Wealth and well-being in a model with discrete time.

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1. Introduction

In "Net National Product, Wealth and Social Well-being"[1], Dasgupta and Mäler analysed indicators of well-being in a model of economic growth. However, they developed their analysis in terms of continuous time, although all empirical applications will and must be carried out in discrete time (typically with a time period of one year). In this appendix, we will develop a similar analysis but in discrete time. In order to be brief, the model we analyse will be the simplest possible, i.e. only one consumption good and only one capital good. Extensions to more complex models will be obvious.

2. A simple growth model with constant population

Consider one economy with one single capital good K, which is the only input in the production of the single consumption good c:

$$\dot{K}_\tau = f(K_\tau) - c_\tau \tag{2.1}$$

Felicity or instantaneous utility at time t is given by $U(c_t)$, and social well-being is given by

$$V_t = \sum_{\tau=t}^{\infty} \frac{U(C(\tau))}{(1+\delta)^{\tau-t}} \tag{2.2}$$

that is by the present value of future felicities with δ as the discount rate.[2] In order have V well defined, δ must be non-negative. Notice that δ is the utility discount rate and not the discount rate applicable to discounting monetary costs and benefits. The importance of this observation is discussed in Dasgupta, Mäler, and Barrett (1999). In any serious application consumption c should be regarded as a vector with components representing all flows that affect current well-being U, including flows of environmental damages, labour supply etc. See Mäler (1991) and Dasgupta and Mäler (2000) for a discussion of such issues.

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[1] P. Dasgupta and K.-G. Mäler, 2000, Net National Product, Wealth, and Social Well-being, *Environment and Development Economics*, 5, 69-93

[2] For a axiomatic derivation of this formulation of social welfare, see Koopmans (1960). See also Dasgupta, Mäler, and Barrett (1999)

One major novelty with Dasgupta's and Mäler's article was that they showed that it is perfectly possible to avoid the established procedure of considering only optimal paths by using an abstract notion of a resource allocation mechanism α. A resource allocation mechanism is a mapping from the initial stock of capital into the feasible time paths of consumption. If we start at time t with K_t, then consumption at time ι is given by

$$C_\tau = \alpha(\tau, t, K_t) \tag{2.3}$$

α incorporates whatever we know about the working of the institutions in the economy, now and in the future. We will mostly (but not always) assume that the resource allocation mechanism is time autonomous, that is

$$\alpha(\tau + s, t + s, K) = \alpha(\tau, t, K) \tag{2.4}$$

for all s. In this case, calendar time does not matter but only the length of time after the nitial point of time. With the resource allocation mechanism α, the social well-being functional (or shorter the value function) can be written

$$V(\alpha(.,t,K_t)) = \sum_{\tau=t}^{\infty} \frac{U(\alpha(\tau,t,K_t))}{(1+\delta)^{\tau-t}} \tag{2.5}$$

Social well-being depends on the initial capital stock K_t and the efficiency of the economy, as revealed by the resource allocation mechanism. We will in sequel keep the resource mechanism constant (although see Dasgupta and Mäler op. cit. for analysis of changing mechanisms). We define the accounting or shadow price of the capital stock as the present value of future changes in felicities following an increase in the current stock of capital with one unit, that is as

$$\frac{\partial V}{\partial t} = p_t \tag{2.6}$$

3. **Wealth and Social well-being**

Let us interpret sustainable development as a development path along which social well-being is never decreasing, that is such that

$$V_{t+1} \geq V_t \tag{3.1}$$

where $V_t = V(\alpha(.,t, K_t)$.

We now find that, assuming that the resource allocation mechanism is time autonomous,

$$V_{t+1} - V_t = \sum_{\tau=t} \frac{U(\alpha(\tau+1, t+1, K_{t+1}))}{(1+\delta)^{\tau-t}} - \sum_{\tau=t} \frac{U(\alpha(\tau, t, K_t))}{(1+\delta)^{\tau-t-1}} =$$

$$= \sum_{\tau=t} \frac{1}{(1+\delta)^{\tau-t}} \{U(\alpha(\tau+1, t+1, K_{t+1})) - U(\alpha(\tau, t, K_t))$$

