero restoration. We will later look at a different formulation of the cost function. As in the previous section, we define the J-function (making the same intertemporal separability assumption as there) as

$$J(X_1, \xi) = \max_{X_2} \left\{ \sum_i \xi_i v(X_2, z_i) - c(X_1, X_2) \right\} \qquad (66)$$

Define $\overline{X_2}$ from

$$\max_{X_2} \sum_i \xi_i v(X_2, z_i) = \sum_i \xi_i v(\overline{X_2}, z_i),$$

that is $\overline{X}_2 = \arg\max \sum_i \xi_i v(X_2, z_i)$. Assume v is diffentiable and define $\underline{X_2}$ from

$$\sum_i \xi_i \frac{\partial v(\underline{X_2}, z_i)}{\partial X_2} + \frac{\partial c(X_1, \underline{X_2})}{\partial X_2} = \sum_i \xi_i \frac{\partial v(\underline{X_2}, z_i)}{\partial X_2} - \gamma = 0$$

$\overline{X_2}$ is the upper bound for the set of X_1 at which no restoration will take place and $\underline{X_2}$ is similarly the lower bound for the set of ∎X_1 at which restoration will take place.

As v is concave, it follows that $\underline{X_2} < \overline{X}_1$. Let us now study the choice of optimal X_2^*, contingent upon the choice of $\overline{X}_1$. We have
  i)   If $0 \leq X_1 \leq \overline{X}_2$, then $X_2^*(X_1) = \overline{X}_2$, and $\partial J/\partial X_1 = 0, dX_2^*/dX_1 = 0$,
  ii) If $\overline{X}_2 \leq X_1 \leq \underline{X}_2$, then $X_2^*(X_1) = X_1$ and $\partial J/\partial X_1 = \sum_i \xi_i \frac{\partial v(X_2^*(X_1), z_i)}{\partial X_2} \frac{dX_2^*}{dX_1}$
  iii) If $\underline{X}_2 \leq X_2$, then $X_2^* = \underline{X}_2$, $\partial J/\partial X_1 = -\gamma$, $\frac{dX_2^*(X_1)}{dX_1} = 0$
  which can be illustrated in a diagram

Obviously, $\partial J/\partial X_1$ is not a concave function , unless $\gamma = -\infty$, but that corresponds to the case we just analyzed, i.e. the pure irreversibility case.

As $\partial J/\partial X_1$ is neither convex nor concave as a function of X_1, it follows from Epstein's theorem that there exist information schemes Y, Y', and Y" and a random variable Z such that both Y' and Y" are more informative than Y and such that the optimal X_1' for Y' exceeds X_1 (the optimal choice for Y) and the optimal X_1" for Y" is less than X_1. Thus, it is impossible to say anything globally on the existence of the "irreversibility effect".

However, if $\partial v/\partial X_2$ is linear in X_2, $\partial J/\partial X_1$ is convex in the interval $[0, \overline{X_2}]$. Let X_1' be the optimal choice of resource use in period 1 if the information scheme is Y' and let X_1" be the corresponding choice if the scheme is Y" and assume Y' is more informative than Y". Furthermore, assume X_1' belongs to the interval $[0, \overline{X_2}]$. Then, if X_1 would have been restricted to that interval, the optimal choice would still have been X_1. It now follows from Epstein's theorem that

$$X_1' \geq X_1".$$

Thus, if the initial resource use is small enough, then the prospect of getting more information in the future will increase the initial use.

25

Figure 3:

In the same way, it is seen that $\partial J / \partial X_1$ is concave in the interval $[\underline{X}_2, \max X_2]$, so that if X_1' is in this interval,

$$X_1' \leq X_1".$$

Finally, it follows that if X_1' belongs to $[\overline{X}_2, \underline{X}_2]$,

$$X_1' = X_1",$$

as $X_2^* = X_1$ on that interval.

Thus, if the initial resource use is larger than $\underline{X}_2$, then the prospect of more information will reduce the initial use, while if the initial resource use is smaller than $\overline{X}_2$ the prospect of more information will increase the initial resource use. We can therefore conclude by stating the following theorem.