Note that because of the assumption of time autonomous resource allocation mechanism it holds that

$$\alpha(\tau+1, t+1, K_{t+1}) = \alpha(\tau, t, K_{t+1})$$

and therefore

(3.2)

$$V_{t+1} - V_t = \sum_{\tau=t} \frac{1}{(1+\delta)^{\tau-t}} \left\{ U(\alpha(\tau, t, K_{t+1})) - U(\alpha(\tau, t, K_t)) \right\} =$$

$$= \sum_{\tau} \frac{U'(\alpha(\tau, t, \tilde{K}))}{(1+\delta)^{\tau-t}}$$

where $\tilde{K}$ is a number between K_t and K_{t+1}. If K_{t+1} and K_t are close, then

$$\sum \frac{U'(\alpha(\tau, t, \tilde{K}_{\tau,t}))}{(1+\delta)^{\tau-t}} \approx p_t$$

and finally

$$V_{t+1} - V_t = p_t(K_{t+1} - K_t) \qquad (3.3)$$

Thus, the change in social welfare from one time period to the next is (approximately) equal to the value of the change in the capital stock.[3] Note that this is not the same as the

[3] In Dasgupta and Mäler, there is a general theory including many assets. Note that with continuous time, we have an exact result while with discrete time, we only have an approximate result. The reason is, of course, that during a non-infinitesimal interval, changes in the capital stock will not be infinitesimal and with the accounting

change in the value of the capital stock, and therefore implies that capital gains should not be included in the calculation of the change in well-being. In a later section, we will look at the situation when the resource allocation mechanism is not time-autonomous, and we will find that there are situations when capital gains should be included!

4. Choice of numeraire

Due to homogeneity in prices and income, economic models do not depend on the choice of a numeraire. However, from a practical point of view, the choice of numeraire may be very important. For example, the accounting price p, which was introduced in the previous section, uses utility as the numeraire, and as utility is not easily observed, it cannot be used for empirical studies. Therefore, we have to look into alternatives.

The simplest way is, perhaps to use consumption in each period as a numeraire. This implies, of course, that the numeraire changes from one period to the next and we have to add rules on how to relate the different numeraires to each other. In order to use this approach, we simply divide the accounting price in each period with the marginal utility of consumption in the same period. Thus, with π_t as the current value price,

$$\pi_t = \frac{p_t}{U'(c_t)} \tag{4.1}$$

We compare utility levels between different time periods by using the utility discount rate δ. Similarly, if we have to compare current value prices in different periods, we should use the consumption rate of discount r as the following analysis shows.

$$\frac{\dot{\pi}_t}{\pi_t} = \frac{\dot{p}_t}{p_t} - \frac{U''(c_t)}{U'(c_t)}\dot{c}_t = \frac{\dot{p}_t}{p_t} - \frac{U''(c_t)}{U'(c_t)}c_t\frac{\dot{c}_t}{c_t} = \frac{\dot{p}_t}{p_t} - \eta g \tag{4.2}$$

where η is the elasticity of marginal utility and g the growth rate of consumption. We know that the market rate of interest r is given by

$$r = \delta - \eta g \tag{4.3}$$

Thus, if we know the consumption rate of interest r(which should be the market (real) rate of interest after taxes) and the utility rate of discount δ(which can be inferred from market rates in periods when g=0), ηg can be estimated and we can from observations of current value prices find the present value prices needed for evaluation of the value of the change in assets.

5. Calculation of the capital stock

In the growth model in the previous section, the capital stock was defined in physical units. For reasons that will become clear in the next section (when we introduce population

price defined as the marginal price, no exact result is possible. On the other hand, if we define the accounting price as a suitable average price over the time interval, an exact result is possible, but hardly useful.

changes), we are not only interested in the value of the change in the capital stock but also of the size of the value of the stock itself. In this section, we will derive a simple expression for the stock, which excludes capital gains. In order to do that, we can follow two different routes:

- Use the definition of the value in the change in the capital stock
- Define a price index so that the deflated price excludes capital gains

5.1 The value function as the stock of capital

The most natural definition of the value of the stock of capital is simply p_tK_t. The problem with this definition is, however, that it includes capital gains. For example,

$$p_{t+1}K_{t+1} - p_tK_t = (p_{t+1}K_{t+1} - p_tK_{t+1}) + (p_tK_{t+1} - p_tK_t) \tag{5.1}$$

The first term on the right side corresponds to the capital gains term, while the second term corresponds to the value of the change in the stock. It is only the second term we are interested in for welfare calculations, as we know from eq , 3.3 that

$$V_{t+1} - V_t = p_t(K_{t+1} - K_t)$$

It follows that

$$V_t = V_0 + \sum_{\tau=0}^{t-1} p_\tau (K_{\tau+1} - K_\tau) \tag{5.2}$$

V_t can therefore be regarded as the desired measure of the capital stock. It differs from the conventional measures by the exclusion of capital gains. We can rewrite this as

$$V_{t+1} = p_tK_{t+1} - \sum_{0}^{t} (p_\tau - p_{\tau-1})K_\tau + V_0 - p_0K_0 \tag{5.3}$$

that is, the value of the capital is equal to the natural definition, $p_{t+1}K_t$ minus a revaluation term that corresponds to past capital gains!

We can now, without difficulty, translate this into a stock of capital evaluated with current value prices or with consumption in the base year as the numeraire by using the technique described in section 4.

6. Non-autonomous resource allocation mechanisms

There are many reasons why a resource allocation mechanism may be non-autonomous. The most frequent reasons seem to be exogenous changes affecting the economy. Such changes may stem from exogenous technical changes, exogenous political changes and exogenous changes is terms of trade. We will here analyse the latter, as it seems to correspond to the situation in Venezuela.

Consider an economy that has a non-renewable resource S. In each time period τ R_τ units are produced and sold on the world market. Assume further that the export revenues are used for importing consumer goods. If the price of the non-renewable resource is q_τ, export revenues is $q_\tau R_\tau$, and thus, consumption will equal $q_\tau R_\tau$, and if felicity is given by $U(q_\tau R_\tau)$, social welfare is given by

$$V_t = \sum_{\tau - t}^{\infty} \frac{U(q_\tau R_\tau)}{(1+\delta)^{\tau - t}} \tag{6.1}$$

The feasibility constraint is

$$\sum_{\tau = t}^{\infty} R_\tau = S_t, \tag{6.2}$$

which implies that $S_{t+1} - S_t = -R_t$.

The production is determined by a resource allocation mechanism

$$\mathrm{R}_\tau = \alpha(\tau, \mathrm{t}, \mathrm{S}_\mathrm{t}) \tag{6.3}$$

and consumption by

$$\mathrm{C}_\tau = \mathrm{q}_\tau \alpha(\tau, \mathrm{t}, \mathrm{S}_\mathrm{t}). \tag{6.4}$$

It is obvious that the resource allocation mechanism for consumption is not time autonomous, although the mechanism for production is. Thus, α is time autonomous, while $p_\tau \alpha$ is not.

We define the accounting price as before, that is

$$p_t = \frac{\partial V}{\partial S_t} \tag{6.5}$$

Note that p and q are in general different prices. If the resource allocation mechanism is optimal, i.e. maximizing V, we can derive a useful relation between q and p. Write down the Lagrangean

$$L = \sum_{\tau = t} \frac{U(q_\tau R_\tau)}{(1+\delta)^{\tau - t}} - p_t (\sum_{\tau = t} R_\tau - S_t) \tag{6.6}$$

and set the partial derivatives with respect to R_τ equal to zero.

$$\frac{U'(q_\tau R_\tau)}{(1+\delta)^{\tau-t}} q_\tau \leq p_t, \qquad \tau = t, t+1, \ldots \tag{6.7}$$

If the utility function is linear, then the resource will be produced in each time period only if q increases at the annual rate δ and p is simply the present value of the future world market prices. In general, when u is strictly concave, production will always be positive and this inequality will be satisfied as equality.

Change in social welfare between two adjacent time periods is given by

$$V_{t+1} - V_t = \sum_{\tau=t+1} \frac{U(q_\tau \alpha(\tau, t+1, S_{t+1}))}{(1+\delta)^{\tau-t-1}} - \sum_{\tau=t} \frac{U(q_\tau \alpha(\tau, t, S_t))}{(1+\delta)^{\tau-t}} =$$

$$= \sum_{\tau=t} \frac{1}{(1+\delta)^{\tau-t}} - \sum_{\tau=t} \frac{U(q_\tau \alpha(\tau, t, S_t))}{(1+\delta)^{\tau-t}} =$$

$$= \sum_{\tau=t} \frac{1}{(1+\delta)^{\tau-t}} \begin{bmatrix} [U(q_{\tau+1}\alpha(\tau+1, t+1, S_{t+1})) - U(q_\tau \alpha(\tau+1, t+1, S_{t+1}))] + \\ [U(q_\tau \alpha(\tau+1, t+1, S_{t+1})) - U(q_\tau \alpha(\tau+1, t+1, S_{t+1}))] \end{bmatrix}$$