Theorem 7 *If it is possible to restore the resource according to the cost function defined in (65), then for "small" initial resource use, an increase in expected forthcoming information will increase that initial resource use, while if the initial resource use is "large", more expected information will reduce the initial resource use.*

This result may have an implication for the current discussion on global warming and emissions of green house gases. It has been claimed that the uncertainty about future impacts from climate change should imply that we would reduce the emissions more than would be desirable if no future information is forthcoming. However, the opposite has also been argued, and it may be that this theorem explains why serious scholars have come to such different conclusions.

Now, the theorem is a result of the discontinuity of the cost function. If we modify the cost function, a different result will emerge. Assume then that the cost function can be written

$$C(X_1, X_2) = \omega(X_1 - X_2) \tag{67}$$

Figure 4:

where $\omega(.)$ is strictly concave, $\omega(0) = 0$, $\omega'(0) = 0$, and $\omega(t) = 0$ for t<0. The $\partial J/\partial X_1$ curve will now look like the curve in the following diagram:

It is now clear that the curve is concave and that the irreversibility effect is global.

Theorem 8 *If the cos tfunction looks like (67) with the properties described above, there will be a global irreversibility effect.*

3.7 The value of information

We can now define the value of one information scheme relative to another, conditional on the decision resource use in the first period as

$$V(X_1, Y", Y') = \sum_j q_j" J(X_1, \pi_j") - \sum_j q_j' J(X_1, \pi_j') \qquad (68)$$

If Y" is more informative than Y', we know by definition that $V(X_1, Y', Y") > 0$. V gives a measure of how much the expected utility from a resource use equal to X_1 will increase, if the forthcoming information changes from Y' to Y". It is natural to define a zero point for this value by choosing the case of no forthcoming information as a reference point. Thus the value of information Y, conditional on the resource use X_1 in the first period as

$$V(X_1, Y) = \sum_j q_j J(X_1, \pi_j) - \sum_j q_j \max_{X_2} \sum_i r_i U(X_1, X_2, z_i).$$

But as $\sum_j q_j = 1$, we have

$$V(X_1, Y) = \sum_j q_j J(X_1, \pi_j) - \max_{X_2} \sum_i r_i U(X_1, X_2, z_i). \qquad (69)$$

In particular, the value of perfect information Y, conditional on the resource use X_1 in the first period is

$$\sum_i r_i \max_{X_2} U(X_1, X_2, z_i) - \max_{X_2} \sum_i r_i U(X_1, X_2, z_i). \qquad (70)$$

The unconditional value of information from the scheme Y is defined as

$$W(Y) = \max_{X_1} \sum_j q_j J(X_1, \pi_j) - \max_{X_2} \sum_i r_i U(X_1, X_2, z_i). \qquad (71)$$

It is easily seen that W(Y) is always non-negative and strictly positive whenever Y is strictly more informative than no information at all, and $\sum_j q_j J(X_1, \pi_j)$ has a unique maximum as a function of X_1.

Assume now that there is a decision maker that does not take the possibility of future information into account. He will thus solve

$$\max_{X_1, X_2} \sum_i r_i U(X_1, X_2, z_i) \qquad (72)$$

However, if he would be paid a subsidy equal to the value of the forthcoming information (conditional on his choice of X_1), he would solve

$$\max_{X_1, X_2} \left\{ \sum_i r_i U(X_1, X_2, z_i) + V(X_1, Y) \right\} = \max_{X_1} \sum_j q_j \max_{X_2} \sum_i \pi_i U(X_1, X_2, z_i)$$

Thus, a decision maker could be made to take forthcoming information into account by being subsidized with an amount equal to the value of information conditional on his choice of resource use in the first period.[10] However, if the decision maker is rational, he should of course have taken this information into account when he makes the decision and if that is the case, the subsidy would only distort the decision. Basically, there is no need for a subsidy, because the only kind of market failure that would cause an individual from considering the possibility of future information is irrationality. This kind of market failures cannot be solved by subsidies.