By using the assumption that α is time autonomous, we can rewrite this into

$$V_{t+1} - V_t = \sum_{\tau=t} \frac{1}{(1+\delta)} \begin{bmatrix} [U(q_{\tau+1}\alpha(\tau, t, S_{t+1})) - U(q_\tau \alpha(\tau, t, S_{t+1}))] + \\ [U(q_\tau \alpha(\tau, t, S_{t+1})) - U(q_\tau \alpha(\tau, t, S_{t+1}))] \end{bmatrix}$$

Once again, by making a linear approximation, we get something that is almost operational (assuming an optimal resource allocation mechanism):

$$V_{t+1} - V_t = \sum_{\tau=t} \frac{1}{(1+\delta)^{\tau-t}} \left[U'(c_t) R_\tau \{q_{\tau+1} - q_\tau\} + U'(c_\tau) \frac{\partial c_\tau}{\partial S_t} \{S_{t+1} - S_t\} \right] =$$

$$= \sum_{\tau=t} \frac{U'(c_\tau) R_\tau}{(1+\delta)^{\tau-t}} \{q_{\tau+1} - q_\tau\} + p_t \{S_{t+1} - S_t\} =$$

$$= \sum_{\tau=t} \frac{U'(c_\tau) q_\tau}{(1+\delta)^{\tau-t}} \left(\frac{q_{\tau+1}}{q_\tau} - 1\right) R_\tau + p_t \{S_{t+1} - S_t\} =$$

$$= p_t \sum_{\tau=t} \left(\frac{q_{\tau+1}}{q_\tau} - 1\right) R_\tau + p_t \{S_{t+1} - S_t\} =$$

$$= p_t \left[\sum_{\tau=t} \left(\frac{q_{\tau+1}}{q_\tau} - 1\right) R_\tau - R_t \right]$$

or as the final result

$$V_{t+1} - V_t = p_t \left[\sum_{\tau=t} \left(\frac{q_{\tau+1}}{q_\tau} - 1 \right) R_\tau - R_t \right] \quad (6.8)$$

The interpretation of this result is obvious. The first term is the present value of future utility changes, due to changes in the world market price, while the second term is, as before, the change in present value of future utilities, due to a change in initial stock (or simply the depletion charge. However, it is possible to go further. Let the average annual relative change in the world market price be ϵ, that is

$$\varepsilon = \frac{\sum_{\tau=t}^{\infty} \left(\frac{q_{\tau+1}}{q_\tau} - 1 \right) R_\tau}{\sum_{\tau=t}^{\infty} R_\tau} \quad (6.9)$$

Then we have

$$V_{t+1} - V_t = p_t \varepsilon S_t - p_t R_t \quad (6.10)$$

Thus in this case, we should include the capital gains generated by price changes in the world market, in order to correctly assess the change in well-being. But it is still true that capital gains resulting from changes in the accounting price p should not be included.
From eq. 6.7. it follows (if there is non-zero production in each period) that

$$p_t = \frac{U'(C_t)}{(1+\delta)^t} q_t \quad (6.11)$$

In the common Hotelling case, $\epsilon=\delta$, and p_t will be constant.

Thus, under fairly general assumptions it is true that

- capital gains (and losses) due to exogenous changes in world market prices should be included in calculations of changes in well-being[4]
- in the Hotelling case, the accounting price will be constant and the term in the formula for the change in well-being that corresponds to capital gains, will be proportional to the capital gains.

[4] This was observed by Sefton and Weale 1996

7. Discoveries and exploration costs

In most applied studies, exploration costs non-renewable resources are included as an investment in the national accounting system (following the recommendation from UN Guidelines 1993). Because of that, additions to reserves are excluded in order to avoid a possible double counting. In this last section of the appendix, this procedure will be critically reviewed. The section is based on Dasgupta, Kriström and Mäler (1995).

In principle, these issues should be analysed in a stochastic framework, as the results from explorations cannot be predicted with certainty. However, for the discussion here, a much more simple approach has been chosen.[5]

Let us postulate the existence of a "discovery function",

$$D_t = N_t(E_t, Z_t, S_t) \tag{7.1}$$

where E_t is the exploration costs during period t, Z_t is the accumulated geological knowledge about the availability of the resource, and S_t is the "proven reserves", and D_t is the amount of the resource found during period t. E_t is obviously a measure of efforts of finding new reserves of the resource, and one would expect that D is not decreasing in E. Z, the accumulated knowledge or the geological human capital can be assumed to be equal to the accumulated past expenditures on exploration, that is

$$Z_{t+1} = \sum_{\tau=-\infty}^{t} E_\tau \tag{7.2}$$

Note that Z_t is a stock and that an increase in this stock may increase new discoveries in the future. Thus exploration costs in period t may increase discoveries in period t directly (represented by the first argument in the discovery function) but they may also increase future discoveries through the increase in geological human capital.

The social welfare is as before

$$V_t = \sum_{\tau=t}^{\infty} \frac{U(C_\tau)}{(1+\delta)^{\tau-t}} \tag{7.3}$$

Furthermore, let us return to the growth model in section 2

$$K_{\tau+1} - K_\tau = f(K_\tau, R_\tau) - C_\tau - E_\tau \tag{7.4}$$

where the production of the resource is used as an input in producing the combined consumption and capital good, which also can be used for the exploration for more deposits of the exhaustible resource.

We now have to formulate the resource allocation mechanism that determines the "control" variables, that is C, R, and E. In order to proceed, se will assume that the economy is optimizing, as this is the easiest but not necessarily the most realistic mechanism to deal with. We will use the discreet maximum principle for that[6]. Define the present value Hamiltonian as

[5] K. Arrow, P. Dasgupta, and K.-G. Mäler are currently working on such a stochastic model, but it is too early to draw any conclusions.

[6] See Mäler??

$$H_\tau = \frac{U(C_\tau)}{(1+\delta)^\tau} + p_\tau(f(K_\tau, R_\tau) - C_\tau - E_\tau)$$
$$+ r_\tau(N(E_\tau, Z_\tau, S_\tau) - R_\tau) + q_\tau E_\tau \tag{7.5}$$

Note that the value of the changes of the assets (or stocks) in period τ is given by the last three terms in the Hamiltonian. Let us write this explicitly:

$$V_{t+1} - V_t = p_\tau(f(K_\tau, R_\tau) - C_\tau - E_\tau)$$
$$+ r_\tau(N(E_\tau, Z_\tau, S_\tau) - R_\tau) + q_\tau E_\tau \tag{7.6}$$

p, r and q are the accounting prices on manufactured capital, the in situ resource and the geological human capital resp. One can show that in an optimizing economy, these prices can be derived from maximizing H and that they will coincide with the definition of accounting prices given in eq. 6.5. Optimal values of the control variables are found by setting the partial derivatives of H with respect to C, R, and E equal to zero:

$$p_\tau = \frac{\frac{dU}{dC_\tau}}{(1+\delta)^\tau} \tag{7.7}$$

$$p_\tau = q_\tau + r_\tau \frac{\partial N}{\partial E_\tau} \tag{7.8}$$

$$r_\tau = p_\tau \frac{\partial f}{\partial R} \tag{7.9}$$

Let us look at two different assumptions with regard to the discovery function.

1. Assume that $\partial N / \partial Z = 0, \; and \; \partial N / \partial E > 0,$, that is accumulated geological knowledge does not determine current discoveries, only current exploration does. Then, obviously, q=0, and it follows that the value of the change in assets is

$$V_{t+1} - V_t =$$
$$= p_t(f(K_t, R_t) - C_t - E_t) + r_t(N(E_t, Z_t, S_t) - R_t) \tag{7.10}$$

In this case exploration costs should not be included in wealth calculations, but discoveries should. However, in an optimizing economy, it is trivial to show that on the margin, the value of discoveries will completely be cancelled by the exploration costs. Thus in an optimizing economy, the rule could also be stated that discoveriew should not be included but exploration costs should. This seems to be the praxis in the World Bank calculating "genuine savings". However, the assumptions underlying this praxis are very strong and probably not very realistic.

2. Assume that $\partial N / \partial Z > 0, and\ \partial N / \partial E \geq 0.$ Now the value of the change in assets is

$$V_{t+1} - V_t = p_t(f(K_t, R_t) - C_t - E_t) + \\ + r_t(N(E_t, Z_t, S_t) - R_t) + q_t E_t \tag{7.11}$$

In this, more realistic, case exploration costs should not be included in the accumulation of manufactured real capitial, but as the increase in geological human capital Z. However, the accounting price for human capital is, according to eq. 7.8 strictly smaller than the accounting price for manufactured real capital as long as there is a positive effect from current explorations on current discoveries. Thus it would, in general be wrong to include current exploration costs at full market value! Furthermore, discoveries should be included in the calculation of the value of change in in situ reserves of the exhaustible resource!