3.8   Uncertainty about irreversibility

Assume now that we don't know whether a decision today will have as a consequence an irreversible change in an environmental resource. Let us therefore assume that there is a positive probability p that the change will be irreversible and a corresponding positive probability 1-p that the change is reversible. In terms of the notations used in previous sections, the optimization problem is

$$\max_{X_1} \left\{ U(X_1 + (1-p) \sum_j q_j \max_{X_2} \sum_i \pi_{i,j} v(X_2, z_i) + p \sum_j q_j \max_{X_2 \geq X_1} \sum_i \pi_{i,j} v(X_2, z_i) \right\}$$
$$(73)$$

The solution to this problem will be compared first, with the solution to the problem when irreversibility is not expected, and second with the solution to the problem when it is known for sure that the change is irreversible. The first of these problems is to determine the solution to

$$\max_{X_1} \left\{ U(X_1 + \sum_j q_j \max_{X_2} \sum_i \pi_{i,j} v(X_2, z_i) \right\}$$

---

[10] This interesting interpretation of the value of information is due to Fisher and Hanemann(Fisher & Hanemann 1986)

and the second problem is exactly the one we have discussed in previous sections. Denote the solution to the first problem X_1^r and the solution to the second problem X_1^i. We know that if more information is expected to come forth in the future,

$$X_1^r \geq X_1^i.$$

Let $\overline{X_1}$ be the solution to the problem when irreversibility is uncertain. Let us for simplicity assume an interior solution. For any choice of X_1, we have for each j the optimal $X_{2,j}$ defined by

$$\max_{X_{2,j} \geq X_1} \sum_i \pi_{i,j} v(X_{2,j}, z_i)$$

The sum is a linear combination of concave functions and is therefore concave. The optimal $X_{2,j}$ will obviously be a function of X_1: $X_{2,j} = X_{2,j}^i(X_1)$. Let $X_{2,j}^r$ be the corresponding solution when there is no irreversibility constraint. If there is no such constraint, $\frac{dX_{2,j}^r}{dX_1} = 0$, otherwise $\frac{dX_{2,j}^r}{dX_1} = 1$. The assumption of an interior solution now yields (suppressing the random variable z_i)

$$\frac{\partial U(\overline{X}_1)}{\partial X_1} + \sum_j q_j \sum_i \pi_{i,j} \frac{\partial v}{\partial X_2} \frac{dX_2^i}{dX_1} = 0 \tag{74}$$

or

$$\frac{\partial U(X_1^i)}{\partial X_1} = -\sum_j q_j \sum_i \pi_{i,j} \frac{\partial v}{\partial X_2} \frac{dX_2^i}{dX_1} \geq$$

$$-p \sum_j q_j \sum_i \pi_{i,j} \frac{\partial v}{\partial X_2} \frac{dX_2^r}{dX_1} = \frac{dU(\overline{X}_1)}{dX_1} \geq 0 = \frac{dU(x_1^r)}{dX_1} \tag{75}$$

The concavity of the U-function now yields that

$$X_1^i \leq \overline{X}_1 \leq X_1^r$$

Thus we have reached the intuitively obvious but very important conclusion that when it is not known for sure that a change is reversible or not, it is better to be cautious and not undertake as much development as would have been optimal if the change had known to be reversible. Note that this conclusion is not dependent on linearity, or a binary choice or even on the possibility of forthcoming information. Whenever, one is uncertain about the reversibility of a change, one should be cautious and not undertake as big change one would have had desired if the change had been known with certainty to be reversible.

If it not known whether a change is going to be irreversible or not, it is still beneficial to be cautious in that one should not undertake as big change as one would have had desired if the change had been known to be reversible. It is easily seen from the inequality above that the optimal amount of development will decrease as the probability of irreversibility goes up.

Finally, it is easily seen that if the utility function is linear or if there is a binary choice, the optimal amount of development will be non increasing with the probability of an irreversible change.

3.9   Option values

Let us now try to integrate the model with temporal resolution of uncertainty which we have been discussing in this part with the discussion in the first part on

option prices and option values. Consider the situation where Q is the measure of environmental resource use and which can take two values Q', implying that the natural asset is preserved, and Q" that it is developed and irrevocably lost. The benefits in the present period are known with certainty and are given by the indirect utility function

$$U(W^1, Q^1) \tag{76}$$

where W^1 is wealth in the first period and Q^1 can take the two values Q' or Q". The present value of future net benefits, given that state i occurs is

$$v^i(W_i, Q^2)/(1 + \delta_{i)} \tag{77}$$

where W_i is the future wealth if state i occurs, v^i is the utility function in state i, δ_i is the discount rate in state i, and Q^2 takes the values Q' or Q". The total present value of benefits, given that state i occurs is

$$U(W^1, Q^1) + \frac{v^i(W_i, Q^2)}{1 + \delta_i} \tag{78}$$

where $Q^2 = Q^1$ if $Q^1 = Q$" (the irreversibility assumption).Assuming the same information structure as in the previous section, the decision problem can be formulated

$$\max_{Q^1} \left\{ U(W^1, Q^1) + \sum_j q_j \max_{Q^2} \sum_i \pi_{i,j} v^i(W_i, Q^2)/(1 + \delta_i) \right\}$$
$$subject\ to\ Q^2 = Q" \ if\ Q^1 = Q" \tag{79}$$

The maximum price, CV, the individual would be willing to pay for keeping the option of deciding in the next period the use of the resource is given by

$$U(W^1 - CV, Q') + \sum_j q_j \max_{Q^2} \sum_i \pi_{i,j} v^i(W_i, Q^2)/(1 + \delta_i) = \tag{80}$$

$$= \quad U(W, Q") + \sum_j q_j \sum_i \pi_{i,j} v^i(W_i, Q")/(1 + \delta_i) \tag{81}$$

The state contingent benefits CV_i of preserving the asset for at least one period are given by

$$U(W^1 - CV_i, Q') + \max_{Q^2} v^i(W_i, Q)/(1 + \delta_i) = U(W^1, Q") + v^i(W_i, Q")/(1 + \delta_i) \tag{82}$$

Thus

$$v^i(W_i, Q") = U(W^1 - CV_i, Q') + \max_{Q^2} v^i(W_i, Q)/(1 + \delta_i) - U(W^1, Q") \tag{83}$$

and substituting this into (62) gives

$$U(W^1 - CV, Q') + \sum_j q_j \max_{Q^2} \sum_i \pi_{i,j} v^i(W_i, Q^2)/(1 + \delta_i) =$$

$$\sum_j \quad q_j \sum_i \pi_{i,j} U(W^1 - CV_i, Q') + \sum_j q_j \sum_i \pi_{i,j} \max_{Q^2} v^i(W_i, Q)/(1 + \delta_i) \tag{84}$$

This simplifies to (remembering that $\sum_j q_j \pi_{i,j} = r_i$)

$$U(W^1 - CV, Q') - \sum_i r_i U(W^1 - CV_i, Q') = \tag{85}$$

$$= \sum_j q_j \left\{ \sum_i \pi_{i,j} \max_{Q^2} v^i(W_i, Q)/(1+\delta_i) - \max_{Q^2} \sum_i \pi_{i,j} v^i(W_i, Q^2)/(1+\delta_i) \right\} \tag{86}$$

or

$$U(W^1 - CV, Q') - \sum_i r_i U(W^1 - CV_i, Q') \leq 0 \tag{87}$$

If, as we assume, the utility function U is concave, it follows that

$$\sum_i r_i U(W^1 - CV_i, Q') \leq U(W^1 - \sum_i r_i CV_i, Q') \tag{88}$$

and finally

$$U(W^1 - CV, Q') \geq U(W^1 - \sum_i r_i CV_i, Q') \tag{89}$$

or

$$CV \leq \sum_i r_i CV_i \tag{90}$$

Thus, the option value as defined by Weisbrod, Cichetti and Freeman[11]