8. Population changes

An analysis of population changes can be approached in two fundamentally different ways:

- population changes are endogenous, that is determined by economic factors
- population changes are exogenous, that is not affected by economic changes

We know from empirical observations that economic factors affect both fertility and mortality rates, and we should therefore analyse population change as endogenous changes. However, there are serious difficulties in formulating an acceptable normative theory for judging the social benefits from changes in the population size. These difficulties have to do with the impossibility of comparing a potential person with an actual person. See Dasgupta (1998) and Dasgupta (2001) for a discussion. We will therefore assume that population changes are exogenous and hope that this is a reasonable assumption. The analysis follows Dasgupta and Mäler (2001

Assume that population grows at rate g in each period, so that population at time t, L_t, is given by

$$L_t = L_0(1+g)^t \qquad 1.$$

A natural way of defining social welfare is as the average utility over all individuals living now and in the future, with future generations utilities discounted. Formally

$$V_t = \frac{\sum_t^\infty L_\tau U(\frac{C_\tau}{L_\tau})(1+\delta)^{-(\tau-t)}}{\sum_t^\infty L_\tau (1+\delta)^{-(\tau-t)}} \qquad 2.$$

In order for V_t to be well-defined, $\delta > g$ (otherwise the sums above will not converge). Denote per capita consumption C_t/L_t as c_t. V is a function of the flow of per capita consumption c and the size of population L.

Assume now that the production function in eq. 1. also depends on labour L and we assume that labour input is equal to the whole population (basically it means that we have full employment, for a case with unemployment, see Mäler 2001) and also on the use of the resource R. We also assume that production is characterized by constant returns to scale, that is a doubling of all inputs will double output. The production function can then be written

$$Y_t = F(K_t, L_t, R_t).$$

Because of constant returns to scale we can rewrite this as (we abstract from the time index for simplicity)

$$Y = LF(K/L, 1, R/L)$$

or with y=Y/L, k=K/L, r=R/L, and f(k, r) = F(k, 1, r)

$$y_t = f(k_t, r_t).$$

Thus, the production system can be expressed completely in per capita units.
We can now proceed exactly as in section 3 and show that the change in V between two adjacent time periods is equal to (with a new notation for the accounting prices)

$$V_{t+1}-V_t=p_t(k_{t+1}-k_t)-q_t r_t+v_t(L_{t+1}-L_t) \qquad 3.$$

Here v is the accounting price for population, that is the contribution of one additional person to well-being. In most cases, one would be entitled to disregard this term.

9. References

Dasgupta, P. (1995), "Population, Consumption, and Resources: Ethical Issues", Ecological Economics, 24, 139-52

Dasgupta, P., B. Kriström, and K.-G. Mäler, (1997), "Should Search Costs and New Discoveries be Included in Net National Product", Beijer Discussion Paper Series, No. 104

Dasgupta, P. and K.-G. Mäler (2000), "Net National Product, Wealth, and Social Well-Being", Environment and Development Economics, 5, 69-93

Dasgupta, P. and K.-G. Mäler (2001a) "Wealth as a Criterion for Sustainable Development", Beijer Discussion Paper Series, No. 139

Dasgupta, P. and K.-G. Mäler (2001b) "Intertemporal Welfare Economics in Imperfect Economies", Beijer Discussion Paper Series, No. 140

Dasgupta, P., K.-G. Mäler, and S. Barrett, (1999), "Intergenerational Equity, Social Discount Rates, and Global Warming", in Discounting and intergenerational equity, eds. P. Portney and J. Weyant, Resources for the Future, Washington DC, 51-79

Koopmans, T.C. (1960) "Stationary Ordinal Utility and Impatience", Econometrica, 28, 287-309

Mäler, K.-G. (1991) "National Accounting and Environmental Resources", Environmental and Resource Economics, 1, 1-15

Mäler, K.-G. (2001) "National Income, Welfare Measurements, and Local Prices", manuscript, The Beijer Institute

Sefton, J. and M. Weale (1996), "The Net National Product and Exhaustible Resources", Journal of Public Economics, 61, 21-48