3.10 Uncertainty about irreversibility

Assume now that we don't know whether a decision today will have as a consequence an irreversible change in an environmental resource. Let us therefore assume that there is a positive probability p that the change will be irreversible and a corresponding positive probability 1-p that the change is reversible. In terms of the notations used in previous sections, the optimization problem is

$$\max_{X_1} \left\{ U\left(X_1 + (1-p)\sum_j q_j \max_{X_2} \sum_i \pi_{i,j} v(X_2, z_i) + p \sum_j q_j \max_{X_2 \geq X_1} \sum_i \pi_{i,j} v(X_2, z_i) \right) \right\} \tag{91}$$

The solution to this problem will be compared first, with the solution to the problem when irreversibility is not expected, and second with the solution to the problem when it is known for sure that the change is irreversible. The first of these problems is to determine the solution to

$$\max_{X_1} \left\{ U\left(X_1 + \sum_j q_j \max_{X_2} \sum_i \pi_{i,j} v(X_2, z_i) \right) \right\}$$

and the second problem is exactly the one we have discussed in previous sections. Denote the solution to the first problem X_1^r and the solution to the second problem X_1^i. We know that if more information is expected to come forth in the future,

---

[11] See ?, Rod (1964), and J.V. Krutilla & Russel (1972).

31

$$X_1^r \geq X_1^i.$$

Let $\overline{X_1}$ be the solution to the problem when irreversibility is uncertain. Let us for simplicity assume an interior solution. For any choice of X_1, we have for each j the optimal $X_{2,j}$ defined by

$$\max_{X_{2,j} \geq X_1} \sum_i \pi_{i,j} v(X_{2,j}, z_i)$$

The sum is a linear combination of concave functions and is therefore concave. The optimal $X_{2,j}$ will obviously be a function of X_1: $X_{2,j} = X_{2,j}^i(X_1)$. Let $X_{2,j}^r$ be the corresponding solution when there is no irreversibility constraint. If there is no such constraint, $\frac{dX_{2,j}^r}{dX_1} = 0$, otherwise $\frac{dX_{2,j}^r}{dX_1} = 1$. The assumption of an interior solution now yields (suppressing the random variable z_i)

$$\frac{\partial U(\overline{X_1})}{\partial X_1} + \sum_j q_j \sum_i \pi_{i,j} \frac{\partial v}{\partial X_2} \frac{dX_2^i}{dX_1} = 0 \tag{92}$$

or

$$\frac{\partial U(X_1^i)}{\partial X_1} = -\sum_j q_j \sum_i \pi_{i,j} \frac{\partial v}{\partial X_2} \frac{dX_2^i}{dX_1} \geq$$
$$-p \sum_j q_j \sum_i \pi_{i,j} \frac{\partial v}{\partial X_2} \frac{dX_2^r}{dX_1} = \frac{dU(\overline{X_1})}{dX_1} \geq 0 = \frac{dU(x_1^r)}{dX_1} \tag{93}$$

The concavity of the U-function now yields that

$$X_1^i \leq \overline{X}_1 \leq X_1^r$$

Thus we have reached the intuitively obvious but very important conclusion that when it is not known for sure that a change is reversible or not, it is better to be cautious and not undertake as much development as would have been optimal if the change had known to be reversible. Note that this conclusion is not dependent on linearity, or a binary choice or even on the possibility of forthcoming information. Whenever, one is uncertain about the reversibility of a change, one should be cautious and not undertake as big change one would have had desired if the change had been known with certainty to be reversible.

If it not known whether a change is going to be irreversible or not, it is still beneficial to be cautious in that one should not undertake as big change as one would have had desired if the change had been known to be reversible. It is easily seen from the inequality above that the optimal amount of development will decrease as the probability of irreversibility goes up.

Finally, it is easily seen that if the utility function is linear or if there is a binary choice, the optimal amount of development will be non increasing with the probability of an irreversible change.

3.11   Conclusions

4   Annex On the equivalence of two representa-

tions of environmental changes.

Assume that λ' has a density $\nu(i)$ (Radon-Nikodym derivative) (Halmos 1965) with respect to λso that

$$\lambda_i' = \nu(i)\lambda_i$$

Assume furthermore that Q_i is increasing in i, $U_i(W_i, Q)$ is increasing in Q and $v(i) > 0$.

Then define

$$Q_i' = U^{i^{-1}}W_i, U^i(W_i, Q_i v(i))$$

where $U^{i^{-1}}$ is the inverse function to $\partial U^i / \partial Q$

Then it is easily seen that

$$\int U^i(W_i, Q_i')d\lambda_i = \int U^i(W_i, Q_i)d\lambda_i'$$

Moreover, assume that the policy change $\lambda \to \lambda'$ means an increase in the probability of states with a high Q, i. e. that $\lambda_i \geq \lambda_i'$. This in turn implies that $v(i) \geq 1$ and it

follows that

$$Q_i' = U^{i^{-1}}(W_i, U^i(W_i, Q_i v(i))) \geq U^{i^{-1}}(W_i, U^i(W_i, Q_i)) = Q_i$$

so that the new Q' variable corresponds to a higher (or unchanged) Q in each state, a result we would demand of an economic reasonable representation.

The converse proposition follows from a simple variable substitution, i.e. let Q' = T(Q) where T is measurable. Then it follows from Halmos[12] that

$$\int U^i(W_i, T(Q_i))d\lambda_i = \int U^i(W_i, Q_i)d(\lambda_i T^{-1})$$

and we choose $d\lambda_i' = d(\lambda_i T^{-1})$.

References

Arrow, K. J. & Fisher, A. C. (1974), 'Environmental preservation, uncertainty, and irreversibility', *American Economic Review* **88**, 312–320.

Arrow, K. & Lind, R. (1972), 'Uncertainty and the evaluation of public investment decisions', *American Economic Review* **60**.

Bishop, R. (1982), 'Option value: An exposition and extrension', *Land Economics* **58**.

Blackwell, D. (1951), Comparisons of experimnents, *in* J. Neyman, ed., 'Proceedings of the Second Berkley Symposium on Mathematical Statistics and Probability'.

---

[12](Halmos 1965), §32

Bohm., P. (1975), 'Option demand and consumer's surplus: A comment', *American Economic Review* **65**.

Bradford, K. & Keleian, H. (1977), 'The value of information for crop forecasting in a market system', *The Review of Economic Studies* **44**.

Cicchetti, C. & A.M. Freeman III, . (1971), 'Option demand and consumer surplus: Further comment', *Quarterly Journal of Economics* **85**.

Epstein, L. (1980), 'Decision making and the temporal resolution of uncertainty', *International Economic Review* **21**.

Fisher, A. C. (1973), 'Environmental externalities and the arrow-lind public investment theorem', *American Economic Review* **63**.

Fisher, A. & Hanemann, M. (1986), 'Option value and the extinction of species', *Advances in Appplied Microeconomics 4* pp. 169–190.

Halmos, P. (1965), *Measure Theory.*

Hanemann, W. (1989), 'Information and the concept of option value', *Journal of Environmental Economics and Management* **16**, 23–37.

Henry, C. (1974), 'Investment decisions under uncertainty: The irreversibility effect', *American Economic Review 64* pp. 1006–1012.

J.V. Krutilla, C.J. Cicchetti, A. F. I. & Russel, C. (1972), Observations on the economics of irreplaceable assets, *in* A. Kneese & B. Bower, eds, 'Environmental Quality Analysis:Theory and Method in the Social Sciences', John Hopkins University Press.

Malinvaud, E. (1972), 'Malinvaud72', *Journal of Economic Theory* **4**.

Marshak, J. & Miyasawa, K. (1968), 'Economic comparability of information systems', *International Economic Review* **9**.

Raiffa, H. & Schleifer, R. (1961), *Applied Statistical Decision Theory.*

Rod, B. W. (1964), 'Collective-consumption services of indivöidual consumption goods', *Quarterly Journal of Economics* **78**.

Schmalensee, R. (1972), 'Option demand and consumer's surplus:valuing price changes under uncertainty', *American Economic Review* **62**.

Weisbrod, B. (1964), 'Collective-consumption services of individual consumption goods', *Quaterly Journal of Economics* **78**.